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Optimal saving and sustainable foreign debt

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This paper develops and discusses an open-economy growth model in a modified Arrow learning-by-doing framework, in which workers learn through experience on the job, thereby increasing their productivity. Applying optimal control to maximize the discounted stream of intertemporal consumption, the model yields domestic saving rates of 18-22 percent of GDP, which are feasible targets in developing and emerging market economies. Sustainable gross foreign debt is in the range of 39-50 percent of GDP. Saving, debt, and growth policies are suggested.

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1. Introduction

High ratios of external debt to gross domestic product (GDP) in several Asian countries, exacerbated by the current COVID19 pandemic, have contributed to the initiation, propagation, and severity of financial and economic crises in the last two and a half decades, reflecting runaway fiscal deficits and excessive foreign borrowing by both public and private sectors.[1] The servicing of large debt stocks has diverted scarce resources from investment and economic growth.

[1] "Amid rising debt risks in low-income developing countries and emerging markets, the IMF and the WB have been implementing a multipronged approach (MPA) to address debt vulnerabilities. Amplification of debt risks owing to COVID19 has upped the urgency to implement the MPA and highlights the importance of debt sustainability and transparency for long-term financing for development. At the same time, it should be noted that countries have limited capacities which are further stretched by COVID19 and that implementation of the MPA by itself may not be sufficient to address debt vulnerabilities and risks from global economic shocks." [IMF 2020]

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Applying and calibrating the formal framework developed by Villanueva [2003] and Mariano and Villanueva [2005] to Philippine data, Villanueva and Mariano [2007] (henceforth VM1) explored the joint dynamics of external debt, capital accumulation, and growth.[2] The relative simplicity of the VM1 model made it convenient to analyze the links between domestic adjustment policies, foreign borrowing, and growth.[3] Using the Golden Rule criterion suggested by Phelps [1966], VM1 calculated the optimal domestic saving rate at 34 percent of GDP and the ratio of gross external debt at 44 percent of GDP, consistent with maximum steady-state real consumption per effective labor. In a comment on the VM1 paper, Lui [2007] noted that the ambitious saving rate of 34 percent may be due to the inattention to consumer preferences and attitudes toward risk. An optimal control procedure explicitly incorporating preference and risk is developed and discussed in Ramsey [1928]-Cass [1965]-Koopmans [1965], henceforth RCK. The RCK setup maximizes the discounted stream of lifetime consumption in search of optimal saving. This Golden Utility level of saving is a function of deep parameters such as, among others, the rate of time preference and the coefficient of relative risk aversion, or its reciprocal, the elasticity of intertemporal substitution.[4] How do these deep parameters affect the optimal saving rate? Is it possible that the relatively high domestic saving rate estimated by VM1 is exaggerated by the absence of explicit consideration of consumer tastes and attitudes toward risk?

To answer the above questions, this paper presents an open-economy growth (henceforth VM2) model, employing the RCK optimal control procedure and modifying the Arrow [1962] learning-by-doing framework in which workers learn through experience on the job, thereby increasing their productivity. The VM2 model finds that the Golden Rule domestic saving rate of 34 percent of GDP estimated by the VM1 model is associated with an implicitly high value of the elasticity of intertemporal substitution or an unrealistically low degree of relative risk aversion. Using a range of elasticities of intertemporal substitution estimated by Szpiro [1986], the VM2 model implies much lower Golden Utility domestic saving rates of 18-22 percent of GDP.[5] This range of optimal saving rates is dynamically

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[2] The VM1 model was developed and discussed in a paper, External Debt, Adjustment, and Growth, presented at the Conference on Fiscal Policy and Management in East Asia, hosted jointly by the National Bureau of Economic Research and the Philippine Institute for Development Studies, and held in Manila on June 23-25, 2005. That paper was subsequently published as Ch. 6 in Ito and Rose (eds.) [2007].

[3] Ito and Rose (eds.) [2007: 4] comment: "Villanueva and Mariano use a model that focuses on external debts, while providing an explicit set of economic dynamics that links borrowing to growth, capital accumulation and productivity. They apply their model to Philippine data. Their key findings are eminently reasonable; they imply that increased saving by the public and private sectors is the only way to escape future disaster. If this seems like common sense, it is; the depressing realization is that increasing savings is still a task beyond the ability of most governments."

[4] Using a Constant Relative Risk Aversion (CRRA) utility function.

[5] Corresponding to the estimated intertemporal substitution elasticity of 0.5, 0.7, and 0.9 from Table 1, Section 5.

efficient and achievable in developing and emerging market economies. The associated sustainable net foreign debt to GDP ratio is in the range of 12-23 percent of GDP.[6] Given the 27 percent average ratio of gross foreign assets to GDP during 1970-2004 for the Philippines [Lane and Milesi-Ferretti 2006], the sustainable gross foreign debt is in the range of 39-50 percent of GDP.[7]

Section 2 is a brief survey of the relevant literature. The model is introduced and explained in Section 3, followed by an analysis and discussion of its transitional and steady-state dynamics in Section 4. Section 5 derives optimal saving rates and sustainable foreign borrowing. Section 6 concludes with implications for saving, debt, and growth policies. As background, an Appendix provides a quick review of the workhorse neoclassical growth (Solow [1956]-Swan [1956] henceforth S-S) model.

2. A brief survey of the literature

The evolution of aggregate growth theory involves three levels: (1) closed *vs.* open economy; (2) fixed-exogenous saving rate vs. optimally derived endogenous saving rate; and (3) exogenous natural rate via exogenous technical change vs. endogenous natural rate via endogenous technical change or endogenous labor participation (see Box 1).[8]

BOX 1. Aggregate growth models: summary features

	C	O	EXS	ENS	EXT	ENT	EXP	ENP
Ramsey [1928]	*			*	*		*	
Solow [1956]	*		*		*		*	
Swan [1956]	*		*		*		*	
Arrow [1962]	*		*		*		*	
Cass [1965]	*			*	*		*	
Koopmans [1965]	*			*	*		*	
Conlisk [1967]	*		*			*	*	
Romer [1986]	*			*		*	*	

[6] For derivation and discussion, see text and notes to Table 1, Section 5.
[7] Net foreign debt = gross foreign debt (liabilities) minus gross foreign assets (US$). Using gross international reserves as a proximate measure of gross foreign assets, Philippine gross foreign assets at end-2019 were 23.3 percent of GDP [BSP 2019]. Taking 17.5 percent of GDP as the sustainable net foreign debt to GDP ratio (corresponding to 0.7 for the intertemporal substitution elasticity in Table 1, Section 5), the associated sustainable gross foreign debt of 40.8 percent of GDP falls within the sustainable gross range of 39~50 percent of GDP. Philippine gross foreign debt at end-2019 was 22.2 percent of GDP [BSP 2019], well below the sustainable gross range, suggesting ample room for additional foreign borrowing.
[8] Box 1 contains definitions of these three levels and related terminology.

	C	O	EXS	ENS	EXT	ENT	EXP	ENP
Lucas [1988]	*			*		*	*	
Otani &Villanueva [1989]		*	*			*	*	
Grossman & Helpman [1990]	*			*		*	*	
Otani & Villanueva [1990]		*	*			*	*	
Romer [1990]	*			*		*	*	
Grossman & Helpman [1991]	*			*		*	*	
Rivera-Batiz & Romer [1991]	*			*		*	*	
Rebelo [1991]	*		*		*		*	
Aghion & Howitt [1992]	*			*		*	*	
Knight, Loayza & Villanueva [1993]	*		*		*		*	
Villanueva [1994]	*		*			*	*	
Barro & Sala-i-Martin [1995]	*			*		*	*	
Villanueva [2003]		*	*			*	*	
Mariano & Villanueva [2005]		*	*			*	*	
Villanueva & Mariano [2007]		*	*			*	*	
Villanueva [2020]	*		*		*			*
Villanueva [2021]	*		*			*	*	

C = Closed, O = Open, EXS = Exogenous Saving, ENS = Endogenous Saving,
EXT = Exogenous Natural Rate via Exogenous Technical Change,
EXP = Exogenous Natural Rate via Exogenous Labor Participation
ENT = Endogenous Natural Rate via Endogenous Technical Change,
ENP = Endogenous Natural Rate via Endogenous Labor Participation,
L (effective labor) = APN, A = technology or productivity multiplier (index number),
P = labor participation $(0 < P \leq 1)$, N = population, $\dot{L}/L = \dot{A}/A + \dot{P}/P + \dot{N}/N$, $\dot{L}/L$ = natural rate,
$\dot{N}/N = n$ = exogenous population growth rate.

Ramsey [1928] began with a closed economy, optimally derived endogenous saving rate, exogenous technical change growth model, joined later by Cass [1965], and Koopmans [1965], henceforth RCK. Next were the closed-economy, fixed-exogenous saving rate, exogenous technical change models of Solow [1956] and Swan [1956], henceforth S-S, and of Arrow [1962]. These were followed by the closed-economy, fixed-exogenous saving rate, endogenous technical change growth models of Conlisk [1967] and Villanueva [1994], and by the open-economy, fixed-exogenous saving rate, endogenous technical change models of Otani and Villanueva [1989], Villanueva [2003], Mariano and Villanueva [2005], and Villanueva and Mariano [2007]. Villanueva [2020] is a closed-economy and fixed-exogenous saving rate growth model that generalizes the S-S model by incorporating an endogenously determined natural rate through endogenous labor participation.[9] Villanueva [2021] presents a closed-economy, fixed-exogenous saving rate, endogenous technical change growth model with two inputs: physical capital stock and combined stock of human and intellectual capital. In flow terms, these correspond to Solow's [1991] physical, human, and intellectual investments. The model finds that a higher saving rate raises both the steady-state and transitional growth rate through increases in physical capital, and human and intellectual capital (higher labor productivity). Finally, the present contribution is an open-economy growth model with optimally derived endogenous saving rates and sustainable foreign borrowing.

S-S is a closed-economy growth model where domestic saving finances aggregate investment. This reference model assumes a fully exogenous natural rate via exogenous labor-augmenting (Harrod-neutral) technical change, which determines the equilibrium or steady-state growth rate of per capita output.[10]

Conlisk [1967] was first to introduce endogenous technical change in a closed-economy neoclassical growth model. Employing a constant-returns, well-behaved neoclassical production function $Y = F(K,L) = Lf(k)$, where Y = GDP, K = capital, and L = effective[11] labor, the Conlisk [1967] model consists of the following relations:

$$\dot{K}/K = sY/K - \delta = s\,f(k)/k - \delta \text{ and } \dot{L}/L = h\,Y/L + \mu + n = hf(k) + \mu + n,$$

where $L = AN$, A = technology or productivity index, N = working population, $k = K/L$, δ = rate of depreciation, μ = rate of exogenous technical or productivity change, n = population growth rate, and a dot over a variable = time derivative, $\dot{K} = (d\,(K)/dt$. A fixed fraction, s, of Y is invested in K and another proportion, h, of Y is used to increase A.[12]

The equilibrium or steady-state growth rate of GDP is,

$$\dot{Y}/Y^* = \dot{K}/K^* = \dot{L}/L^* = h\,Y/L^* + \mu + n = hf(k^*) + \mu + n,$$

which is a positive function of the equilibrium capital-labor ratio k^*. The latter is a function of all the model's structural parameters s, h, μ, n and δ, and of the form of the intensive production function $f(k^*)$.

[9] All other growth models in Box 1 assume exogenous labor participation.

[10] A major strength of the S-S model is its rich transitional dynamics, elegant simplicity, as well as empirical relevance. For details, see Appendix.

[11] In efficiency units. Denoting K as capital and L as effective labor, if a 2020 man-hour is equivalent as an input in the production function to two man-hours in the base period, say, 2000, then the ratio K/L is the amount of capital per half-hour 2020 or per man-hour 2000.

[12] Representing expenditures, for example, on secondary and tertiary education, on-the-job training, and health [Villanueva 1994]. The proportion h is a composite parameter that translates expenditures in dollars into units of L in man-hours.

Villanueva [1994] developed and discussed a variant of the Conlisk [1967] model, combining it with a modified Arrow [1962] learning-by-doing model wherein experience on the job plays a critical role in raising labor productivity, that is,

$$\dot{A}/A = \varnothing k + \mu, \qquad 0 < \varnothing < 1$$

where $\varnothing$ is a learning coefficient. The idea is that as the *per capita* stock of capital with embodied advanced technology gets larger, the learning experience makes workers more productive.[13] Together with $\dot{N}/N = n$, the equilibrium or steady-state growth rate of GDP is,

$$\dot{Y}/Y^{*} = \dot{K}/K^{*} = \dot{L}/L^{*} = \varnothing k^{*} + \mu + n,$$

which is similar to the Conlisk growth expression above, with $hf(k^{*})$ replaced by $\varnothing k^{*}$. In efforts to explain the endogenous technical change in the context of an optimal choice of the consumption path, the literature on endogenous growth exploded during the eighties and nineties, beginning with contributions by Romer [1986], Lucas [1988], Romer [1990], Grossman and Helpman [1990], Rivera-Batiz and Romer [1991], Aghion and Howitt [1992], and Barro and Sala-i-Martin [1995]. These endogenous growth models conclude that the economy's steady-state output can grow as fast as or faster than, the capital stock, and public policies with regard to saving and investment affect long-run economic growth. In the *AK* model [Rebelo 1991], output grows at the same rate as the capital stock *K*, equal to *sA*, where s (larger than the saving rate of the S-S model by the amount of investment in human capital) is the fraction of income saved and invested, and *A* is a technological constant. There are the R&D models of Romer [1986], Grossman and Helpman [1991], Aghion and Howitt [1992], and Barro and Sala-i-Martin [1995], in which firms operating in imperfectly competitive markets undertake R&D investments that yield increasing returns, which are ultimately the source of long-run per capita output growth. Among all classes of closed-economy growth models, the equilibrium properties of fixed [Conlisk 1967 and Villanueva 1994] and optimally derived saving rate-endogenous growth models are similar.[14] The next development was to open up Conlisk's [1967] and Villanueva's [1994] growth models to foreign trade and global lending. An early attempt was made by Villanueva [2003]. The fixed-saving rate model of Villanueva [2003] is an open-economy variant of Conlisk's [1967] endogenous technical change model and Arrow's [1962] learning-by-doing model wherein on-the-job experience plays a critical role in raising labor productivity over time.

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[13] Using $L = AN$ and $k = K/L$, rewrite the above equation as $\dot{A} = \varnothing\left(K/N\right) + \mu A$.

[14] Lucas [1988] specifies effective labor $L = uhN$, where h is the skill level, u is the fraction of non-leisure time devoted to current production, and $1 - u$ to human capital accumulation. The $(1 - u)h$ variable is VM2 model's variable A in $L = AN$ in Section 3. This variable is T in Villanueva [1994], defined as technical change or labor productivity multiplier.

In Villanueva [2003], the aggregate capital stock is the accumulated sum of domestic saving and net external borrowing (current account deficit). At any moment, the difference between the expected marginal product of capital, net depreciation, and the marginal cost of funds in the international lending market determines the proportionate rate of change in the external debt-capital ratio.[15] When the expected net marginal product of capital matches the marginal cost of funds at the equilibrium capital-labor ratio, the proportionate increase in net external debt (net external borrowing) is fixed by the economy's equilibrium output growth, and the external debt/output ratio stabilizes at a constant level. Although constant in long-run equilibrium, the external debt ratio shifts with changes in the economy's propensity to save out of national disposable income, the marginal cost of funds in the world lending market, depreciation rate, growth rates of the working population, and exogenous technical change, and the parameters of the risk premium, production, and technical change functions.

A major shortcoming of the Villanueva [2003] model is its inability to pin down the saving rate and equilibrium external debt to GDP ratio that is consistent with maximum consumer welfare. The Villanueva-Mariano [2007] (VM1) model corrected this shortcoming by employing Phelp's [1966] *Golden Rule* maximization criterion. On the balanced growth path, if consumption per unit of effective labor (or any monotonically increasing function of it) is taken as a measure of the social welfare of society, the domestic saving rate that maximizes the consumption per unit of effective labor is chosen. Consistent with this, the optimal outcome is a sustainable ratio of net external debt to total output. Using parameters for the Philippines to calibrate the model, the VM1 growth model's steady-state solution is characterized by a constant capital-effective labor ratio, an optimal domestic saving rate, and a unique external debt-capital ratio.[16] The latter ratio interacts with long-run growth and domestic adjustment and is determined jointly with other macroeconomic variables, including a country's set of structural parameters.

A weakness of the VM1 growth model is its lack of micro-foundation, a criticism leveled by Lui [2007]. The RCK setup is suitable to determine unique values of the optimal saving rate and foreign debt to GDP ratio. As Box 1 shows, all the micro-founded optimally derived saving–rate growth models are closed-economy models. The VM2 model extends a micro-founded growth model such as RCK to an open economy with access to foreign trade and global lending. Most importantly, the VM2 model incorporates a modified Arrow learning-by-doing feature. Imports of capital goods with embodied advanced technology allow learning-by-doing to raise labor productivity and, thus, long-run growth (see Figure 2, Section 4, and related discussion).

[15] The marginal cost of funds is the risk-free interest rate plus a risk premium. For details, see Equation (8) and footnote 31, Section 3.

[16] For research on the sustainability of external debt using various statistical procedures, see Manasse and Schimmelpfenning [2003], Reinhart et al. [2003], Kraay and Nehru [2004], Patillo et al. [2004], and Manasse and Roubini [2005]. For an excellent survey, see Kraay and Nehru [2004].

3. The VM2 model

Before presenting the VM2 model, the following background is a useful summary of the closed-economy RCK model and the original Arrow learning-by-doing framework. In the RCK model with a *CRRA* utility function and fully exogenous labor-augmenting technical change and population growth, the equations for the optimal growth of consumption, c, and capital intensity, k, consistent with maximum discounted stream of intertemporal consumption are:

$$\dot{c}/c = 1/\theta\left[f'(k) - \delta - \rho - \theta(n+\mu)\right];\ \dot{k} = f(k) - c - (\delta + n + \mu)k,$$

wherein $\mathrm{c}=C/L, \mathrm{k}=K/L, \mathrm{f(k)}=F(K,L)/L, F(.)$ = unit-homogeneous production function, C = is consumption, K = physical capital, L = effective labor = AN,[17] A = labor productivity or technology index, N = population, ρ = time preference or discount rate, θ = degree of relative risk aversion, δ = capital's depreciation rate, $\dot{A}/A = \mu$, and $\dot{N}/N = n$. The asymptotic (equilibrium) values c^* and $k^*$are the roots of the above equations equated to zero:

$$f'(k^*) - \delta - \rho - \theta(n+\mu) = 0;\ f(k^*) - c^* - (\delta + n + \mu)k^* = 0$$

Given a specific form of $f(k^*)$, the first equation solves for k^*. Plugging k^* in the second equation solves for c^*. Given k^* and a well-behaved, *constant-returns* neoclassical production function, the equilibrium growth rate of per capita output, $\dot{Y}/Y^* - n$ is fixed entirely by the rate of exogenous Harrod-neutral technical change, μ, and is independent of consumer preferences and technology.[18] Nearly six decades ago, Arrow [1962] proposed a learning-by-doing growth model,

$$\dot{A}/A = \varnothing\left(\dot{K}/K\right) + \mu,\quad 0 < \varnothing < 1, \tag{1}$$

in which $\varnothing$ is a learning coefficient. Equation (1) states that the proportionate growth in labor productivity is the sum of the learning coefficient $\varnothing$ multiplied by the proportionate growth in the capital stock plus a constant rate of exogenous technical or productivity change μ. The faster the growth of the capital stock, the more intensive the learning experience on the job, and the higher the growth in labor productivity is.

[17] Generally, the L definition should be $L = APN$, where P is the labor participation rate, which measures the percentage of the population in the labor force ($0 < P \leq 1$). The working population is PN. When $P = 1$, $L = AN$. Whatever P is, it is usually assumed in current literature as an exogenous constant, whose rate of change is zero. For an endogenous and variable P, see Villanueva [2020].

[18] $\dot{Y}/Y^* = \dot{K}/K^* = \dot{L}/L^* = \dot{A}/A + \dot{N}/N = \mu + n$

Given Equation (1), definition $L = AN$, and assumptions, $\dot{N}/N = n$, $\dot{A}/A = \mu$, a constant steady-state capital intensity $k^* \left(= K/L\right)^*$ implies the following equilibrium growth rate of output:

$$\dot{K}/K^* = \dot{L}/L^* = \dot{A}/A^* + n = \dot{Y}/Y^* = g^{y*} = (\mu + n) / (1 - \varnothing).^{19} \quad (2)$$

Although a multiple of the S-S equilibrium output growth rate (μ + n), equilibrium output growth, $\dot{Y}/Y^* = g^{y*}$ remains equal to a constant involving only three parameters μ, $\varnothing$, and n. That is, g^{y*} is independent of the preference and risk parameters ρ and θ, and the form of the intensive production $f(k^*)$.[20] Besides, the Arrow model has the property that, $(d[g^{Y*} - n] / dn = [\varnothing/((1-\varnothing))] > 0$, i.e., an increase in the population growth rate n raises the long-run growth rate of per capita output, $g^{y*} - n$.[21] This prediction is counterintuitive and rejected by empirical evidence.[22]

Turning now to the VM2 growth model, assume the following institutional arrangements of an open and perfectly competitive economy with rational agents. One good is produced that is partly consumed and the remainder exported, using an aggregate production function with inputs of labor, and imported capital goods with embodied advanced technology. Enterprises rent capital from households and hire workers to produce output in each period. Households own the physical capital stock and receive income from working, renting capital, and managing the enterprises. To finance imports of capital goods, households use export earnings and borrow from abroad.[23]

The VM2 model's key innovation is a modification of Arrow's learning-by-doing equation as follows:

$$\dot{A} = \varnothing\left(K/N\right) + \mu A \qquad (3)$$

The difference between Arrow Equation (1) and VM2 Equation (3) is the endogenous component [the first term on the right-hand side (RHS)]. Both equations

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[19] The unit-homogeneous production function $Y = F(K, L)$ is subject to constant returns to K and L jointly, implying balanced growth in Y, K, and L.

[20] Refer back to the basic RCK model, second paragraph of the current section.

[21] Subtracting n from both sides of Equation (2) yields $(g^{y*} - n) = \left[\mu/((1-\varnothing))\right] + \left[\varnothing/((1-\varnothing))\right]n$.

[22] See Conlisk [1967], Otani and Villanueva [1990], Knight et al. [1993] and Villanueva [1994].

[23] The numeraire is the foreign price of the imported capital good. Thus, if Pd is the price of the domestic consumer good, Pf is the price of the foreign good, and e is the exchange rate expressed as quantity of local currency units per unit of foreign currency, Pd/ePf is multiplied by residents' saving to obtain domestic saving (in constant dollars). Foreign borrowing denominated in foreign currency is deflated by Pf to get the real value. Similarly, the marginal real cost of foreign borrowing is the sum of the world interest rate and risk premium in foreign currency less the rate of change in Pf. Since model simplicity and long-run growth are our primary concerns, the VM2 model abstracts from the effects of movements of these variables by arbitrarily assigning unitary values to these price and exchange rate indices without loss of generality. Incorporation of these variables in the VM2 model is straightforward and is done in Otani and Villanueva [1989]. Imports of capital goods are financed by the European Ex-Im Bank and American Ex-Im Bank, global commercial banks, and international and regional development banks.

have an exogenous component in the second term on the RHS involving the labor-augmenting technical change or productivity parameter μ. In the first term's endogenous component, instead of assuming that learning-by-doing is proportional to the growth rate of the aggregate capital stock, $\dot{K}/K$ as in Arrow Equation (1), the VM2 model assumes that the endogenous component is proportional to the level of the aggregate capital stock per capita, K/N. This is particularly relevant to developing countries whose K is imported and embodies the most advanced technology produced by the advanced industrial countries. A large stock of K/N enables workers in developing countries to engage in learning-by-doing on a significant scale.[24] In these countries, starting from a low level of K/N, even a very high growth rate of the capital stock would barely make a dent on learning-by-doing to have significant effects on labor productivity and growth rate of aggregate per capita output. The R&D sector in developing countries is virtually nonexistent. Owing to its large real resource (including financial) costs, R&D development is left for the rich industrial countries to pursue. The resource-poor developing countries have a cheaper alternative: Import capital goods with embodied advanced technology, learn from using these goods in the production process, and thereby raise labor productivity and long-run growth. The presence of learning through experience on the job has three major consequences: First, equilibrium or steady-state growth becomes endogenous and is influenced by preferences, technology, and government policies. Second, the speed of adjustment to growth equilibrium is faster, and enhanced learning-by-doing further reduces adjustment time.[25] Third, capital's income share is higher than the optimal saving rate to compensate capital for the additional GDP growth generated by endogenous growth and learning by doing.[26]

The VM2 model's aggregate production function adopts the S-S model's assumption of constant returns to K and L jointly, and diminishing returns to K and L separately, and in the context of perfectly competitive markets with full wage-price flexibility.[27] Like the S-S, Conlisk [1967], and Villanueva [2003] models, the VM2 model employs a well-behaved unit-homogeneous neoclassical production function $Y = F(K, L) = Lf(k)$, where Y, K, L, and k have been defined earlier, subject to the Inada [1963] conditions: $\lim \partial F / \partial K = \infty$ as $K \rightarrow 0$; $\lim \partial F / \partial K = 0$ as $K \rightarrow \infty$; $f(0) \geq 0$; $f'(k) > 0$ and $f'(k) < 0$ for all $k > 0$.

The Cobb-Douglas production function, used to calibrate the VM2 model, satisfies these conditions.

[24] The empirical results from Villanueva [1994] suggest that the learning coefficient $\varnothing$ is positively influenced by the openness of the economy (sum of exports and imports) and expenditures on education and health, and negatively by fiscal deficits, all three variables expressed in percent of GDP.

[25] See Villanueva [1994] for analytical approach and simulation that explain reduced adjustment time towards the steady state.

[26] See Section 5 for proof.

[27] Unlike the models of Romer [1986], Grossman and Helpman [1991], Aghion and Howitt [1992], and Barro and Sala-i-Martin [1995] that are subject to increasing returns to capital operating in imperfect markets.

From the definition $L = AN$, noting that $\dot{N} = nN$,

$$\dot{L}/L = g^L = \dot{A}/A + n. \qquad (4)$$

Substituting Equation (3) into Equation (4),

$$\dot{L}/L = g^L = \varnothing k + \mu + n. \qquad (5)$$

In the steady-state, $\mathrm{k} = \mathrm{k}^*$ (a constant), and from the *constant-returns* assumption,

$$\dot{K}/K^* = \dot{L}/L^* = \dot{Y}/Y^* = g^{Y*} = \varnothing k^* + \mu + n, \qquad (6)$$

i.e., the steady-state growth rate of per capita output $= g^{Y*} - n = \varnothing k^* + \mu$. Comparing Equations (2) and (6), the key difference is the presence of equilibrium capital intensity k^* in the expression for the equilibrium growth rate of per capita output in the VM2 model, and the absence of k^* in the growth equation of the Arrow model. The VM2 model solves for optimal values of k^*, c^*, and d^* [Equations (24)-(26)] in an open-economy RCK optimal control setup using a CRRA utility function, wherein k^*, c^*, d^* and $g^{Y*} - n$ are functions of consumer tastes, technology, and policy parameters. Besides this key property, the VM2 model implies a more empirically plausible prediction (opposite to Arrow's) that an increase in the population growth rate depresses the long-run growth rate of per capita output, $d(g^{Y*} - n)/dn = \varnothing(\partial k^*)/\partial n < 0$[28] because $\partial k^*/\partial n < 0$ as shown in Figure 4, Section 4.

The budget constraint of a representative household is:

$$C + \dot{K} + \delta K = rK + wL + \Pi + \dot{D} - iD, \qquad (7)$$

C = consumption, K = physical capital, D = net foreign debt (foreign liabilities less foreign assets)[29], r = capital's rental rate, w = real wage rate, Π = total profit in managing and owning the enterprises, and i = real effective interest rate.

Equation (7) is the budget constraint that total uses of funds equal total sources of funds. Total uses are consumption and gross investment. Total sources are GDP and foreign borrowing net of interest payments. Restating Equation (7), $rK + wL + \Pi = C + \dot{K} + \delta K - (\dot{D} - iD)$, where $(\dot{D} - iD) = M - X =$ balance of payments identity, M = imports (of capital goods), X = exports, $M - X$ = current account balance, and $\dot{D}$ = change in net foreign liabilities. Gross capital formation $\dot{K} + \delta K = rK + wL + \Pi - C + \dot{D} - iD$, i.e., capital accumulation is financed by an unconsumed output that is exported, and by foreign borrowing net of interest payments on debt.

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[28] Subtract n from both sides of Equation (6) and take its derivative with respect to n. For empirical evidence that $d\left(g^{Y*} - n\right)/dn < 0$, see Conlisk [1967], Otani and Villanueva [1990], Knight et al. [1993] and Villanueva [1994].

[29] If foreign assets exceed foreign liabilities, D is negative.

The real interest i is the global real interest rate i^f plus a risk premium equal to a proportion λ of the foreign debt stock $d = D/L$.[30]

$$i = i^f + \lambda d \qquad 0 < \lambda < 1 \qquad (8)$$

The second term on the RHS of Equation (8) is the risk premium representing the combined effects of risk factors and financial markups that foreign lenders take into account prior to extending loans. When i^f is held constant, a higher debt stock d, by raising the probability of default, increases the risk premium and thus i.

Dividing both sides of Equation (7) by L,

$$c + \dot{k} + (\delta + g^L)k - \dot{d} - g^L d = rk + w + \pi - id, \quad (9)$$

wherein,

$$\dot{d} = \beta(r - \delta - i)d \qquad 0 < \beta \leq \infty, \qquad (10)$$

and, as before, lower case letters are expressed as ratios to effective labor L, and g^L is given by Equation (4).

Equation (10) postulates that foreign borrowing is undertaken in response to a positive differential between the expected capital's net marginal product and the effective real interest rate, with the coefficient β measuring the response speed (an aggregate lending offer function from global lenders).

Inserting Equation (10) into Equation (9):

$$c + \dot{k} + (\delta + g^L)k - \beta(r - \delta - i)d - g^L d = rk + w + \pi - id \quad (11)$$

The representative household maximizes a discounted stream of lifetime consumption C, subject to constraints Equations (9) and (10), in which instantaneous utility is of the CRRA form (for brevity, time t is suppressed for all variables):

$$N(0)^{(1-\theta)} \int_0^\infty \frac{\left(\frac{C}{L}\right)^{(1-\theta)}}{(1-\theta)} A^{(1-\theta)} e^{-\rho * t} \mathrm{dt} \qquad (12)$$

For the integral to converge, the standard restriction $\rho^* = \rho - (1-\theta)\mathrm{n} > 0$ is imposed. In maximizing Equation (12) subject to Equations (9) and (10), each household takes as parametrically given the time paths *of* r, w, π, i and A. When making decisions about consumption, capital accumulation, and net foreign borrowing, the representative household is small enough to affect r, w, π, i and A.

[30] The risk-free interest rate is i^f . The risk premium is i - i^f. The LIBOR (to be ended in 2021 and replaced by new benchmark rates), US Prime Rate, US Federal Funds Rate, or US Treasury, deflated by changes in an appropriate price index in the United Kingdom or United States of America, typically represents the risk-free interest rate. The risk premium is country-specific and a positive function of a country's external debt burden and other exogenous factors capturing market perceptions of country risk.

The household's Hamiltonian is

$$H = e^{-p*t}\left[\frac{c^{1-\theta}}{(1-\theta)}\right]A^{(1-\theta)} + \varphi_1[rk + w + \pi - c - id - (\delta + g^L)k$$

$$+\beta(r-\delta-i)d + g^L d] - \varphi_2[\beta(r-\delta-i)d] \tag{13}$$

After substituting Equations (3), (4), and $\rho^* = \rho - (1-\theta)$n, the first-order conditions are,

$$\dot{c}/c = (1/\theta)\left[\left(r - \delta - \rho - \theta n - \theta\left(\varnothing k + \mu\right)\right)\right] \tag{14}$$

$$\dot{k} = rk + w + \pi - c - id + \beta(r-\delta-i)d + g^L d - (\delta + g^L)k \tag{15}$$

$$\dot{d} = \beta(r - \dot{\delta} - i)d \tag{16}$$

The economy-wide resource constraint is:

$$C + \dot{K} + \delta K = F(K, L) - iD + \dot{D} \tag{17}$$

Dividing both sides by L,

$$c + \dot{k} + (\delta + g^L)k = f(k) - id + \dot{d} + g^L d. \tag{18}$$

In competitive equilibrium, $r = f'(k)$, and $w = f(k) - kf'(k)$, implying $\pi = 0$. Substituting these expressions for r, w, and π into Equations (14)—(16) and (18), the optimal time paths for c, k, and d are:

$$\dot{c}/c = (1/\theta)[(f'(k) - \delta - \rho - \theta n - \theta(\varnothing k + \mu) \tag{19}$$

$$k = f(k) - c - id + \beta\left[(f'(k) - \delta - i)\right]d + g^L d - (\delta + g^L)k \tag{20}$$

$$\dot{d} = \beta(f'(k) - \delta - i)d, \tag{21}$$

wherein $g^L = \varnothing k + \mu + n$ and $i = i^f + \lambda d$. The transversality conditions are:

$$\lim_{t\to\infty} e^{-\rho*t}\varphi_1 k = 0 \tag{22}$$

$$\lim_{t\to\infty} e^{-\rho*t}\varphi_2 d = 0 \text{ [31]} \tag{23}$$

[31] The time paths for φ_1 and φ_2 are given by $\dot{\varphi}_1 = \rho^*\varphi_1 - \partial H/\partial k$, $\dot{\varphi}_2 = \rho^*\varphi_2 - \partial H/\partial d$, wherein $\partial H/\partial k$ and $\partial H/\partial d$ are functions of k and d. As a standard condition, the no-Ponzi game is imposed, i.e., non-negative present values of the household holdings k and d.

In the absence of learning by doing $\varnothing = 0$ and net foreign debt ($d = 0$), the above model reduces to the closed-economy RCK model that allows for population growth n and fully exogenous technical progress μ, with the key property that the equilibrium growth rate of per capita output is fixed entirely by μ and is independent of preferences, technology[32], and policy.[33]

The system (19-21) represents the reduced model in c, k, d, and time t. The asymptotic (equilibrium) values c^*, k^*, and d^* are the roots of Equations (19)-(21) equated to zero:

$$f'(k^*) - \delta = \rho + \theta\mu + \theta n + \theta\varnothing k^* \quad (24)$$

$$f'(k^*) - \delta = i = i^f + \lambda d^* \quad (25)$$

$$f'(k^*) - c^* + (d^* - k^*)(\varnothing k^* + \mu + n) - d^*(i^f + \lambda d^*) - \delta k^* = 0 \quad (26)$$

The model's phase diagrams, shown as Figures 1-4, are based on calibrated values specified in Equations (24)-(26), using the following parameters: $\alpha = 0.3$, $\delta = 0.04$, $\mu = 0.005$, $\theta = 1.4$[34], $\varnothing = 0.01$, $\rho = 0.03$, $\beta = 1$, $\lambda = 0.25$, $i^f = 0.05$, and $n = 0.02$.

The parameter α = exponent in the Cobb-Douglas production function $f(k) = k^\alpha$. The other parameters are: δ = capital's depreciation rate, μ = rate of exogenous labor-augmenting technical change, θ = coefficient of relative risk aversion, $\varnothing$ = learning coefficient, ρ = rate of time preference, β = speed of adjustment of foreign borrowing to the gap between capital's net marginal product and the effective cost of foreign borrowing, λ = linear response of the borrowing spread to the debt stock i^f = world interest rate, and n = growth rate working population. The solutions are $k_0^* = 2.7$ and $s^* = 0.20$.[35] For comparison, the following solutions for the VM1 model are $k^{**} = 6.8$ and $s^{*'} = 0.34$. Note that the lower steady-state value of capital intensity is consistent with the prediction that the *Golden Utility* capital intensity level is lower than the *Golden Rule* level (see Figure 1). While the latter maximizes consumption per effective labor at c^{**}, the former maximizes consumer utility at c_0^* and is dynamically efficient.

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32 Form of the production function $f(k^*)$.

33 The optimal paths for c and k are: $\dot{c}/c = 1/\theta\left[f'(k) - \delta - \rho - \theta(n+\mu)\right]$ and $\dot{k} = f(k) - c - (\delta + n + \mu)k$. These are identical expressions for the optimal growth of consumption and capital intensity delineated at the beginning of the current section.

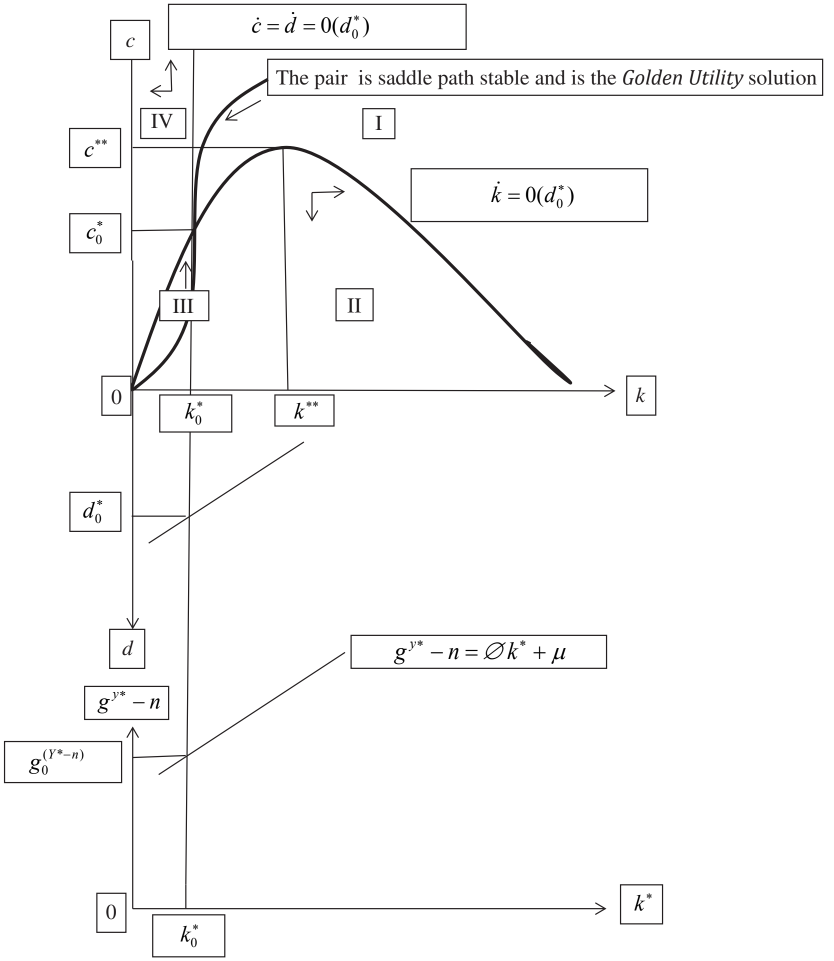
34 Corresponding to the estimate of 0.7 for the intertemporal substitution elasticity [Szpiro 1986] shown in Table 1, Section 5.

35 Microsoft's Solver tool is used to solve the first-order conditions. The program searches the optimal k^*, c^*, and d^* such that Equations (24)-(26), are met. The Solver tool uses the Generalized Reduced Gradient nonlinear optimization code developed by Leon Lasdon and Allan Waren.

Figure 1 is the phase diagram of the VM2 model. The upper panel plots the $\dot{k} = 0$ curve and the $\dot{d} = \dot{c} = 0$ curve in k, c space. Equations (24)-(25) imply:

$$i^f + \lambda d^* = \rho + \theta\mu + \theta n + \theta \varnothing k^*. \qquad (27)$$

FIGURE 1. Long-run equilibrium



This is the $\dot{d} = \dot{c} = 0$ curve. For a given d, say d_0^*, it is a vertical line in the k, c space in the upper panel of Figure 1. The bell-shaped curve represents the $\dot{k} = 0$ relationship, which is drawn for a given level of d, say d_0^* [Equation (26)]. This curve's slope has the property that $\partial c / \partial k \geq 0$ for $k \leq k^{**}$, and $\partial c / \partial k < 0$ for $k > k^{**}$. The middle panel plots the $\dot{d} = 0$ line in the k, d space with a negative slope. That is to say, when d rises above d_0^* and pushes up the real interest rate above capital's net marginal product at k_0^*, $\dot{d} < 0$ and d tends to fall. For d to remain constant, capital's net marginal product must increase, requiring k *to* decrease below k_0^*. Thus, the $\dot{d} < 0$ line slopes downward. The lower panel plots the growth rate of per capita output $g^{Y^*} - n = \varnothing k^* + \mu$. This curve slopes upward.

Figure 1 shows the equilibrium values k_0^*, c_0^* in the upper panel, d_0^* in the middle panel, and $g_0^{(Y^*-n)}$ in the lower panel. The pair (k_0^*, c_0^*) is saddle path stable and is the *Golden Utility* solution.[36] While c_0^* is below the maximum *Golden Rule* [Phelp 1966] level at c^{**}, c_0^* maximizes intertemporal utility and is thus dynamically efficient. The equilibrium capital intensity k_0^* is a function of all the parameters of the VM2 model, including the parameters of the utility function, namely the discount rate and the coefficient of relative risk aversion or its reciprocal, the elasticity of intertemporal substitution, and other parameters, including the learning coefficient, the global real interest rate, and the parameters and form of the production function. Since the equilibrium growth rate of per capita output $g^{Y^*} - n$ equals $\varnothing k^* + \mu$, any public policy that enhances the equilibrium capital intensity k^* and the learning coefficient $\varnothing$ raises the long-run growth rate of per capita output. This stands in sharp contrast to the open-economy RCK model with *no* learning by doing ($\varnothing = 0$), wherein $g^{Y^*} - n = \mu$.[37]

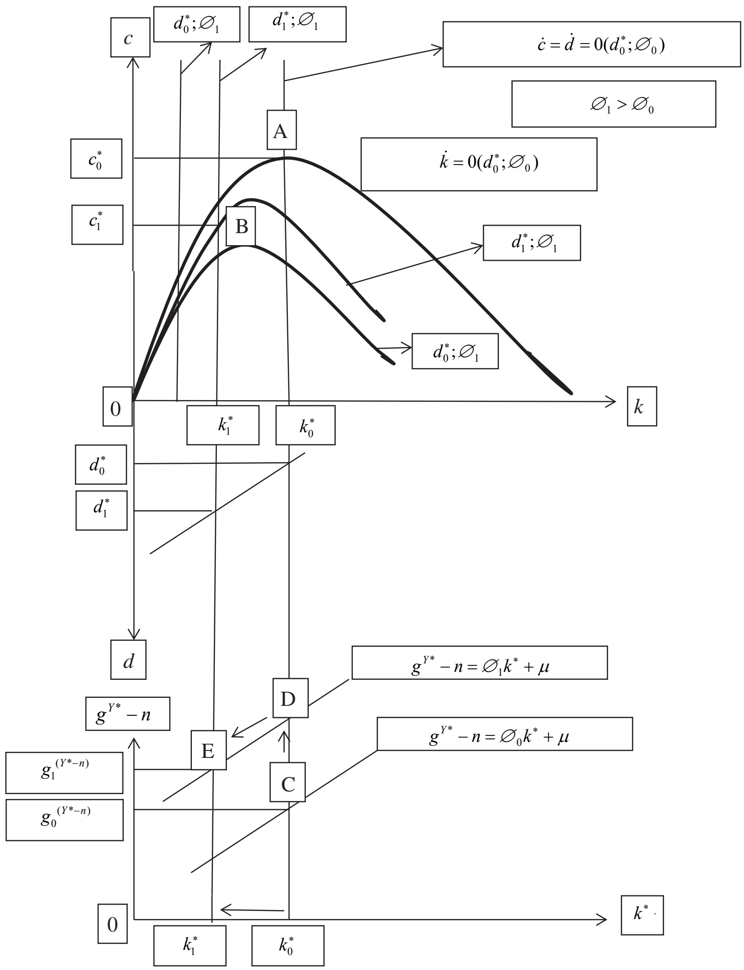
4. Comparative dynamics

Figure 2 illustrates the growth effect of an increase in learning by doing. In the upper and middle panels, the intersection at $A(k_0^*, c_0^*)$ shows the initial equilibrium point corresponding to a given level of the learning coefficient $\varnothing_0$ and equilibrium debt stock d_0^*. In the lower panel, the equilibrium growth rate of per capita output is and $g_0^{(Y^*-n)}$ given by the capital intensity level k_0^*.

[36] The transversality conditions rule out quadrants II and IV in Figure 1.

[37] In this special case, the $g^{Y^*} - n$ line in the lower panel of Figure 1 turns horizontal with intercept equal to μ.

FIGURE 2. Growth effect of an increase in learning by doing



Now, assume that public policy subsidizes on-the-job training at enterprises, resulting in an increase in the learning coefficient from $\varnothing_0 \rightarrow \varnothing$. From Equations (3), (5), and (19), labor productivity and g^L go up, slowing consumption growth, i.e., $\dot{c}/c < 0$. For $\dot{c}/c = 0$, k^* has to decrease, so that capital's marginal product rises. In the upper panel of Figure 2, the $\dot{c} = \dot{d} = 0(d_0^*;\varnothing_0)$ curve shifts leftward to $\dot{c} = \dot{d} = 0(d_0^*;\varnothing_1)$. In the middle panel, when k^* declines, capital's marginal product goes up, encouraging larger amounts of foreign borrowing, that is, $d > 0$. For $d = 0$, the real marginal cost of borrowing must increase, and so must d^*, as shown in the middle panel. Going back to the upper panel, as d^* goes up from d_0^* to d_1^*, $\dot{c} = \dot{d} = 0(d_0^*;\varnothing_1)$ the shifts rightward to $\dot{c} = \dot{d} = 0(d_1^*;\varnothing_1)$.

What happens to the $\dot{k} = 0$ curve? In the upper panel, when the learning coefficient increases, the higher effective labor growth implies $\dot{k} < 0$; for $\dot{k} = 0$, consumption has to fall; and the $\dot{k} = 0$ curve shifts downward to $\dot{k} = 0(d^*{}_0; \varnothing_1)$. However, increased foreign borrowing leads to higher debt stock at d_1^*, finances higher levels of consumption and capital intensity, so that the $\dot{k} = 0$ curve shifts upward to $\dot{k} = 0(d^*{}_0; \varnothing_1)$. The new equilibrium shifts to point B $(k^*{}_1, c^*{}_1)$, with both equilibrium consumption per effective worker and equilibrium capital intensity lower than at point A.

In the lower panel, the increase in the learning coefficient from $\varnothing_0$ to $\varnothing_1$ shifts the per capita output growth curve upward from $g_0^{(Y^*-n)} - n = \varnothing_0 k_0^* + \mu \rightarrow g_1^{(Y^*-n)} = \varnothing_1 k^* + \mu$. Equilibrium shifts from $C(k_0^*, g_0^{(Y^*-n)})$ to $E(k_1^*, g_1^{(Y^*-n)})$, characterized by lower capital intensity and higher per-capita output growth.

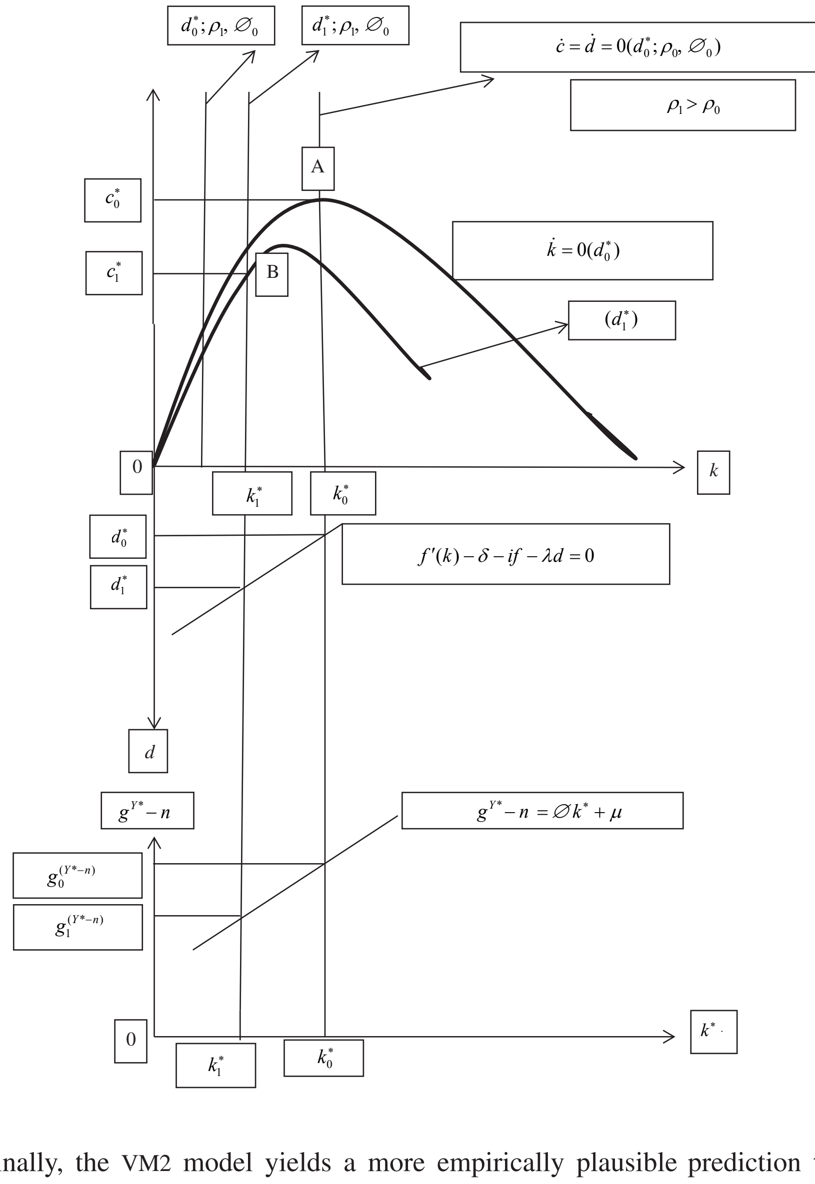
At the old equilibrium capital intensity k_0^*, the transition jumps from C to D, the latter characterized by per capita output growth temporarily higher than the next equilibrium rate $g_1^{(Y^*-n)}$ at E. As capital intensity goes down from k_0^* to k_1^*, per capita output growth declines toward $g_1^{(Y^*-n)} > g_0^{(Y^*-n)}$.[38] Thus, an increase in the learning coefficient leads to a short-run overshooting of the new and higher long-run per capita output growth rate.

Figure 3 illustrates the effects of higher discounting of future consumption or higher degree of relative risk aversion (lower elasticity of intertemporal substitution). The initial equilibrium is at point A $(k^*{}_0, c^*{}_0)$, shown in the upper panel, corresponding to an initial value for ρ_0 and a given level of the debt stock $d^*{}_0$, shown in the middle panel. The equilibrium growth rate of per capita GDP at $g_0^{(Y^*-n)}$, corresponding to capital intensity $k^*{}_0$, is shown in the lower panel. The increase in ρ shifts the $\dot{c} = \dot{d} = 0$ curve to the left, lowering both c^* and k^*. A lower k^* implies a higher d^*, as shown in the middle panel. When d^* rises from $d^*{}_0$ to $d^*{}_1$, the $\dot{c} = \dot{d} = 0$ curve shifts to the right, while the $\dot{k} = 0$ curve shifts downward. The new equilibrium intersection is at point B $(k^*{}_1, c^*{}_1)$. Both the equilibrium consumption per effective worker and equilibrium capital intensity are lower. In the lower panel, as k^* falls from $k^*{}_0$ to $k^*{}_1$, learning by doing drops and so does the equilibrium rate of per capita output, from $g_0^{(Y^*-n)}$ to $g_1^{(Y^*-n)}$.[39] The economic explanation is that higher discounting of future consumption means higher degree of relative risk aversion due, for instance, to increased uncertainty and macroeconomic and financial instability. The domestic saving rate falls, lowering the equilibrium capital intensity, lower imports of advanced capital goods and the associated decline in learning by doing. The equilibrium growth rate of per capita output goes down. The opposite economic adjustment follows from a reduction in uncertainty aided, for example, by a strong set of policies aimed at strengthened macroeconomic and financial stability.

[38] Adjustment is traced by the segment DE.

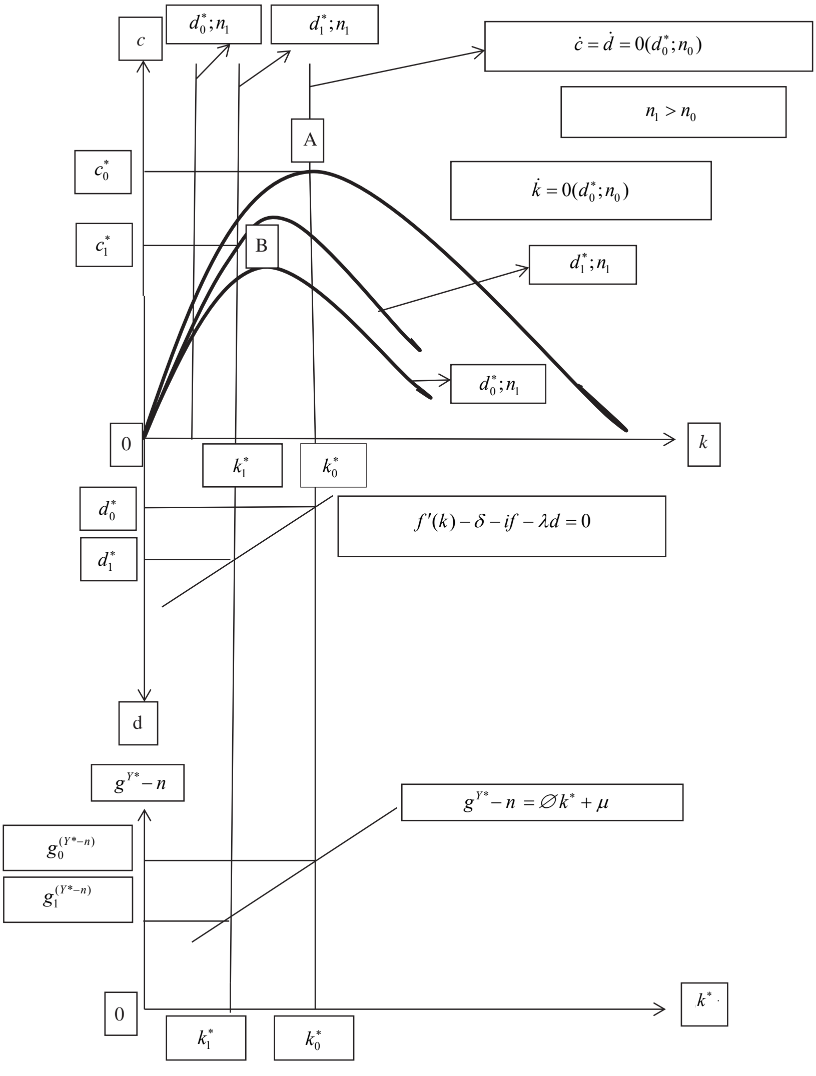
[39] Reflecting a downward movement along the $(g^{Y^*} - n)$ curve.

FIGURE 3. Growth effect of higher discounting or lower intertemporal substitution elasticity



Finally, the VM2 model yields a more empirically plausible prediction that an increase in population growth lowers the steady-state growth rate of per capita output, as illustrated in Figure 4, a result that is particularly relevant to developing countries.

FIGURE 4. Growth effect of an increase in population growth



In the upper panel, an increase in n from n_0 to n_1 shifts the $\dot{c} = \dot{d} = 0$ curve to the left and the $\dot{k} = 0$ curve downwards. As k falls, d rises; a higher d^* shifts the $\dot{c} = \dot{d} = 0$ curve to the right and the $\dot{k} = 0$ upwards. The new equilibrium settles at point B, characterized by a lower c^* and k^* and, as shown in the middle panel, a higher d^*. In the lower panel, the decline in the equilibrium stock of capital per effective worker cuts learning by doing and leads to lower equilibrium growth rates of productivity and per capita output.[40]

[40] The growth effect of a higher depreciation rate of capital is similar.

5. Optimal saving and sustainable foreign debt

Assuming a Cobb-Douglas production function $f(k) = k^{\alpha}$ and from Equations (19)-(20), the endogenously derived optimal saving rate is given by:

$$s^* = \{[\{i^* - g^{Y*})(d^* / k^*) + \delta + g^{Y*}] / (\rho + \delta + \theta g^{Y*})\}\alpha, \qquad (28)$$

in which $g^{Y*} = \varnothing k^* + \mu + n$ and $i^* = i^f + \lambda d^*$. If $d^* = 0$ (closed economy), $\rho > 0$, $\theta > 1$, and $\varnothing = 0$ (all technical change is exogenous), then

$$s^{**} = \{(\mu + n + \delta) / [(\theta(\mu + n) + \delta + \rho)]\}\alpha. \qquad (29)$$

Evaluated in the steady state, the fraction in braces of Equation (28) is in range 0.60-0.74, depending on the elasticity of intertemporal substitution, and the fraction in braces of Equation (29) falls in range 0.55-0.67, so that s^* is in range 0.18-0.22 and s^{**} in range 0.17-0.20 (see Table 1). The optimal saving rate in the presence of partly endogenous learning by doing is larger than the optimal saving rate in a world of entirely exogenous learning by doing. The intuitive reason is this: learning-by-doing uses some portion (about a percentage point of GDP) of society's resources (endogenous variable), so a larger proportion of society's income must be saved for this purpose.

As noted in Section 1, the sustainable net foreign debt to GDP ratio is in range of 12-23 percent of GDP. Lane and Milesi-Ferretti's [2006] estimate for the Philippine average ratio of gross foreign assets to GDP during 1970-2004 is 27 percent, implying that the sustainable gross foreign debt is in range of 39-50 percent of GDP.[41]

TABLE 1. Sensitivity of optimal results[a]

	Elasticity of intertemporal substitution[b]		
	0.5	0.7	0.9
s^*	0.1805	0.1963	0.2236
s^{**}	0.1659	0.1813	0.2000
d^* / y^*	0.2277	0.1754	0.1228
d^* / k^*	0.1237	0.0868	0.0541
$(i^* - i^f)^c$	0.0729	0.0584	0.0427
$g^{Y*} - n$	0.0289	0.0323	0.0370

s^* = optimal saving ratio in an open economy with partly endogenous technical change,
s^{**} = optimal saving ratio in an open economy with fully exogenous technical change,
d^* / y^* = debt-GDP ratio, d^* / k^* = debt/capital ratio, i^* = real interest rate, i^f = real global interest rate, $g^{Y*} - n$ = per capita output growth rate.

[a] Based on maximization of a discounted stream of lifetime consumption [Equation (12)], subject to Equations (9)-(10), in which instantaneous utility is of the CRRA form. Microsoft's Solver tool is used to solve the first-order conditions for a maximum.

[b] Estimates from Szpiro [1986].

[c] Risk premium [Equation (8)], reflecting combined effects of risk factors and financial markups.

[41] Refer back to footnote 7 for the most recent [2019] Philippine gross foreign assets and debt ratios.

If $\rho = 0$, $\theta = 1$ (utility function is ln c), and $\varnothing \geq 0$, then the optimal saving rate is:

$$s^{***} = \alpha, \qquad (30)$$

the S-S result. In the RCK framework, if the time preference discount is close to zero (but not zero) and the utility function is ln c, the saving rate must be set equal to the income share of capital, whether or not learning by doing is partly endogenous. It is also true that $s^{***} > s^{*} > s^{**}$.

As the elasticity of intertemporal substitution increases, Table 1 reveals the following:

- Optimal saving ratio and the equilibrium growth rate of per capita GDP rise; and
- The debt to GDP ratio declines.[42]

When $\rho = 0, \theta = 1$, and $\varnothing \geq 0$, Equation (28) says that the optimal saving rate is not only a function of the deep parameters $\rho, \theta, \varnothing, \mu, \delta$, and n, as well as k^*, but must be set equal to a fraction of capital's income share α, with the fraction equal to $\{[\{i^* - g^{Y*})(d^* / k^*) + \delta + g^{Y*}] / (\rho + \delta + \theta g^{Y*})\}$.[43] An alternative interpretation is that capital's income share should be a multiple of the optimal saving rate in order to compensate capital for the additional GDP growth generated by endogenous growth and learning by doing. Setting capital's income share equal to the saving rate, implicit in the S-S model, would be welfare-reducing because of the under-compensation of capital.

6. Concluding remarks

This paper has developed and discussed the VM2 model, which is an open-economy growth model, employing the RCK optimal control setup and modifying Arrow's learning-by-doing framework in which workers learn through experience on the job to raise their productivity.

- The VM2 model produces empirically plausible and testable predictions about per capita GDP growth effects of parameters describing preferences, technology, and population growth, as well as public policies that affect equilibrium capital intensity and, directly or indirectly, the model's parameters, especially the extent of learning-by-doing associated with the economy's stock of capital per effective labor. Such predictive hypotheses can be tested empirically using panel data.[44]
- The high *Golden Rule* domestic saving rate of 34 percent of GDP reported in the VM1 model is associated with high elasticity of intertemporal substitution (a low degree of relative risk aversion). Lower empirical estimates of the

[42] Households become less risk-averse, so they save more and borrow less, raising the growth rate of per capita GDP, and lowering the debt to GDP ratio. To be precise, a higher saving rate increases the equilibrium capital-labor ratio, lowering the marginal product of capital, foreign borrowing, and the debt to GDP ratio.
[43] In Figure 1, this condition is associated with maximum utility at c^*_0.
[44] For the mechanics and application of a panel data procedure, see Knight et al. [1993].

elasticity of intertemporal substitution imply lower *Golden Utility* domestic saving rates of 18-22 percent of GDP that are dynamically efficient and feasible targets for most governments in Asia and emerging markets. The sustainable gross foreign debt is in the range of 39-50 percent of GDP.

- The domestic saving rate should be set below the share of capital in total output, owing to positive externalities arising from learning by doing associated with capital intensity. Equivalently put, income going to capital as a share of total output should be a multiple of the amount saved and invested in order to compensate capital for the additional output generated by endogenous growth and induced learning-by-doing.
- Fiscal consolidation and strong incentives for private saving are essential to achieving maximum per capita GDP growth. Reliance on foreign savings (foreign borrowing) has limits, particularly in a global environment of high interest rates and risk premiums.
- When real borrowing costs are positively correlated with rising external indebtedness, foreign borrowing is even more circumscribed, and efficient foreign debt management is critically important.
- When risk spreads are large despite the high expected marginal product of capital, there is a role for public policies to achieve and maintain macroeconomic and financial stability to mollify risk-averse global lenders.
- The international community should increase aid including subsidized loans earmarked for imports of advanced capital goods, workers' education, on-the-job training, and health, provided that economic policies are sound.
- In view of current low global interest rates and the actual gross foreign debt remaining well below the sustainable level (as a ratio to GDP) in the Philippines, there is room for additional foreign borrowing to cover imports of advanced capital goods, subsidies to on-the-job training at enterprises, and costs of controlling the COVID19 pandemic and other public health expenditures. Such measures will increase learning-by-doing and labor productivity, leading to a short-run overshooting of a long-run, higher rate of per capita GDP growth.
- The record of the Philippines on fiscal consolidation, external accounts surplus and high and sustained growth is remarkable, earning high marks from credit-rating agencies. Thus, a temporary breach of the limits on foreign borrowing in the current environment of COVID19 is allowed. It is expected that such a breach disappears as the COVID pandemic passes.

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APPENDIX: A QUICK REVIEW OF THE S-S GROWTH MODEL

The S-S model consists of the following relationships:

$$Y = K^{\alpha} L^{(1-\alpha)} \tag{1}$$

$$L = AN \tag{2}$$

$$I = S = sY \tag{3}$$

$$\dot{K} = I - \delta K \tag{4}$$

$$\dot{A}/A = \lambda \tag{5}$$

$$\dot{N}/N = n \tag{6}$$

$$k = K / L \tag{7}$$

Y = GDP, K = capital, L = effective labor, A = exogenous Harrod-neutral labor-augmenting productivity multiplier, N = population, k = capital intensity, α = output elasticity with respect to capital, $1-\alpha$ = output elasticity with respect to effective labor[45], s = gross fixed saving to income ratio, δ = depreciation rate, λ = change in A, and n = population growth rate.

Y is produced according to a Cobb-Douglas production in Equation (1), using K and L as inputs.[46] Equation (2) defines L as the product AN.[47] Equation (3) expresses the warranted rate in which investment is equal to saving, the latter being a fixed proportion, s, of income Y. Equations (4)-(6) are dynamic equations for the state variables K and L. Dividing Equation (4) by K, using Equations (1), (3) and (7),

$$\frac{\dot{K}}{K} = S\frac{Y}{K} - \delta = sk^{(\alpha-1)} - \delta \tag{8}$$

Equation (8) is termed the warranted rate. Time differentiating Equation (2) and substituting Equations (5) and (6) yield,

$$\frac{\dot{L}}{L} = \lambda + n. \tag{9}$$

Equation (9) is termed the natural rate.

Time differentiating Equation (7) and substituting Equations (8) and (9) yield the proportionate change in the capital intensity k,

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} = \frac{\dot{L}}{L} = sk^{(\alpha-1)} - (\lambda + n + \delta) \tag{10}$$

From Equations (1) and (7), output in intensive form is:

$$Y/L = k^{\alpha} \tag{11}$$

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[45] Under assumed marginal factor productivity pricing and wage-price flexibility, the parameters α and $(1-\alpha)$ represent the income shares of capital and labor, respectively.

[46] Any function $Y = F(K,L)$ satisfying the Inada [1963] conditions (Section 3) will suffice.

[47] Refer back to footnote 17.

Time differentiating Equation (11) and substituting Equation (9) yield the (instantaneous or transitional) growth rate of output at any positive k(t):

$$\dot{Y}/Y = (\lambda + n) + \alpha\, k/k \tag{12}$$

Substituting Equation (10),

$$\dot{Y}/Y\, \alpha[sk^{(\alpha-1)} - \delta] + (1-\alpha)(\lambda + n) \tag{13}$$

which is the growth of Y weighted by the income shares of capital α and labor $(1-\alpha)$.[48]

In the steady state, k is constant at $k^*(k/k = 0)$[49], and by the constant-returns assumption,

$$\dot{K}/K* = \dot{L}/L* = \dot{Y}/Y* = g* = \lambda + n. \tag{14}$$

Equation (14) is the steady state output growth rate, at which the warranted and natural rates are equal, and the economy is on a full-employment, balanced growth path.[50]

Substituting Equation (8) into Equation (14), setting $k = k^*$,

$$sk*^{(\alpha-1)} = \lambda + n + \delta. \tag{15}$$

Solving for the equilibrium capital intensity,

$$k^* = \left[\frac{s}{(\lambda + n + \delta)}\right]^{\frac{1}{(1-\alpha)}}. \tag{16}$$

Equation (16) states that the equilibrium (steady state) capital intensity k^* is a positive function of the saving rate s, and a negative function of λ, n, and δ.

From Equations (1), (2), (5), (7), and (16), equilibrium per capita output is given by:

$$\frac{Y}{N}* = A(0)e^{\lambda t}\, \frac{s}{(\lambda + n + \delta)}^{\frac{1}{(1-\alpha)}}. \tag{17}$$

In the S-S model, even though the steady state output growth is exogenously fixed by effective labor growth $\lambda + n$, independent of the saving rate s, the steady state per capita output $Y/N*$ is a positive function of the saving rate s.

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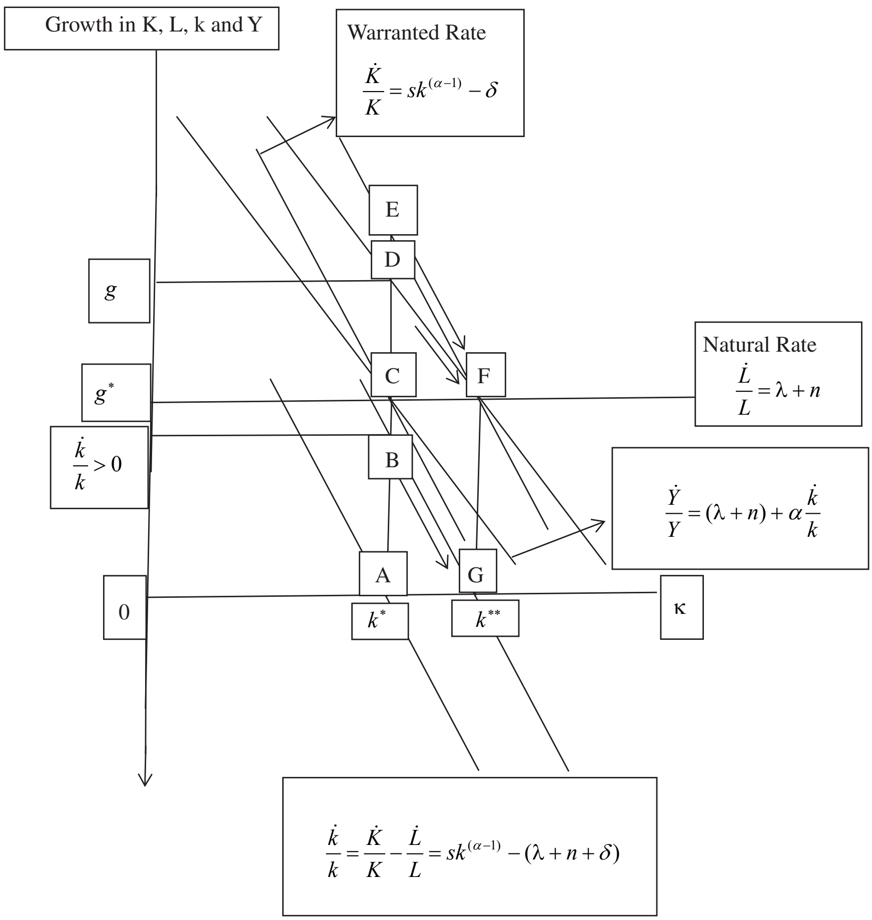
[48] Alternatively, Equation (13) may be derived by time differentiating Equation (1) and substituting Equations (8) and (9) into the result.

[49] The Inada [1963] conditions enumerated in Section 3 ensure a unique and globally stable k*.

[50] This is the S-S solution to the knife-edge problem posed by Harrod [1939]-Domar [1946], who employ a fixed output-capital ratio to conclude that balanced growth, macroeconomic stability, and full employment are not assured and may happen only by accident. S-S offers a variable output-capital ratio [Equation (8)], i.e., a fully adjusting warranted rate, as a solution. Another solution is found in Villanueva [2020] via a fully adjusting natural rate through endogenous labor participation, complementing a fully adjusting S-S warranted rate.

Figure 5 is the phase diagram showing the S-S model's equilibrium behavior and growth dynamics. It illustrates the steady-state and transitional growth effects of an increase in the saving rate. The vertical axis graphs the rates of change in output = $\dot{Y}/Y$ [Equation (12)], warranted rate = $\dot{K}/K$ [Equation (8)], natural rate = $\dot{L}/L$ [Equation (9)], and capital intensity = $\dot{k}/k$ [Equation (10)]. The horizontal axis measures the level of capital intensity = k [Equation (7)].[51]

FIGURE 5. Equilibrium and growth dynamics
S-S Model: effects of an increase in saving rate



[51] The $\dot{K}/K$ line and $\dot{k}/k$ line are downward-sloping and parallel to each other because they have a common slope $s(\alpha-1)k^{(\alpha-2)}$. Both lines are steeper than the $\dot{Y}/Y$ line, whose slope is equal to $\alpha\ s(\alpha-1)k^{(\alpha-2)}$, where α is a positive fraction.

The steady-state (equilibrium) of the S-S model occurs at points A(k^*, 0) and C(k^*, g^*), at which the warranted and natural rates are equal (warranted rate line intersects natural rate line at point C), k^* is equilibrium capital intensity (the $\frac{\dot{k}}{k}$ line intersects the k-axis at point A), and g^* is equilibrium (steady-state) output growth (reading off the $\frac{\dot{Y}}{Y}$ line). Equilibrium at point A(k^*, 0) is unique and globally stable, ensured by the Inada (1963) conditions. Any capital intensity k different from k^* will bring k back to k^* because of the adjustments of the output-capital ratio and hence, of the saving-capital ratio $Y / K = sk^{(\alpha-1)}$, as capital's marginal and average products deviate from their equilibrium values at k^*. The warranted rate adjusts to the natural rate to bring balanced growth back to points A(k^*, 0) and C(k^*, g^*).

Assume an increase in the saving rate *s*, say, through fiscal policy (via higher public sector saving rate). The warranted rate, output growth, and capital intensity growth lines will shift upward to the right, while the natural rate remains stationary. The new steady-state occurs at points G(k^{**}, 0) and F(k^{**}, g^*), characterized by a higher equilibrium capital intensity with the *same* equilibrium output growth rate because the natural rate is fixed at $\lambda + n = g^*$. More interesting is the transition to the new steady state. At the starting capital intensity k^*, in the short-run, a higher saving rate raises the warranted rate to point E (segment k^*E), which is larger than the natural rate (segment k^*C). Capital intensity growth turns positive (segment k^*B). Consequently, output growth goes up to g (segment k^*D, reading off the output growth line), temporarily higher than g^*. As capital intensity increases, the marginal returns on investment decline. The output-capital ratio falls, decreasing the warranted rate. This downward adjustment of the warranted rate (along the segment EF) continues until it equals the natural rate at F, at which point the growth rate of output reverts to its original rate g^* (traced by the segment DF). Meanwhile, the growth rate of capital intensity turns less and less positive until it is zero at G (traced by the segment BG), characterized by a higher level of capital intensity at k^{**}.

Figure 5 shows that, although the steady-state output growth rate is fixed at $\lambda + n$, invariant with respect to the saving rate *s,* the output growth at any time is a function of *s* and all the other structural parameters of the model δ, λ, and *n.*[52] From Equations (10) and (13) and Figure 5, an increase in the saving rate s will raise the growth rate of capital intensity $\dot{k}/k$ and the transitional output growth rate $\dot{Y}/Y$. This rich dynamics is a major strength of the S-S model.[53]

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[52] In Figure 5, map the $\dot{k}/k$ line onto the $\dot{Y}/Y$ line.

[53] As noted earlier, this transitional dynamics is absent in the AK model of the new endogenous growth theory [Rebelo 1991]. Solow's illustrative example of the growth effects of an increase in productivity (invention of a computer), quoted by Villanueva [2021], is an example of such transitional dynamics. The VM2 model preserves the S-S transitional dynamics. The new VM2 result is a higher steady state output growth rate in response to a higher saving rate, a generalization of the S-S model (see Villanueva [2020] for the same result in a closed economy).