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Toward a general neoclassical theory of economic growth

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The Harrod-Domar (H-D) growth model assumes a fixed capital-output ratio, signifying absence of substitutability between capital and labor, leading to a "knife-edge" problem wherein balanced growth of capital (fixed warranted rate) and labor (fixed natural rate) occurs only by accident, preventing the attainment of macroeconomic stability with full employment. The neoclassical Solow-Swan (S-S) growth model provides an elegant solution to the H-D problem by endogenizing the warranted rate via the saving-investment relation, wherein capital growth is a function of a fully adjusting income-capital ratio (inverse of the H-D capital-output ratio)—allowing for smooth substitutability between capital and labor while keeping the natural rate exogenously fixed. The S-S model implies a positive, albeit *temporary* output growth effect of a higher saving rate. The present paper extends the capital-labor ratio's influence onto the natural rate via effects on labor productivity through a modified Arrow learning by doing framework, and via labor participation through real wage adjustments. Thus, the positive output growth of a higher saving rate, although *temporary* in the short run as in the S-S model, is *permanent* in the long run through adjustments in both the warranted and natural rates—a generalization of the Solow-Swan model.

JEL classification: E130, O410
Keywords: Harrod-Domar, neoclassical growth model, Solow-Swan, warranted rate, natural rate, balanced growth, learning by doing, labor participation

1. Introduction

The basic Solow [1956]-Swan [1956] or S-S growth model provides an elegant solution to the "knife-edge" Harrod [1939]-Domar [1946] or H-D problem[1] by endogenizing the warranted rate via the saving-investment relation, wherein capital growth is a function of a fully adjusting income-capital ratio (inverse of the H-D capital-output ratio)—allowing smooth substitutability between capital and labor while keeping the natural rate exogenously fixed. This makes the warranted

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[1] Explained below.

rate a negative[2] function of the capital-labor ratio, allowing short-run adjustments in output or income during the transition to the exogenously fixed natural rate—sum of labor augmenting productivity/technical change and working population growth. The present paper extends the effects of the capital-labor ratio to the natural rate via capital intensity effects on labor augmenting productivity and labor participation. Thus, equilibrium growth is obtained through adjustments in both warranted and natural rates—a generalization of the S-S growth model.

The model is not meant to take account of aggregate demand—neither is the S-S model. Like the S-S model, it is a supply-side model, albeit a generalized one (where the natural rate is made endogenous via endogenous labor productivity and labor participation). Questions on real GDP, inflation, and unemployment, as opposed to *capacity or potential* GDP, are not addressed. For a merger of the textbook short-run macroeconomics of aggregate demand and the S-S textbook long-run macroeconomics of aggregate supply, as called for by Solow [2022] in the 1992 addendum to his Nobel Prize speech , and for a numerical application to the Philippines, see Villanueva et al. [2023], Chapters 7 and 10, respectively.

To put this paper in the simplest perspective, assume a constant-returns (unit-homogeneous) aggregate production function,

$$Y = K^{\alpha} L^{(1-\alpha)}, \tag{1}$$

where Y is output, K is capital stock, $L = APN$ is effective labor (in efficiency units), A = a labor productivity or technology multiplier, P = labor participation rate, N = total population, α = elasticity of output with respect to capital, $1-\alpha$ = output elasticity with respect to labor, and $0 < \alpha < 1$ is a constant. Conlisk [1968] had shown that in a general production function $Y = F(K, L)$ with constant rates of factor-augmenting technical change attached to K and L and subject to degree β returns to scale,[3] the existence of a well-behaved and balanced growth equilibrium[4] requires a unitary elasticity of substitution $\varepsilon(k) = (\dot{k}/k)/(\dot{u}/u)$ where $u = F_2/F_1$ and $k = K/L$.[5] If F is Cobb-Douglas ($\beta = 1$) as in (1), then $\varepsilon(k) = 1$ and α is the constant income share of capital.[6]

Income growth at any moment of time is equal to the weighted sum of capital growth and labor growth, the weights being α and $1-\alpha$, respectively (a dot over a variable denotes time derivative, i.*e*., $\dot{K} = dK/dt$):

$$\frac{\dot{Y}}{Y} = \alpha \frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L}, \tag{2}$$

[2] Owing to diminishing marginal product of capital. The H-D growth model assumes a fixed capital-output ratio, signifying absence of substitutability between capital and labor.

[3] $\beta < 0$ signifies decreasing returns to scale, $\beta = 1$ constant returns to scale, and $\beta > 1$ increasing returns to scale.

[4] Defined as g^* = where $(\dot{K}/K)^* = (\dot{L}/L)^* = (\dot{Y}/Y)^* = (\dot{A}/A)^* + (\dot{P}/P)^* + (\dot{N}/N)^*$, " * " indicates equilibrium and $(\dot{K}/K)^*$, $(\dot{A}/A)^*$, $(\dot{P}/P)^*$, and $(\dot{Y}/Y)^*$ are functions of capital intensity $k^* = K/L$, as postulated in Section 2.

[5] This requirement applies to growth models with *increasing returns to capital* (β >1), e.g., Romer [1986], Lucas [1988], Grossman and Helpman [1990, 1991], Rivera-Batiz and Romer [1991], Barro and Sala-i-Martin [1995], and Aghion and Howitt [1998]. See discussion in Conlisk [1968].

[6] See Chapter 6 of Villanueva et al. [2023] and Conlisk [1968].

Capital growth is the *warranted rate* and labor growth is the *natural rate*. The warranted rate (saving-investment) is a monotonically declining function of the capital-labor ratio,

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta, \tag{3}$$

where s is the fixed gross saving/income ratio and δ is a constant depreciation rate.[7] From the definition $L = APN$, labor growth or the natural rate, is given by

$$\frac{\dot{L}}{L} = \frac{\dot{A}}{A} + \frac{\dot{P}}{P} + \frac{\dot{N}}{N}. \tag{4}$$

Assume, as in the S-S model, that $\dot{A}/A = \lambda$, $\dot{P}/P = 0$, and $\dot{N}/N = n$, where λ and n are constants. In the steady state, k is constant at k^*,[8] implying that

$$\frac{\dot{K}}{K}^* = s\frac{Y}{K}^* - \delta = \frac{\dot{L}}{L}^* = \lambda + n \tag{5}$$

and by the constant-returns assumption, using (1) and definition, $g^* = \lambda + n$,

$$\frac{\dot{K}}{K}^* = sk^{*(\alpha-1)} - \delta = \frac{\dot{L}}{L}^* = \frac{\dot{Y}}{Y}^* = g^* = \lambda + n, \tag{6}$$

where g^* defines the equilibrium growth rate of income Y, or the balanced growth path.

The knife-edge H-D problem ($\dot{P}/P = 0$ by assumption) is expressed by the condition

$$\frac{s}{v} - \delta \gtreqless \lambda + n, \tag{7}$$

where $v = K/Y$ is the fixed H-D capital-output ratio in (5). Since both sides are constants, equilibrium growth is not assured.[9]

The S-S model solves the knife-edge H-D problem by employing a neoclassical production function with smooth substitutability between capital and labor, i.e., $1/v$ is a monotonically decreasing function of k, such that the warranted rate fully adjusts from any initial level of k, making balanced growth possible.[10] However, equilibrium income growth g^* remains exogenous because the natural rate, being fixed at $\lambda + n$, serves as a bottleneck to the growth process, making the positive growth effect of a higher saving rate s temporary.[11]

The present paper extends the capital-labor ratio influence on the natural rate via capital intensity effects on labor productivity ($\dot{A}/A$) through a modified

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[7] The income-capital ratio Y/K declines with k owing to diminishing marginal product of capital—(3).

[8] The Inada [1963] conditions assure the existence of a unique value of k^*.

[9] Equality signifies simultaneous achievement of balanced growth, macroeconomic stability, and full employment. Inequality signifies either inflationary spiral or continuous deflation with unemployment.

[10] Refer to footnote 7.

[11] As Solow [1991:4] calls it.

Arrow [1962] learning by doing framework [Villanueva 1994], and on labor participation ($\dot{P}/P$) through real wage [Villanueva 2020]. Thus, equilibrium income growth is obtained through adjustments in both the warranted rate and in the natural rate.

The 1960s through 1990s saw attempts to solve the S-S model's exogeneity of the natural rate $\dot{A}/A + \dot{P}/P + \dot{N}/N$, by making labor-augmenting technical change $\dot{A}/A$ endogenous.[12] An early learning by doing model by Arrow [1962] assumes that learning has a positive effect on the equilibrium growth of output. If labor productivity A changes according to $\dot{A}/A = \varnothing(\dot{K}/K)$, where $\varnothing$ is a learning coefficient, then equilibrium output growth g^* is a multiple of the S-S steady-state growth rate $n + \lambda$, since $0 < \varnothing < 1$ by assumption:

$$\frac{\dot{K}}{K}^* = \frac{\dot{L}}{L}^* = \frac{\dot{Y}}{Y}^* = g^* = (n + \lambda) / (1 - \varnothing). \tag{8}$$

Note the absence of the saving rate s in (8). However, if the Arrow [1962] model is interpreted as a change in learning $\dot{A}/A$ being proportional to the capital-labor ratio $k = K/L$,[13] and not to the growth rate of the capital stock $\dot{K}/K$, then the present paper is the Arrow model extended to the case of endogenous labor participation $\dot{P}/P$ (elaborated below).

Nelson and Phelps [1966], Conlisk [1967], and Villanueva [1994] advanced early growth models with endogenous labor-augmenting technical change, deriving the key result that an increase in the saving rate raises equilibrium output growth. Agénor [2004:466-471] refers to Villanueva's [1994] model as a variant of the Conlisk [1967] model and "an extension of Arrow's [1962] learning by doing model,…[wherein] the productivity of workers increases when the relative availability of capital goods (for instance, the stock of high-performance computers) rises", leading to enhanced equilibrium growth effects of saving and investment rates. More precisely, Villanueva [1994] interprets Arrow's [1962] learning by doing model by positing $\dot{A}/A = \theta k + \lambda$, where $\theta > 0$ is a learning coefficient and λ is an exogenous labor-augmenting productivity/technical change term.

Subsequent contributions constructed increasingly complex models. Romer [1986; 1990] posited a knowledge-producing sector, alongside a goods-producing sector. The stock of knowledge is a non-rival good—its use in one sector does not preclude its use in the other sector. Lucas [1988] proposed models emphasizing human capital accumulation through schooling and learning by doing, but he abstracted from the economics of demography.[14] Grossman and Helpman [1991] focused on innovation financed by investments in industrial research. Rebelo's [1991] AK model assumed that all productive inputs, including human capital,

[12] For an engaging history of endogenous growth theory, see Warsh [2007]. Solow [1991] has been critical of endogenous growth models with their emphasis on endogenous technical change and increasing returns. See Chapter 1 of Villanueva et al. [2023] and the third paragraph of the present introductory section.

[13] This is my interpretation of the Arrow [1962] model.

[14] Lucas [1988:6] admits that this is a serious omission.

are reproducible.[15] Aghion and Howitt [1998] highlighted imperfect markets in the research and development (R&D) sector and Schumpeterian creative destruction. The knowledge-innovation-R&D sector is assumed to be subject to increasing returns so that growth does not vanish. Conlisk [1967] had shown that increasing returns to capital yield explosive growth, which rarely or temporarily happens in the real world.[16] He had demonstrated that a growth model with endogenous labor-augmenting technical change and an aggregate production function that is subject to diminishing returns to capital is consistent with positive and permanent growth effects of an increased saving rate (or of any change in the other model parameters with expected signs).

In all the above growth models, the labor participation rate *P* is an exogenous constant fraction or percentage by assumption.[17] Another solution to the knife-edge problem, besides the S-S model's variable capital-output ratio implicit in a well-behaved neoclassical production function with smooth factor substitution and wage-rice flexibility and endogenous labor-augmenting productivity multiplier [Villanueva 1994], is a fully-adjusting natural rate via an endogenously-determined labor participation rate *P* [Villanueva 2020]. In a carefully researched IMF empirical study, Grigoli et al. [2018:18] found robust results indicating that, among others, an increasingly educated[18] labor force influenced significantly and positively the labor participation rates in 36 advanced economies. Referring to the US in particular, the Congressional Budget Office (CBO) [2018] issued a working paper on labor participation, containing similar explanatory variables included in the IMF study, and arriving at similar statistical results. The study noted that the US labor participation rate began an uninterrupted decline in the latter half of the 1990s, coinciding with the aging and retirement of baby boomers. In 2007, the labor participation rate stood at 66 percent. A decade later, in 2017 Q4, it fell to 63.2 percent. The CBO projects that the US labor participation rate will continue to decline and will be 60.2 percent in 2028 Q4. The projected increase in educational attainment, which has a positive effect on the labor participation rate, will not be enough to offset the continued decline attributed to aging and retirements and to the stagnation in real wages, among other factors.

Motivated by the empirical findings of Grigoli et al. [2018] and the CBO [2018], Villanueva [2020] postulated that the proportionate change in labor

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[15] Output $Y = AK$, where Y is constant returns to capital K, implying that Y always grows at the same rate as K, and is equal to s^*A, where s^* is the fraction of income saved for investment in physical *and* human capital ($s^* > s$; s is income saved for investment in physical capital) and A is a technological constant. This property is in sharp contrast to the transitional growth dynamics in the S-S model.

[16] However, see Conlisk [1968] and the discussion in the third paragraph of the present introductory section.

[17] Whether it is 70 percent or any other percentage, the rate of change in P is assumed to be zero. The labor participation rate and unemployment rate are metrics used to gauge the health of the labor market. The key difference between the two indicators is that the participation rate measures the percentage of people who are in the labor force, while the unemployment rate measures the percentage within the labor force that is currently unemployed.

[18] Workers with secondary and tertiary degrees.

participation consists of exogenous components including aging and retirements and endogenous components including aggregate income per man-hour and real wages. The objective was to generalize the equilibrium property of the S-S model by making the natural rate fully adjusting through endogenous labor participation [Villanueva 2020], additional to endogenous learning by doing that improves labor productivity [Villanueva 1994].

The rest of the paper is organized as follows. Section 2 presents and discusses the general neoclassical growth model, followed by the analytics of the temporary and permanent growth effects of changes in the structural parameters, notably, the saving rate, learning coefficient, labor-augmenting productivity/technical change, and components of labor participation. Section 3 concludes.

2. A general neoclassical growth model

Equations (9)-(16) below comprise the general neoclassical growth or extended (*e*) model:

$$Y = K^{\alpha} L^{(1\text{-}\alpha)} \tag{9}$$

$$L = APN \tag{10}$$

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta \tag{11}$$

$$\frac{\dot{A}}{A} = \theta k + \lambda \tag{12}$$

$$\frac{\dot{P}}{P} = \beta + \rho\left(\frac{Y}{L}\right) + \omega RW \tag{13}$$

$$\frac{\dot{N}}{N} = n \tag{14}$$

$$RW = \frac{\partial Y}{\partial L} \tag{15}$$

$$k = \frac{K}{L} \tag{16}$$

where $Y = GDP$; K = capital; L = effective labor; A = Harrod-neutral productivity or technical change multiplier; P = labor participation rate; N = population base; RW = real wage rate; k = capital/labor ratio ; α = output elasticity with respect to capital; (1-α) = output elasticity with respect to labor; s = gross saving rate; δ = depreciation rate; θ = learning coefficient; λ = constant rate of exogenous Harrod-neutral technical change; β = exogenous, noneconomic determinants of labor participation; and ρ, ω, n = constant parameters.

Equation (9) repeats (1) with the same properties. Equation (10) defines effective labor L as the product term APN. Equations (11)-(14) are the dynamic relationships governing rates of change in K, A, P, and N. Equation (15) is a profit maximization condition that the real wage be set equal to labor's marginal product. Finally, (16) defines capital intensity as the ratio of K to L.

Equation (11), the warranted rate, repeats Equation (3). The derivation of the natural rate $\dot{L}/L$ is as follows: Villanueva [1994] interprets Arrow's [1962] learning by doing model by positing $\dot{A}/A = \theta k + \lambda$, as in (12), where $\theta > 0$ is a learning coefficient. The idea is that as the per capita stock of capital K/N with embodied advanced technology gets larger, the learning experience makes workers more productive.[19]

Reflecting the empirical findings of Grigoli et al. [2018] and the CBO [2018], (13) states that the proportionate change in labor participation P is the sum of an exogenous component β and endogenous components $\rho(Y/L)$ and ωRW, while (14) expresses the standard rate of exogenous population growth. The exogenous term β includes aging and retirements; changes in labor market policies and institutions, e.g., tax-benefits (tax credits and unemployment benefits); and a host of non-economic variables identified in the aforementioned empirical studies.[20] The endogenous terms are: (a) the portion of aggregate income per man-hour (Y/L) spent on secondary and tertiary education and its effect on the number of graduates, and the latter's effect on the labor participation rate;[21] and (b) the real wage RW that, under profit maximization, is equal to labor's marginal product $\partial Y/\partial L = (1-\alpha)k^{\alpha}$. Equation (16) defines k as the capital/labor ratio. Increases in the percentage ρ of aggregate income per man-hour (Y/L) spent on secondary and tertiary education and in the real wage ($\omega > 0$) are expected to raise the rate of labor participation. There are eight equations with eight endogenous variables—Y, K, L, A, P, N, RW, and k (time t is suppressed).

2.1. Reduced model

Using (9) and (16), (11) becomes

$$\frac{\dot{K}}{K} = sk^{(\alpha-1)} - \delta. \tag{17}$$

Differentiating (9) with respect to L and substituting (15)-(16) yield

$$RW = (1 - \alpha)k^{\alpha}. \tag{18}$$

[19] Using $L = AN$, letting $P = 1$, rewrite the above equation as $\dot{A} = \theta(K/N) + \lambda A$, or $(\dot{A}/A) = \theta k + \lambda$, where $k = K/L$.

[20] If β denotes aging and retirements, then the growth effect is negative; if β denotes tax credits and unemployment benefits, then the growth effect is positive.

[21] The coefficient $\rho > 0$ is a composite parameter reflecting the fraction of aggregate income spent on secondary and tertiary education and its effect on the number of graduates, and the latter's effect on labor participation.

Using (9) and (16) yields

$$\frac{Y}{L} = k^{\alpha\text{-}1}. \tag{19}$$

Equations (18)-(19) into (13) yield

$$\frac{\dot{P}}{P} = \beta + [\rho + \omega\,(1\text{-}\alpha)]k^{\alpha}. \tag{20}$$

Time differentiating (10), using (12), (14), and (20), yields

$$\frac{\dot{L}}{L} = \theta k + [\rho + \omega\,(1\text{-}\alpha)]k^{\alpha} + \lambda + n + \beta. \tag{21}$$

The equilibrium growth rate of per capita income is

$$g^{*} - n = \theta k_e^{*} + [\rho + \omega\,(1\text{-}\alpha)]k_e^{*\alpha} + \lambda + \beta, \tag{22}$$

where k^{*} = equilibrium capital intensity, and e refers to the extended model.

Time differentiating (16) and substituting (17) and (21) into the result yield the rate of change of capital intensity at any time,

$$\frac{\dot{k}}{k}(e) = sk^{(\alpha\text{-}1)} - \theta k + [\rho + \omega\,(1\text{-}\alpha)]k^{\alpha} + (\lambda + n + \beta + \delta). \tag{23}$$

Time differentiating (19) and substituting (22) into the result yield the growth rate of per capita income at any moment of time,

$$\frac{\dot{L}}{L} - n = g - n = \theta k_e^{*} + [\rho + \omega\,(1\text{-}\alpha)]k_e^{*\alpha} + \lambda + \beta + \alpha\frac{\dot{k}}{k}(e). \tag{24}$$

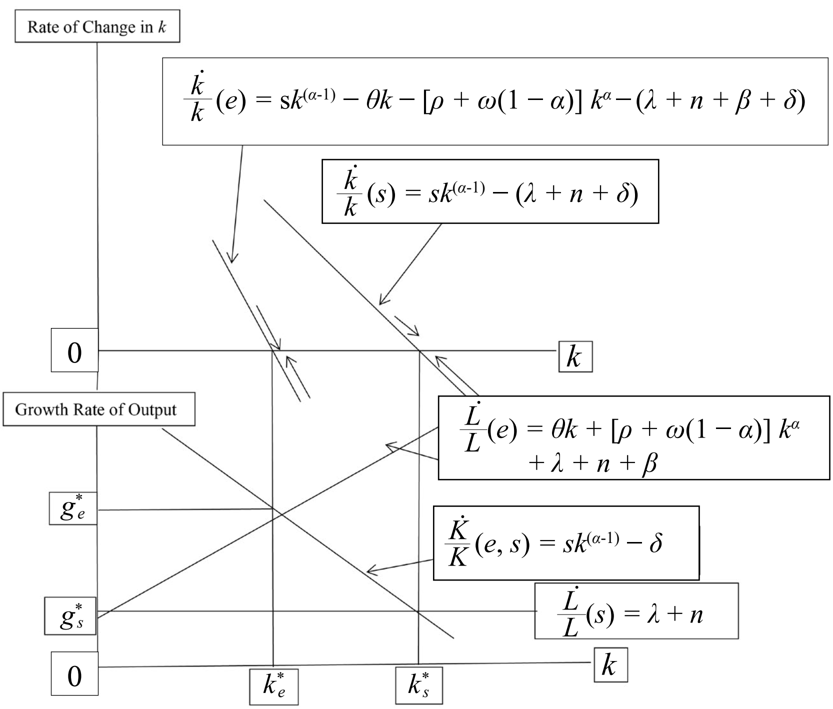
where $\dot{k}/k(e)$ is given by (23). In equilibrium, $\dot{k}/k(e) = 0$ and (24) reduces to (22).

The reduced models in $(\dot{k}/k)$, $(\dot{K}/K)$, and $(\dot{L}/L)$ in the S-S (denoted by s) and extended (denoted by e) models are shown in Figure 1.[22] The upper part shows the proportionate rate of change in the capital-labor ratio and the lower part shows the growth rate of output. In both parts, the horizontal axis shows the level of capital intensity. Given the Inada [1963] conditions, the $\dot{k}/k$ line in either model is downward sloping and intersects the k-axis at some positive k, such as k_s^{*} or k_e^{*}. When the $\dot{k}/k$ line intersects the k-axis, the equilibrium capital intensity is k_s^{*} in the S-S model and k_e^{*} in the extended model. In either model, for $k < k^{*}$, $\dot{k}/k >$ 0, k increases until it reaches k^{*} at which it becomes constant. For $k > k^{*}$, $\dot{k}/k < 0$, and k decreases until it goes back to k^{*} at which it becomes constant. As capital intensity changes, diminishing marginal and average productivity of capital (in either model) and, in the extended model, positive dependence of labor

[22] Figures 1 to 4 are phase diagrams. Phase diagramming is a powerful tool in analyzing growth models not explicitly involving time.

productivity on learning by doing (a positive function of capital intensity) and of labor participation on capital intensity, provide the economic rationale behind the proportionate changes in capital intensity and in the warranted and natural rates. Specifically, with reference to the lower part of Figure 1, the downward sloping warranted rate line in either model owes to diminishing marginal and average capital productivities as k increases. The horizontal natural rate line in the S-S model ($\dot{L}/L(s)$) reflects the full exogeneity of labor-augmenting productivity. The upward sloping natural rate line in the extended model ($\dot{L}/L(e)$) reflects the positive dependence on capital intensity of efficient labor growth via learning by doing and of labor participation via the real wage (a positive function of capital intensity). As capital intensity rises, more intensive learning leads to greater labor productivity. Higher aggregate income per man-hour translates into higher spending on secondary and tertiary education, higher number of graduates, and higher labor participation. As capital intensity rises, labor's marginal product (real wage) goes up, encouraging higher labor participation and, hence, a larger natural rate. In the lower part of Figure 1, equilibrium growth rates of output g_s^* and g_e^* respectively, in the S-S and extended models, are indicated when the warranted and natural rates are equal at points k_s^* and k_e^*. Note that $g_e^* > g_s^*$ because of endogenous learning by doing and endogenous labor participation in the extended model.

FIGURE 1. Extended (e) and Solow-Swan (s) models



2.2. Temporary and permanent growth

Table 1 shows the permanent effects of changes in the S-S and extended model's parameters on capital intensity and growth rate of income. In the S-S model, only the rates of exogenous labor-augmenting change and population growth have permanent growth effects. In the extended model, higher values for the saving rate, learning coefficient, expenditures on secondary and tertiary education, real wage, exogenous labor-augmenting productivity/technical change, and population growth have growth effects, while aging/retirements and physical capital depreciation impact negatively on growth. On the balanced growth path of the extended model, a constant equilibrium capital/labor ratio means that the warranted rate is equal to the natural rate, and by the constant returns assumption, to the growth rate of per capita output/income as well,

$$\frac{\dot{K}}{K}^{*} = \frac{\dot{L}}{L}^{*} = \frac{\dot{Y}}{Y}^{*} = g^{*} - n = \theta k_e^{*} + [\rho + \omega\,(1\text{-}\alpha)]k_e^{*\alpha} + \lambda + \beta,$$

which is (22). At any moment of time, the output growth rate is a weighted average of the warranted and the natural rates,

$$\frac{\dot{Y}}{Y} = \alpha(sk^{(\alpha\text{-}1)} - \delta) + (1 - \alpha)\{\theta k + [\rho + \omega(1\text{-}\alpha)]\, k^{\alpha} + \lambda + n + \beta\}.$$

There is a divergence between the warranted and natural rates in the short-run transition to equilibrium. In the S-S model, in equilibrium, the natural rate is equal to a constant term $\lambda + n$. In the transition to equilibrium, output growth adjustment falls only on the warranted rate as capital intensity adjusts to its equilibrium value. In the extended model, the equilibrium natural rate adjusts as well to a moving capital/labor ratio.

TABLE 1. Permanent growth

		s	θ	ρ	ω	β	λ	n	δ
k^*	S-S	+	0	0	0	0	-	-	-
	Extended	+	-	-	-	-	-	-	-
g^*	S-S	0	0	0	0	0	+	+	0
	Extended	+	+	+	+	+	+	+	-

Notes:
a. s = gross saving rate; θ = learning coefficient; ρ = the portion of aggregate income per man-hour (Y/L) spent on secondary and tertiary education and its effect on the number of graduates, and the latter's effect on the labor participation rate; ω = effect of real wage on labor participation; β = effects on labor participation of aging and retirements, changes in labor market policies and institutions, e.g., tax-benefits (tax credits and unemployment benefits), and a host of non-economic variables; λ = effect of rate of exogenous labor-augmenting productivity/technical change; n = rate of exogenous population growth; and δ = rate of depreciation; k^* = equilibrium capital intensity; g^* = equilibrium output or income growth.

b. The equilibrium capital intensity k^* is the root of $\dot{k}/k\ (e) = sk^{*(\alpha\text{-}1)} - \theta k^{*} - [\rho + \omega(1\text{-}\alpha)]\, k^{*\alpha} - (\lambda + n + \beta + \delta) = \psi(k^{*}; s, \theta, \rho, \omega, \beta, \lambda, n, \delta) = 0$.

c. In the above table, for the extended model $\partial k^{*}/\partial s = -\psi'_s / \psi'_{k*} > 0$, since $\psi'_s > 0$, and $\psi'_{k*} < 0$. The equilibrium output growth rate is $g^* = sk^{*(\alpha\text{-}1)} - \delta = H(k^*; s, \delta)$, or $g^{*} = \theta k^* + [\rho + \omega(1\text{-}\alpha)]\, k^{*\alpha} + \lambda + n + \beta$ $= J(k^{*};\theta,\rho,\omega,\lambda,n,\beta)$. Taking partial derivatives, $\partial g^{*}/\partial s = H'_{k*}\, H'_s > 0$; $\partial g^{*}/\partial \delta = H'_{k*} - H'_{\theta} = H'_{k*} -1 < 0$; $\partial g^{*}/\partial \theta = J'_{k*}\, J'_{\theta*} > 0$. The same procedure was used to derive the signs of the other parameters in the above table.

2.2.1. Growth effects of higher saving rate

Figure 2 shows the effects of an increase in the saving rate on equilibrium capital intensity and equilibrium output growth in the S-S and extended models. The starting equilibrium positions are points $B(k_s^*, g_s^*)$ for the S-S model and $A(k_e^*, g_e^*)$ for the extended model. A higher saving rate shifts the warranted rate line to the right in either model. The new equilibrium positions are indicated by point *C* in the S-S model and point *D* in the extended model. In both models, the capital/labor ratio goes up, albeit the new ratio remains lower in the extended model than in the S-S model, owing to positive labor participation in the extended model. The key difference is that the new equilibrium output growth increases in the extended model but remains unchanged in the S-S model.

The short-run dynamics of the S-S model is taken up first, followed by that of the extended model. During the transition between equilibrium points *B* and *C*, the S-S output growth rate is momentarily higher than the natural rate g_s^* at point *E* because of a higher warranted rate.[23] As noted, Figure 2 repeats the lower panel of Figure 1 (see (17) and (21)). The capital/labor ratio begins to rise from k_s^* to $k_s^{*\prime}$, which slows the warranted rate. Since the natural rate is independent of the capital/labor ratio, only the warranted rate adjusts downward along the segment *EC*.[24] Over time, labor becomes a bottleneck, and the output growth rate slows to the constant natural rate $g_s^* = \lambda + n$ at *C*. At this point, the capital/labor ratio stops rising and stabilizes at a new and higher-level $k_s^{*\prime}$. Thus, the output growth rate effect of a higher saving rate is temporary, and a higher equilibrium capital/labor ratio is the only permanent effect.

In the extended model, following the increase in the saving rate, equilibrium shifts from *A* to *D*. At the starting position *A*, capital grows faster than labor (by the segment *AF*), and the capital/labor ratio rises from k_e^* to $k_e^{*\prime}$. The marginal and average products of capital fall, lowering the level of saving per unit of capital, thus slowing the warranted rate downward along the segment *FD*. On the other hand, the natural rate, instead of remaining constant as in the S-S model, rises because of enhanced learning by doing and higher labor participation associated with a rising capital/labor ratio.[25] This process continues until the warranted and natural rates are again equal via a continuous increase in capital intensity at the new long-run equilibrium *D*, at which point the warranted rate would have fallen to the new and higher value of the natural rate, equal to the new and higher equilibrium output growth rate $g_e^{*\prime}(> g_e^*)$. Thus, the higher output growth effect of a larger saving rate is both temporary (like in the S-S model) and permanent (unlike in the S-S model), the latter owing to the existence of endogenous learning by doing and endogenous labor participation, making the natural rate respond positively to an increase in capital intensity.

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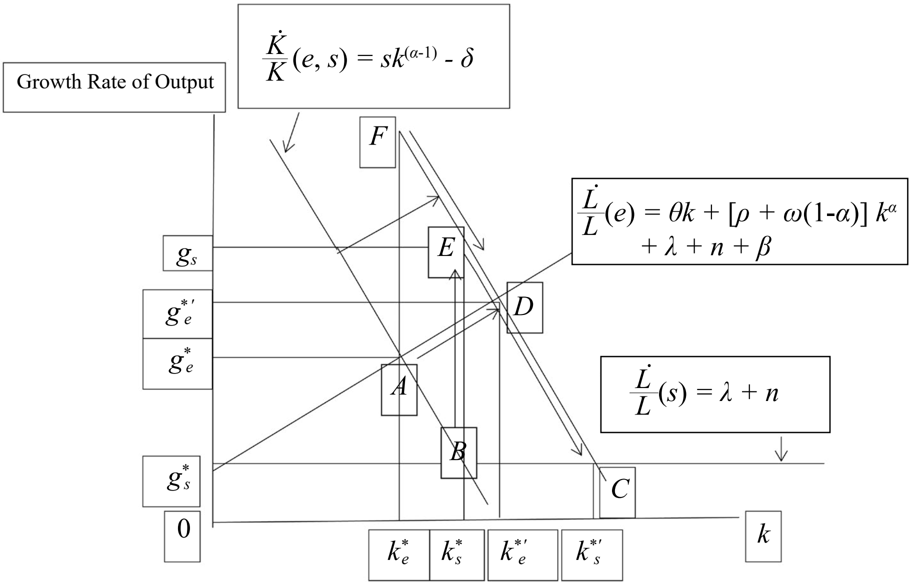
[23] The output growth rate at $E = \alpha g_s + (1\text{-}\alpha) g_s^*$.

[24] The output growth rate adjustment is traced by the segment *BEC* in terms of the weighted average of the warranted and natural rates.

[25] The natural rate adjustment is traced by the segment *AD*.

FIGURE 2. Temporary and permanent growth effects of higher saving rate



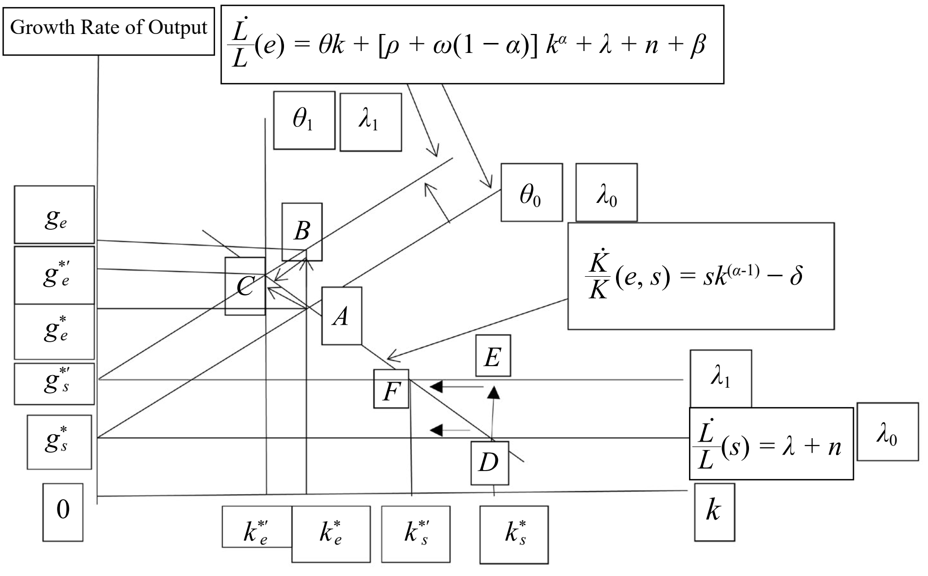
2.2.2. Growth effects of higher labor-augmenting productivity, enhanced learning by doing

Figure 3 illustrates the temporary and permanent growth effects of enhanced learning by doing and higher exogenous labor-augmenting labor productivity in the S-S and extended models. The growth effects of higher labor-augmenting productivity are taken up first, followed by the growth effects of enhanced learning by doing. The starting equilibrium positions are points $D(k_s^*, g_s^*)$ for the S-S model and $A(k_e^*, g_e^*)$ for the extended model. Higher exogenous labor-augmenting productivity λ shifts the natural rate of the S-S model upward to the $\dot{L}/L = \lambda_1 + n$ line, a parallel shift from the previous line. The extended model's natural rate shifts upward to the left. The new equilibrium positions are indicated by point *F* in the S-S model and point *C* in the extended model. In either model, the capital/labor ratio goes down, albeit the new ratio remains lower in the extended model than in the S-S model, owing to positive labor participation and learning by doing in the former. The key difference is that, while the new equilibrium output growth increases to the higher rate of $g_s^{*\prime} = \lambda_1 + n$ in the S-S model, in the extended model the new equilibrium output growth increases to an even higher rate equal to $g_e^{*\prime} = g_s^{*\prime} + \theta k_e^{\prime *} + [\rho + \omega(1\text{-}\alpha)]k_e^{*\prime}\, \alpha + \lambda_1 + n + \beta$ (point *C*).

The short-run (temporary) dynamics of the S-S model is taken up first, followed by that of the extended model. Note that at any moment of time the output growth rate is given by $\dot{Y}/Y = \lambda + n + \alpha\, \dot{k}/k$. Begin with enhanced learning by doing or higher labor-augmenting productivity. Before the steady-state transition between equilibrium points *D* and *F* begins, starting from k_s^* the output growth rate jumps to $\dot{Y}/Y = \alpha g_s^* + (1 - \alpha)\, g_s^{*\prime} = \lambda_1 + n + \alpha\, \dot{k}/k < g_s^{*\prime}$ because $\dot{k}/k < 0$ at k_s^*—the natural rate is higher than the warranted rate by segment *ED*. Capital intensity begins to fall from k_s^* to $k_s^{*\prime}$, resulting in $\dot{k}/k$ turning less and less negative, thus raising output growth until $\dot{k}/k = 0$ at point *F*, wherein the new permanently higher S-S growth rate $\lambda_1 + n$ is reached.

In the extended model, following the increase in λ from λ_0 to λ_1, equilibrium shifts from *A* to *C*. At the starting position A, labor grows faster than capital, and the capital/labor ratio declines from k_e^* toward $k_e^{*\prime}$. The extended model's output growth rate adjustment is traced by the weighted average of segments *BC* and *AC*, as capital intensity falls from k_e^* to $k_e^{*\prime}$. The marginal and average products of capital rise, raising the level of saving per unit of capital, accelerating the warranted rate upward along the segment *AC*. On the other hand, the natural rate, instead of remaining constant at $\lambda_1 + n$ as in the S-S model, slows from *B* to *C* because of a lower labor participation rate associated with a declining capital/labor ratio. This process continues until the warranted and natural rates are again equal via a continuous fall in the capital/labor ratio at the new long-run equilibrium *C*, at which point the warranted rate would have risen to the new value of the natural rate, equal to the new and higher equilibrium output growth rate $g_e^{*\prime}(> g_e^*)$. Thus, whereas in the S-S model the higher output growth is temporary, it is permanent in the extended model because of the existence of endogenous learning by doing and endogenous labor participation.

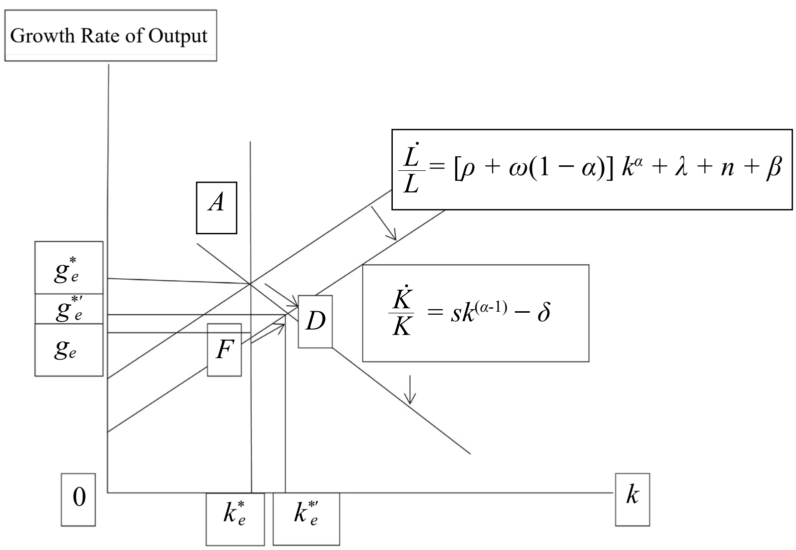
FIGURE 3. Temporary and permanent growth effects of enhanced learning by doing ($\theta_1 > \theta_0$) or higher exogenous labor-augmenting productivity ($\lambda_1 > \lambda_0$)



2.2.3. Growth effects of lower labor participation

Figure 4 illustrates the temporary and permanent growth effects of a decline in labor participation from the CBO [2018] forecast—a lower ρ, ω, or β. The initial equilibrium is at point A, with capital/labor ratio k_e^* and output growth g_e^*. Following the fall in labor participation, the natural rate line shifts downward to the right. Equilibrium shifts from A to D. The capital/labor ratio goes up from k_e^* to $k_e^{*\prime}$, and the equilibrium output growth goes down from g_e^* to $g_e^{*\prime}$. The increase in the equilibrium capital/labor ratio owes to lower effective labor induced by a lower rate of labor participation. The fall in equilibrium output growth is the result of a lower natural rate line along with an unchanged warranted rate line. Notice that there is temporary overshooting of the lower output growth rate at F (in relation to the new permanent growth rate at D). At the starting capital intensity k_e^* the natural rate has a precipitous drop to g_e ($< g_e^{*\prime}$) at F, following the decline in labor participation.[26] As the capital-labor ratio begins to rise from k_e^* to $k_e^{*\prime}$, the natural rate recovers along the segment FD, while the warranted rate falls along the segment AD because of diminishing marginal and average products of capital.

FIGURE 4. Temporary and permanent growth effects of a decline in labor participation (lower *ρ, ω, or β*)



[26] In Figure 4, before k has time to adjust, the output growth rate drops to g_e. Thus, a decline in labor participation results in a short-term contractionary overshooting (a sharp drop of short-run output growth, temporarily even lower than the permanently lower output growth at $g_e^{*\prime}$($< g_e^*$)).

This process continues until the warranted and natural rates are again equal via a continuous increase in the capital/labor ratio at the new long- run equilibrium *D*, where the permanent output growth $g_e^{*\prime}$ is lower than the initial prevent the decline in, and to encourage higher labor participation through vigorous implementation of public policies on education, on-the-job training, upgrading skills for a digital economy, real wage increases in line with labor productivity, and other labor market participation initiatives identified by Grigoli et al. [2018] and the CBO [2018].

3. Conclusion

This paper's main conclusion is that endogenous learning by doing and endogenous labor participation ensure a fully adjusting natural rate. Together with a fully adjusting warranted rate of the S-S model, the equilibrium growth rate of output and capital intensity are functions of all the structural parameters reflecting saving, learning by doing, and labor participation. There are temporary and permanent growth effects of increases in the saving rate and in the coefficient of learning by doing. A growth-oriented policy includes measures to raise public and private saving rates, as well as expenditures on education and health care aimed at raising labor productivity.

The CBO [2018] projected decline in the labor participation rate over the next decade will result in a permanently lower per capita output growth path, with temporary recessionary overshooting. Policies to restore the previous growth path or to achieve a permanently higher growth path involve avoiding the projected fall in labor participation by aggressive and calibrated spending on secondary and tertiary education, on-the-job training, and skills upgrade to a full-fledged digital economy (a higher ρ), steady increases in real wages in line with labor productivity (a higher ω), vigorous labor market participation activities and more generous tax-benefits (higher β), in order to offset the negative effects of aging and retirements (lower β).

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References

Agenor, P. [2004] *The economics of adjustment and growth*. Cambridge, MA: Harvard University Press.

Aghion, P. and P. Howitt [1998] *Endogenous growth theory*. Cambridge, MA: MIT Press.

Arrow, K. [1962] "The economic implications of learning by doing", *Review of Economic Studies* 29:155-173.

Barro, R. and X. Sala-i-Martin [1995] *Economic growth*. NY: McGraw-Hill.

Congressional Budget Office (CBO) [2018] "CBO's projection of labor force participation rates", *Working Paper No*. 2018-04.

Conlisk, J. [1967] "A modified neoclassical growth model with endogenous technical change", *Southern Economic Journal* 34:199-208.

Conlisk, J. [1968] "Non-constant returns to scale in a neoclassical growth model", *International Economic Review* 9:369-373.

Domar, E. [1946] "Capital expansion, rate of growth, and employment", *Econometrica* 14:137-147.

Grigoli, F., Z. Koczan, and P. Tapalova [2018] "Drivers of labor participation: macro and micro evidence", *International Monetary Fund Working Paper* WP/18/150. Washington, DC.

Grossman, G. and E. Helpman [1990] "Comparative advantage and long-run growth", *American Economic Review* 80:796–815.

Grossman, G. and E. Helpman [1991] *Innovation and growth in the global economy*. Cambridge, MA: MIT Press.

Harrod, R. [1939] "An essay in dynamic theory", *The Economic Journal* 49:14-33.

Inada, K-I. [1963] "On a two-sector model of economic growth: comments and generalization", *Review of Economic Studies* 30:119-127.

Lucas, R. [1988] "On the mechanics of economic development", *Journal of Monetary Economics* 22:3-42.

Nelson, R. and E. Phelps [1966] "Investment in humans, technological diffusion, and economic growth", *American Economic Review* 56:69-75.

Rebelo, S. [1991] "Long-run policy analysis and long-run growth", *Journal of Political Economy* 99:500-21.

Rivera-Batiz, L. and R. Romer [1991] "International trade with endogenous technical change", *NBER Working Paper* 3594. Washington: National Bureau of Economic Research.

Romer, P. [1986] "Increasing returns and long-run growth", *Journal of Political Economy* 94:1002-37.

Romer, P. [1990] "Endogenous technical change", *Journal of Political Economy* 98:s71-s102.

Solow, R. [1956] "A contribution to the theory of economic growth", *Quarterly Journal of Economics* 70:65-94.

Solow, R. [1991, November 12] *Policies for economic growth* [Ernest Sturc memorial lecture]. *SAIS*, Johns Hopkins University.

Solow, R. [2022]. *Nobel Prize outreach AB* [Nobel Prize lecture].

Swan, T. [1956] "Economic growth and capital accumulation", *Economic Record* 32:334-362.

Villanueva, D. [1994] "Openness, human development, and fiscal policies: effects on economic growth and speed of adjustment", *IMF Staff Papers* 41:1-29. Reprinted in D. Villanueva et al. [2023] "Chapter 6", *Economic adjustment and growth: theory and practice*. Singapore: World Scientific.

Villanueva, D. [2020] "A modified neoclassical growth model with endogenous labor participation", *Bulletin of Monetary Economics and Banking* 23(1):83-100. Reprinted in D. Villanueva et al. [2023] "Chapter 3", *Economic adjustment and growth: theory and practice*. Singapore: World Scientific.

Villanueva, D., R. Mariano, and C. Guinigundo [2023] *Economic adjustment and growth: theory and practice*. Singapore: World Scientific.

Warsh, D. [2007] *Knowledge and the wealth of nations: a story of economic discovery*. New York: WW Norton and Company.

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