Volume LXII No. 2
December 2025

p-ISSN 1655-1516
e-ISSN 2984-8156

The Philippine Review of Economics

ARTICLES IN THIS ISSUE

BOOK REVIEW

IN MEMORIAM

A joint publication of the
University of the Philippines
School of Economics
and the **Philippine Economic Society**

The Philippine Review of Economics

A joint publication of the UP School of Economics (UPSE)
and the Philippine Economic Society (PES)

Aims and Scope: *The Philippine Review of Economics* (PRE) invites theoretical and empirical articles on economics and economic development. Papers on the Philippines, Asian and other developing economies are especially welcome. Book reviews will also be considered.

The PRE is published jointly by the UP School of Economics and the Philippine Economic Society. Its contents are indexed in Scopus, the *Journal of Economic Literature*, EconLit, and RePec. PRE's readership includes economists and other social scientists in academe, business, government, and development research institutions.

Publication Information: The PRE (p-ISSN 1655-1516; e-ISSN 2984-8156) is a peer-reviewed journal published every June and December of each year. A searchable database of published articles and their abstracts is available at the PRE website (http://pre.econ.upd.edu.ph).

Subscription Information:
Subscription correspondence may be sent to the following addresses:

- css@pssc.org.ph and pes.eaea@gmail.com
- PSSC Central Subscription Service,
  PSSCenter, Commonwealth Avenue, 1101, Diliman,
  Quezon City, Philippines.
  2/F Philippine Social Science Center, Commonwealth
  Avenue, Diliman, Quezon City 1101
- PHONE: (02) 8929-2671, FAX: 8924-4178/8926-5179

Submissions: Authors may submit their manuscripts to the addresses below:

- pre.upd@up.edu.ph
- The Editor, The Philippine Review of Economics,
  School of Economics, University of the Philippines,
  Diliman, Quezon City, 1101.

Manuscripts must be written in English and in MS Word format. All graphs and tables must be in Excel format. Submission of a manuscript shall be understood by the PRE as indicating that the manuscript is not under consideration for publication in other journals. All submissions must include the title of the paper, author information, an abstract of no more than 150 words, and a list of three to four keywords. Complete guidelines can be viewed in the PRE's website.

Acknowledgments: The PRE gratefully acknowledges the financial support towards its publication provided by the Philippine Center for Economic Development (PCED). The *Review* nonetheless follows an independent editorial policy. The articles published reflect solely the editorial judgement of the editors and the views of their respective authors.

The Philippine Review of Economics

Vol. LXII No. 2
December 2025

p-ISSN 1655-1516
e-ISSN 2984-8156
DOI: 10.37907/ERP5202D

PRE The Philippine Review of Economics
62(2):125-129. DOI:10.37907/6ERP5202D

A stylized version of the Aghion-Howitt growth model

Delano S. Villanueva*

This short note introduces a compact, self-contained 3 × 3 reduced version of the Aghion-Howitt (AH, Schumpeterian, or creative destruction) model. This stylized reduction aims to capture the three core mechanisms in the AH intuition: (i) innovation (endogenous technological progress), (ii) capital accumulation or production, and (iii) allocation of labor to research & development (R&D) which responds to relative returns. This piece writes the three-equation system, explains every symbol, and outlines the steady-state or balanced-growth conditions as well as the Jacobian for local stability analysis. Developing and emerging economies such as the Philippines, where the R&D sector is actively growing, can benefit from understanding the AH model.

JEL classification: E27, O31, O410
Keywords: Aghion–Howitt model, endogenous growth, creative destruction, research and development, emerging economies

1. Introduction

After more than three decades, Philippe Aghion and Peter Howitt were awarded the Nobel Prize in Economic Sciences for their model of economic growth through creative destruction. Solow [1956] and Swan [1956] presented the neoclassical growth model with exogenous labor-augmenting technology. Conlisk [1967] modified this model with endogenous technical change, well before Romer [1990], Lucas [1988] and Villanueva [2023; 2021; 2020a; 2020b; 1994; 1971].[1]

This note introduces a compact, self-contained 3 × 3 reduced version of the Aghion-Howitt (AH, Schumpeterian, or creative destruction) model. It is a stylized reduction aimed at capturing the three core mechanisms in the AH intuition: (i) innovation (endogenous technological progress), (ii) capital accumulation or production, and (iii) allocation of labor to research & development (R&D)

* Former adviser, International Monetary Fund. Address all correspondence to dansvillanueva@gmail.com.

1 Romer made investments in research and development (R&D) endogenous variables, while Lucas considered investments in education endogenously determined. Villanueva [1971; 2021] incorporated Romer and Lucas in a productivity sector that includes new capital construction and made (i) labor-augmenting technical change endogenous via adopting Arrow's [1962] learning-by-doing [Villanueva 1994; 2020b; and 2023] and (ii) labor participation endogenous via real wage adjustments [Villanueva 2020a].

which responds to relative returns. This short piece writes the three-equation system, explains every symbol, and outlines the steady-state or balanced-growth conditions as well as the Jacobian for local stability analysis. Developing and emerging economies such as the Philippines, where the R&D sector is actively growing, can benefit from understanding the AH model.

2. The model

2.1. State variables

- $A(t)$: aggregate technology or "quality" level (scale variable)
- $k(t)$: capital per worker
- $s(t) \in [0,1]$: share of labor allocated to R&D (innovation intensity)

2.2. Parameters (all positive):

- $\alpha \in (0,1)$: capital share in production
- $\sigma \in (0,1)$: saving rate or the fraction of non-R&D output invested
- $\chi > 0$: productivity of R&D or how effective R&D is in producing technological growth
- $\delta \geq 0$: depreciation of capital
- $n \geq 0$: exogenous population and labor growth or dilution rate
- $\beta > 0$: speed of adjustment of the R&D labor share
- $\kappa > 0$: parameter compressing the production side return to a comparable unit

The functional form for output per worker is:

$$y(t)=A(t)k(t)^{\alpha}.$$

2.3. Dynamics (reduced system)

2.3.1. Technology (innovation)

$$\dot{A}(t) = \chi s(t)A(t) \tag{1}$$

Innovation increases the technology stock proportionally to current technology and the R&D share s. This is the simplest multiplicative specification consistent with endogenous (span-of-control style) growth.

2.3.2. Capital per worker

Assume a fraction $1-s$ of labor produces output and a constant fraction σ of that output is saved (invested). Then investment per worker is $\sigma(1-s)Ak^{\alpha}$. Capital per worker grows net of depreciation and dilution:

$$\dot{k}(t) = \sigma(1 - s(t))A(t)k(t)^{\alpha} - (\delta + n)k(t). \quad (2)$$

2.3.3. R&D labor share (selection/return adjustment)

It is assumed that s responds to the relative expected return to R&D versus production. In reduced form, a logistic or replicator adjustment captures this:

$$\dot{s}(t) = \beta s(t)(1 - s(t))(\chi - \kappa k(t)^{\alpha-1}). \quad (3)$$

2.4. Interpretation

- χ is the marginal return to R&D (per unit of s, expressed in the same units as the production return)
- $\kappa k(t)^{\alpha-1}$) is a reduced-form marginal return to allocating labor to production (marginal product of labor in production scaled into the same units). If χ exceeds that production return, s tends to rise; otherwise, it tends to fall. The logistic pre-factor $s(1 - s)$ pins s to the [0,1] interval and provides diminishing adjustment when s is near 0 or 1. As this is a convenient, standard, and reduced form, one can replace this with any other plausible adjustment rule.

3. Comments on the structure

- Equation 1 is the canonical multiplicative specification of AH style growth: growth in technology is driven by the intensity of R&D and is proportional to the current level.
- Equation 2 is standard capital accumulation with the twist that only the non-R&D fraction produces output.
- Equation 3 encodes profit or return incentives that move labor into or out of R&D. The specific form is a reduced form. A micro foundation would compute expected profits from R&D versus from production and derive a best-response s (or an Euler condition).

4. Balanced growth and steady states

Because A multiplies many terms, the model typically does not have a finite stationary A^* unless $s^* = 0$. Instead, we look for a balanced growth path (BGP) where A and k grow at the same constant rate g and s converges to a constant s^*. From (1):

$$\dot{A}/A = g = \chi s^*. \quad (4)$$

If k grows at the same rate g, then $\dot{k}/k = g$. From (2), dividing by k:

$$g = \sigma(1 - s^*)Ak^{\alpha\text{-}1} - (\delta + n).$$

However, on a BGP the level, $Ak^{\alpha\text{-}1}$ must be constant. Solve for $k^{\alpha\text{-}1}$ using (2), set $\dot{k}/k = g$:

$$g + (\delta + n) = \sigma(1 - s^*)Ak^{\alpha\text{-}1} \rightarrow k^{\alpha\text{-}1} = \frac{g + (\delta + n)}{\sigma(1 - s^*)A}. \qquad (5)$$

From (3), at an interior steady share $s^* \in (0,1)$,

$$0 = \chi - \kappa k^{\alpha\text{-}1} \rightarrow k^{\alpha\text{-}1} = \chi/\kappa. \qquad (6)$$

Combine (5) and (6) to eliminate $k^{\alpha\text{-}1}$:

$$\frac{\chi}{\kappa} = \frac{g + (\delta + n)}{\sigma(1 - s^*)A}.$$

Use (4) $g = \chi s^*$ to substitute:

$$\frac{\chi}{\kappa} = \frac{\chi s^* + (\delta + n)}{\sigma(1 - s^*)A}.$$

Solve for the ratio $A(1 - s^*)$ (or for s^* if desired). Rearranging gives one scalar relation between A and s^*; to get numerical s^*, normalise one scale (e.g., set $A = 1$ at one date or solve from microfoundations). The important message: interior s^* exists where marginal returns equalise (Eq. 6), and growth rate is $g = \chi s^*$.

Special corner solutions: $s^* = 0$ gives zero long-run innovation (then A constant) and the model reduces to Solow-style capital dynamics; $s^* = 1$ is extreme full R&D and production collapses. In the AH model, $0 < s^* < 1$.

5. Linearization/Jacobian (for local stability)

Write the vector $X = (A, k, s)$. The Jacobian J evaluated at a generic point A, k, s (compute partial derivatives of RHS of Equations 1 to 3):

$$J = \begin{bmatrix} \chi s & 0 & \chi A \\ \sigma(1 - s)\,\alpha A k^{\alpha\text{-}1} & \sigma(1 - s)\,A\alpha k^{\alpha\text{-}1} - (\delta + n) & -\sigma A k^{\alpha} \\ 0 & -\beta s(1 - s)\kappa(\alpha - 1)k^{\alpha\text{-}2} & \beta(1 - 2s)(\chi - \kappa k^{\alpha\text{-}1}) \end{bmatrix}$$

Given reasonable values of the elements of J, all eigenvalues of J have negative real parts so that equilibrium is locally asymptotically stable.

Acknowledgments: I am indebted to Professor Thorvaldur Gylfason for valuable comments.

References

Aghion, P. and P. Howitt [1992] "A model of growth through creative destruction", *Econometrica* 60:323-351.

Arrow, K. [1962] "The economic implications of learning by doing", *Review of Economic Studies* 29:155-173.

Conlisk, J. [1967] "A modified neoclassical growth model with endogenous technical change", *The Southern Economic Journal* 34:199-208.

Lucas, R. [1988] "On the mechanics of economic development", *Journal of Monetary Economics* 22:3-42.

Romer, P. [1990] "Endogenous technological change", *Journal of Political Economy* 98:S71-S102.

Solow, R. [1956] "A contribution to the theory of economic growth", *The Quarterly Journal of Economics* 70:65-94.

Swan, T. [1956] "Economic growth and capital accumulation", *Economic Record* 32:334-362.

Villanueva, D. [1971] "A note on Professor Fei's 'Per capita consumption and growth' ", *The Quarterly Journal of Economics* 75:704-709.

Villanueva, D. [1994] "Openness, human development and fiscal policies", *IMF Staff Papers* 41:1-29.

Villanueva, D. [2020a] "A modified neoclassical growth model with endogenous labor participation", *Bulletin of Monetary Economics and Banking* 23:83-100.

Villanueva, D. [2020b] "Optimal saving and sustainable foreign debt", *The Philippine Review of Economics* 57:170-199.

Villanueva, D. [2021] "Capital and growth", *Bulletin of Monetary Economics and Banking* 24:285-312.

Villanueva, D. [2023] "Toward a general neoclassical theory of economic growth", *The Philippine Review of Economics* 60:64-80.

The Philippine Economic Society

Founded 1961

The Philippine Economic Society (PES) was established in August 1962 as a nonstock, nonprofit professional organization of economists.

Over the years, the PES has served as one of the strongest networks of economists in the academe, government, and business sector.

Recognized in the international community of professional economic associations and a founding member of the Federation of ASEAN Economic Associations (FAEA), the PES continuously provides a venue for open and free discussions of a wide range of policy issues through its conference and symposia.

Through its journal, the *Philippine Review of Economics* (PRE), which is jointly published with the UP School of Economics, the Society performs a major role in improving the standard of economic research in the country and in disseminating new research findings.

At present, the Society enjoys the membership of some 500 economists and professionals from the academe, government, and private sector.

- Lifetime Membership – Any regular member who pays the lifetime membership dues shall be granted lifetime membership and shall have the rights, privileges, and responsibilities of a regular member, except for the payment of the annual dues.
- Regular Membership – Limited to individuals 21 years of age or older who have obtained at least a bachelor's degree in economics, or who, in the opinion of the Board of Directors, have shown sufficient familiarity and understanding of the science of economics to warrant admission to the Society. Candidates who have been accepted shall become members of the Society only upon payment of the annual dues for the current year.
- Student Membership – This is reserved for graduate students majoring in economics.

For more information, visit: economicsph.org.