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EXCHANGE RATE DYNAMICS UNDER ALTERNATIVE INTERVENTION RULES

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This paper examines the effects of two alternative intervention rules, the real exchange rate rule and the nominal exchange rate rule, on the behavior of a small open economy. These rules are found to differ in terms of the resulting exchange-rate jumps, deviations from purchasing power parity, and relationships between interest rate and exchange rate movements. Thus, even though these rules have the same steady-state effects, in the short run it makes a difference as to which rule is pursued since they lead to different exchange-rate jumps and hence to different paths that the economy will follow.

1. Introduction

Recent papers on exchange rates have incorporated short-run intervention rules into models assuming rational expectations. However, these papers usually focus on a particular rule, either the policy of moderating the nominal exchange rate or the policy of moderating the real exchange rate.[1] The objective of this paper is to analyze the dynamic behavior of a small economy under each of the two intervention rules.[2] Using a modified Dornbusch (1976) model characterized by imperfect asset substitution and full sterilization, we show that these rules differ in terms of the resulting exchange-rate jumps, deviations from purchasing power

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[1]Intervention may also be classified as contemporaneous (as in this paper) or noncontemporaneous (as in Bludell-Wignall and Masson (1985) where it becomes one of the sources of dynamics). Another distinction is, of course, between sterilized and nonsterilized intervention; for the general case of partially-sterilized intervention and the special cases of fully-sterilized and non-sterilized interventions under a real exchange rate rule, see Natividad and Stone (1990).

[2]Mussa (1985) also analyzes the effects of these two alternative intervention rules using a modified Dornbusch model characterized by perfect asset substitution, nonsterilization, and moving equilibrium (due to price adjustment being a function of deviation from anticipated changes in purchasing power parity).

parity, and relationships between interest rate and exchange rate movements. Thus, even though these two rules have the same steady-state effects, in the short run it makes a difference as to which policy is pursued since these rules lead to different exchange-rate jumps and hence to different paths that the economy will follow. The choice therefore between the two rules is crucial because they have different implications for the dynamics of the model.

2. The Model

The model is a slightly modified version of the Dornbusch (1976) model and is described by:

(1.1) $y = y_o + \gamma y - \sigma i + \delta(e\text{-}p\text{+}p_f)$

(1.2) $dp/dt = \pi(y - y^*)$

(1.3) $m - p = \Phi y - \beta i$

(1.4) $i = i_f + E(de/dt) - (1/\Phi)(f_o + f_1 nfa - f_2 r)$

(1.5) $E(de/dt) = de/dt$

where y = log of income or output; i, i_f = domestic and foreign interest rates; e = log of nominal exchange rate measured in units of domestic currency per unit of foreign currency; p, p_f = logs of domestic and foreign price levels; $e\text{-}p\text{+}p_f$ = log of real exchange rate; m = log of money supply; r = log of reserves; nfa = log of net foreign assets; $f_o + f_1 nfa - f_2 r$ = log of net private foreign assets; and, * denotes a long-run equilibrium value. All parameters are positive, and $0 < \gamma < 1$.

Except for (1.4), the model is the same as that as that of Dornbusch. The goods market is described by (1.1) and (1.2), and the money market by (1.3). Equation (1.4) is the foreign exchange market equilibrium condition. It embodies the assumption of perfect capital mobility so that the return on domestic assets, i, always equals the net return on foreign assets, $i_f + E(de/dt) - (1/\Phi)(f_o + f_1 nfa - f_2 r)$, as well as the assumption of imperfect substitution between domestic and foreign assets, as shown by the risk-premium term.[3] Equation (1.5) is the perfect foresight assumption that the expected and actual exchange-rate changes over time are equal.

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[3]Equation (1.4) is derived from an inverted net private foreign asset demand function, $f^d = -\Phi(i - i_f - E(de/dt))$, and an equilibrium condition, $f^d = f$, where $f(f^d)$ is the net private foreign asset supply demand (see Frankel, 1983). It can also be derived using either the mean-variance approach (Black, 1985) or an optimization model (Turnovsky, 1985). To be able to incorporate intervention, we have used the identity $f = f_o + f_1 nfa - f_2 r$.

We complete the model with the specification of two alternative policy functions which have the same intervention parameter u:

(1.6) $r = r_o - u\,((e\text{-}p\text{+}p_f) - (e^*\text{-}p^*\text{+}p_f))$
(1.7) $r = r_o - u\,(e\text{-}e^*)$

where $0 < u < \infty$. The terms $-u((e\text{-}p\text{+}p_f) - (e^*\text{-}p^*\text{-}p_f))$ and $-u(e\text{-}e^*)$ represent the short-run intervention rules of changing reserves in response to deviation of the real exchange rate and the nominal exchange rate, respectively, from their long-run equilibrium values. We assume full-sterilization so that the money supply is exogenous and cannot be affected by changes in reserves.[4] Note that when asset substitution is perfect, $\Phi = \infty$, and the model reduces to the Dornbusch model where short-run intervention cannot affect the dynamics of the system.

Under each rule, the steady-state is attained when $E(de/dt) = de/dt) = 0$ and $dp/dt = 0$ and is described by:

(2.1) $e^* = p^* - p_f + (\sigma/\delta)i^* - (1/\delta)\,y_o + ((1\text{-}\gamma)/\delta)y^*$
(2.2) $p^* = m - \phi y^* + \beta i^*$
(2.3) $i^* = i_f - (1/\Phi)\,(f_o + f_1 nfa - f_2 r^*)$
(2.4) $r^* = r_o$

where y^* is fixed at the natural level. Thus, the steady-state is invariant with respect to the type of intervention rule.

In the long run, the system is neutral with respect to a change in the money supply in the sense that $de^* = dp^* = dm$ and hence $d(e^*\text{-}p^*\text{+}p_f) = 0$. However, it is nonhomogeneous with respect to shocks that affect the long-run equilibrium interest rate, such as changes in i_f, in the sense that such shocks yield $de^* > dp^*$ and therefore $d(e^*\text{-}p^*\text{+}p_f)$ is nonzero.

3. Real Exchange Rate Rule

The dynamics of the model ((1.1) to (1.5) and (1.6)) under the real exchange rate rule (RR rule hereafter) can be described by the short-run static equations:

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[4]The money supply function is assumed to have the following form: $m = m_1 c + m_2 r$ where the domestic credit function is given by $c = c_o - c_1 r$ (see Makin, 1981). For full sterilization to occur, $(m_2 - m_1 c_1)$ must equal zero.

(3.1) $i^{RR} - i^* = (\varnothing\delta / V)(e^{RR} - e^*) + (((1-\gamma) - \varnothing\delta)/ V)(p^{RR} - p^*)$

(3.2) $y^{RR} - y^* = (\beta\delta / V)(e^{RR} - e^*) - ((\sigma + \beta\delta)/V)(p^{RR} - p^*)$

and the dynamic matrix equation:

$$(3.3) \quad \begin{bmatrix} de^{RR}/dt \\ dp^{RR}/dt \end{bmatrix} = \begin{bmatrix} a_{11} & - a_{12} \\ a_{21} & - a_{22} \end{bmatrix} \begin{bmatrix} (e^{RR} - e^*) \\ (p^{RR} - p^*) \end{bmatrix}$$

where

$a_{11} = (\varnothing\delta + VFu)/V > 0 \qquad a_{21} = \pi\beta\delta/V > 0$

$a_{12} = ((1-\gamma) - \varnothing\delta - VFu)/V \qquad a_{22} = \pi(\sigma+\beta\delta)V < 0$

$V = \phi\sigma + (1-\gamma)\beta > 0 \qquad F = (1/\Phi)f_2 > 0$

and superscript "*RR*" denotes a variable under the RR rule.[5] Since the determinant ($\det(A) = R_1R_2 = a_{11}a_{22} - a_{21}a_{12} = -\pi(\sigma Fu + \delta)/V$) of the matrix in (3.3) is unambiguously negative, whatever the sign of the trace ($tr(A) = R_1 + R_2 = a_{11} + a_{22}$ the two roots:

(4) $R_1, R_2 \; [tr(A) \pm ((-tr(A))^2 - 4(det\,(A)))^{1/2}]/2$

are real and opposite in sign ($R_1 < 0, R_2 > 0$), implying that the steady-state is a saddlepoint.

Starting from an initial steady-state, we now consider an increase in the money supply. Since the price level is sticky, the exchange rate must first jump to place the system on the path converging toward the new steady-state where $e = e^*$ and $p = p^*$. It can be shown that the stabilizing exchange-rate jump is:

(5.1) $de^{RR}(0)/dm = de^*/dm - [a_{12}/a_{11} - R_1)]d(p^{RR}(0) - p^*/dm > 0$

where $-d(p^{RR}(0) - p^*)/dm = dp^*/dm = de^*/dm = 1$ and $-a_{12}/(a_{11} - R_1)$ is the slope of the convergent arm.[6] The other impact effects, using (5.1), (3.1), (3.2) and (1.6), are:

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[5]Equations (3.1) and (3.2) are derived by expressing (1.1) and (1.3) in deviation forms and then solving them simultaneously for (i^{RR}-i^*) and (y^{RR}-y^*). The de/dt equation in (3.3) is derived by substituting (1.6), (3.1), and (1.5) into (1.4) and then expressing the resulting equation in deviation form while the dp/dt equation is derived by substituting (3.2) into (1.2).

[6]The exchange rate path is given by: $e^{RR}(t) = e^* + b_{11}K_1 exp\,(R_1 t) + b_{12}K_2 exp\,(R_2 t)$, where $b_{ij} = 1$ and K_j is some constant. The condition that the coefficient associated with the positive root must equal zero ($K_2 = c_{21}(e^{RR}(0) - e^*) + c_{22}(p^{RR}(0) - p^*) = 0$, where $c_{21} = (a_{11} - R_1)/(R_2 - R_1$ and $c_{22} = a_{12}/(R_2 - R_1)$) yields (5.1).

$$(5.2) \quad d(i^{RR}(0) - i^*)/dm = [-(1-\gamma) + \phi\delta(1 + a_{12}/(a_{11}-R_1))]/V \lesseqgtr 0$$
$$(5.3) \quad d(y^{RR}(0) - y^*)/dm = [\sigma + \beta\delta(1 + a_{12}/(a_{11} - R_1))]/V > 0$$
$$(5.4) \quad d(r^{RR}(0) - r^*)/dm = -u[d((e^{RR}(0) - p^{RR}(0) + p_f) - (e^* - p^* + p_f))/dm] = -u(1 + a_{12}/(a_{11}-R_1)) < 0$$

where $di^*/dm = dy^*/dm = dr^*/dm = 0$.

Equation (5.1) and, in particular, a_{12} determine the sign of the slope of the convergent arm and therefore whether the exchange rate will exhibit overshooting or undershooting. Specifically, if at the initial exchange rates and short-run price level the change in the short-run domestic interest rate is less (greater) than the change in the net return on foreign assets, then $a_{12} > 0$ (< 0) and the exchange rate will have to overshoot (undershoot) its new long-run equilibrium value, i.e., $e^{RR}(0) >$ ($<$) e^*, so as to maintain equilibrium in both the money and the foreign exchange markets.[7] In the borderline case where these changes are equal, $a_{12} = 0$ and $e^{RR}(0) = e^*$: there is neither overshooting nor undershooting.

After all adjustments have been made, it can be seen from (5.2) to (5.4) that at $t = 0$, income rises, reserves fall, and the domestic interest may fall, remain the same, or rise. Following the jump, the system moves along the stable path and along this path:

$$(6) \quad de^{RR}/dt = R_1(e^{RR}(t) - e^*) \lesseqgtr 0$$

where $-R_1$ is the system's speed of adjustment under the RR rule.[8] Thus, if $a_{12} \lesseqgtr 0$, then $e^{RR}(t) \lesseqgtr e^*$ and $de^{RR}/dt \lesseqgtr 0$.

We now examine the effects of an increase in the size of intervention parameter. Given that the shock is in the form of monetary expansion, then:

$$(7) \quad d(de^{RR}(0)/dm)/du = d[d(e^{RR}(0) - e^*)/dm]/du = d[d((e^{RR}(0) - p^{RR}(0) + p_f) - (e^* - p^* + p_f))/dm]/du = -[F/(a_{11} - R_1)][1 + (a_{12}/(a_{11} - R_1))(1/2)(1 + \Omega)] < 0$$

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[7]At the initial short-run domestic interest rate and price level, the change in domestic interest rate equals $-((1-\gamma) - \phi\delta)/V$ (see 3.1)) while the change in the net return on foreign assets equals $-Fu$ (see (1.4), (1.5) and (1.6)).

[8]We get (6) by differentiating the $e^{RR}(t)$ in note 6 with respect to time and noting that $e^{RR}(0) - e^* = (e^{RR}(t) - e^*)/\exp(R_1 t)$.

where

$$0 < \Omega = (tr(A) + 2\pi\sigma/V))/ \\ ((-tr\ (A) + 2\pi\sigma/V\ ))^2 + 4a_{21}\ ((1-\gamma) + \pi\sigma)/V\ ))^{1/2} < 1$$

and $d(de^*/dm)/du = d(dp^*/dm)/du = 0$ and $dp^{RR}(0)/dm = dp_f/dm = 0.$

Equation (7) shows that, when $a_{12} > 0$ (i.e., when the response would have been overshooting), a stronger RR rule reduces all of the following: the extent of short-run nominal depreciation, the extent of real exchange rate overshooting, and the absolute deviation of the nominal exchange rate from its equilibrium value. However, when $a_{12} < 0$ (i.e., when the response would have been undershooting), a stronger RR rule reduces the extent of short-run depreciation and the extent of real exchange rate overshooting but increases the absolute deviation of the nominal exchange rate from its equilibrium value.

4. Nominal Exchange Rate Rule

The steady-state of the model ((1.1) to (1.5) and (1.6') under the nominal exchange rate rule (NR rule hereafter) is also a saddlepoint. It can be shown that under this rule the impact effects of an increase in the money supply are:

$$\begin{aligned}
&(8.1) \quad & d(e^{NR}(0) - e^*)/dm &= a_{12}'/(a_{11} - R_1') \\
&& &= d(e^{RR}(0) - e^*)/dm + \varepsilon \gtrless 0 \\
&(8.2) & d(i^{NR}(0) - i^*)/dm &= (R_1' - Fu)d(e^{NR}(0) - e^*)/dm \\
&& &= d(i^{RR}(0) - i^*)/dm + \phi\delta\varepsilon \gtrless 0 \\
&(8.3) & d(y^{NR}(0) - y^*)/dm &= d(y^{RR}(0) - y^*)/dm + \beta\delta\varepsilon > 0 \\
&(8.4) & d(r^{NR}(0) - r^*)/dm &= -u[d(e^{NR}(0) - e^*)/dm] \\
&& &= -u(a_{12}'/(a_{11} - R_1')) \gtrless 0
\end{aligned}$$

where

$$\begin{aligned}
a_{12}' &= (1-\gamma) - \phi\delta = a_{12} + Fu \\
R_1' &= [(a_{11} + a_{22}) - ((-(a_{11} + a_{22}))^2 - 4\,(a_{11}a_{22} - a_{21}a_{12}'))^{1/2}]/2 < 0 \\
\varepsilon &= [(F/(a_{11} - R_1)][1 - (a_{12}/Fu)((R_1 - R_1'))/(a_{11} - R_1))] > 0
\end{aligned}$$

and $-R_1'$ (which is greater than $-R_1$ is the system's speed of adjustment under the NR rule, $-a_{12}'/(a_{11}-R_1')$ is the slope of the convergent arm, superscript NR denotes a variable under the NR rule and, again, $-\ d(p^{NR}(0)-p^*)/dm = dp^*/dm = de^*/dm = 1$ and $di^*/dm = dy^*/dm = dr^*/dm = 0$.[9]

Equation (8.1) shows that the condition for overshooting (undershooting) is $a_{12}' > 0$ (< 0) and that the borderline case of $e^{NR}(0) - e^* = 0$ occurs when $a_{12}' = 0$. Notice that $d(e^{NR}(0)-e^*)/dm > d(e^{RR}(0) - e^*)/dm$ and, since the steady-state effects are the same under both rules, it follows that $de^{NR}(0)/dm > de^{RR}(0)\, dm > 0$. This is so because, at the initial exchange rates and short-run price level, the change in the return on domestic assets (equals - $((1-\gamma)-\phi\delta)/V$ which may be negative, zero or positive) is the same under both rules (see ((1.1) and (1.3)) while the change in the net return on foreign assets is zero under the NR rule but negative (equals - Fu) under the RR rule (see (1.4), 1.5), (1.6) and (1.6')); thus, the short-run depreciation needed to equilibrate the asset markets must be larger under the NR rule.[10] Since the extent of undershooting (overshooting) is smaller (larger) under the NR rule, it follows that the transitional depreciation (appreciation) is smaller (larger in absolute value or sharper) under this rule, *i.e.*, $de^{NR}/dt < de^{NR}/dt$.

Notice also from (8.2) that $d(e^{NR}(0)-e^*)/dm$ and $d(i^{NR}(0)-i^*)/dm$ are always opposite in sign, since $R_1' - Fu) < 0$. Under the NR rule, $d(e^{NR}(0)-e^*)/dm$ is only dampened by intervention; in contrast, under the RR rule, the sign of $d(e^{RR}(0) - e^*)/dm$ may be reversed so that there is no longer an inverse relationship between $d(i^{RR}(0) -i^*)$ and $d(e^{RR}(0) -e^*)$.

Under both rules, monetary expansion results in real exchange rate overshooting. However, the extent of this overshooting is greater under the NR rule: $d((e^{NR}(0) - p^{NR}(0)+p_f - (e^*-p^*+p_f))/dm > d((e^{RR}(0)-p^{RR}(0)+p_f - (e^*p^*+p_f))/dm > 0$; since such overshooting has a positive effect on income, then, $d(y^{NR}(0) - y^*)/dm > d(y^{RR}(0) - y^*)/dm > 0$ (see(8.3)). Equation (8.4) shows that $d(r^{NR}(0)-r^*)/dm \ - \gtreqless\ 0$ as $a_{12} \gtreqless 0$ or as $e^{NR}(0) - e^* \gtreqless e^*$, whereas (5.4) shows that $d(r^{RR}(0) - \gtreqless r\,)/dm < 0$ since there is always real exchange rate shooting. This implies that $d(r^{NR}(0)-r^*)/dm > d(r^{RR}(0) - r^*)/dm$.

[9]The derivations of (8.1) to (8.4) and of (9) below are similar to those under the RR rule.

[10]At the initial exchange rates and short-run level, the change in the return on domestic assets under the NR rule (see (1.1) and (1.3)) is the same as that under the RR rule while the change in the net return on foreign assets is zero (see (1.4), (1.5) and (1.6')).

The effect of an increase in u under this rule is given by:

$$(9) \quad d(de^{NR}(0)/dm)/du = d[d(e^{NR}(0) - e^*)/dm]/du$$
$$= d[d((e^{NR}(0) - p^{NR}(0) + p_f - (e^* - p^* + p_f))/dm]\, du$$
$$= -(F/2)\,(a_{12}'/(a_{11} - R_1)^2)\,(1 + \Omega') \gtrless 0 \text{ as } a_{12} \gtrless 0$$

where

$$\Omega' = ((a_{11} - a_{22})/((-a_{11} - a_{22})^2 + 4a_{21}a_{12}'))^{1/2} \gtrless 0 \text{ as } a_{12}' \gtrless 0$$

Equations (9) and (7) show that stronger NR and RR rules have the same (opposite) effects when $a_{12}' > a_{12} > 0$ ($a_{12} < a_{12}' < 0$). Thus, the effect of an intervention rule on the stability of the system depends on whether the exchange rate response would have been overshooting or undershooting. Mussa (1985), on the other hand, shows that the overall effect of an intervention policy on the "stability" of the system depends on the relative importance of different disturbances (internal and external, monetary or real) as sources of "instability."

Finally, these rules also differ in the sense that u does not affect a_{12}' but affects a_{12}, implying that the NR (RR) rule may only dampen (may not only dampen but may also reverse) the slope of the convergent arm. This implies that the NR rule which is essentially a policy of leaning against the wind may only dampen the movement of the nominal exchange rate. In contrast, under the RR rule u can be set equal to $((1 - \gamma - \phi\delta)/\text{VF}$ so that a12 = 0 (there is neitherovershooting nor undershooting) or it can even be set such that $a_{12} < 0$ (there is undershooting).

5. Conclusion

This paper has studied the effects of adopting two alternative intervention rules using a model characterized by imperfect asset substitution, full sterilization and stationary long-run equilibrium. We have considered a shock in the form of an increase in the money supply and have shown that the NR rule results in greater (though not necessarily in absolute value) initial deviations and, therefore, a faster speed of transitional adjustment ($-R_1' > R_1$) than the RR rule.[11]

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[11]In particular, $d(e^{NR}(0)-e^*)/dm > d(e^{RR}(0)-e^*)/dm$, $d(i^{NR}(0)-i^*)/dm > d(i^{RR})(0)-i^*)/dm$, $d(y^{NR}(0)-y^*)/dm > d(y^{RR}(0)-y^*)/dm$, and $d(r^{NR}(0)-r^*)/dm > d(r^{RR}(0)-r^*/dm$. These and the other results also hold for any shock (internal or external) that raises the equilibrium price level; the only difference is that an external shock such as a change in i_f affects i^* and therefore $(e^*-p^*+p_f)$.

Specifically, we have shown that: (1) short-run depreciation is larger, overshooting (undershooting) is greater (smaller in absolute value) and purchasing-power-parity deviation is larger under the NR rule than under the RR rule; (2) interest-rate and exchange-rate deviations are always opposite in signs under the NR rule but not necessarily so under the RR rule; and, (3) the RR (NR) rule may reverse (may only dampen) the slope of the convergent arm. We have also examined the effects of stronger intervention rules and have shown that a stronger RR (NR) rule: (1) dampens overshooting but reinforces undershooting (moderates both overshooting and undershooting) and (2) always reduces (may reduce or increase) the deviation from purchasing power parity. These results indicate that the choice between the two rules is important for theoretical and empirical applications.

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