THE PHILIPPINE REVIEW OF ECONOMICS
VOL. XLVI NO. 1 JUNE 2009 PP. 87-102

Optimal cash-in-advance contracts under weak third-party enforcement

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We explore the implications of North's weak third-party enforcement (TPE) on the structure of the ex ante incentives-compatible contracts that require an advance payment by the principal to the agent. This generates appropriable quasi-rent, which the agent can appropriate. To deal with this, we introduce a stronger constraint, the augmented participation constraint, reflecting the quality of TPE that prompts a distinction between insider reservation and outsider reservation utility. We show that a falling TPE raises the agent's insider reservation utility, reducing the principal's profit and his willingness to contract. When the cash-in-advance commitment is endogenous, its optimal level falls as TPE falls. TPE erosion thus leads to either the nonexistence of or the flight from more productive contracts and exchange, leading to North's observation of poorer economic performance.

JEL classification: 022, 026, 611
Keywords: Cash-in-advance contracts, agent opportunism, weak TPE, flight to spot contracts

1. Introduction

Many contracts in the real world involve some form of advance payment made good either in cash (e.g., linked credit-and-output contracts or even contract killing) or in kind (e.g., training cum employment). The principal P pays a percentage of the agreed fee with the expectation that the agent A will perform a service at some later date. When the agreed outcome is observed, the rest of the fee is made good and the contract terminates. We call this contract

*The author thanks Douglas Marcouiller and Emmanuel de Dios for their comments. He also thanks the Philippine Center for Economic Development and the Centre for Regulation and Competition, Manchester University, for financial support. Errors are the author's alone.

"cash-in-advance". Typical is the Shapiro and Stiglitz [1984] agency game. Under its wing also come all contracts involving a promise to repay at some future date in currency other than effort or service. Thus, a loan contract is also a cash-in-advance contract; so are all exchanges characterized by "separation of the quid and the quo" according to Greif [2001] who illuminated contracting in the Middle Ages when third-party enforcement (TPE) was clearly weak.

This contract must confront the very real possibility of ex post opportunism, i.e., the agent absconds with the cash advance and does not supply the service (and a priori the outcome). The cash advance falls into the category of appropriable quasi-rent [Klein, Crawford, and Alchian 1978] and attracts ex post opportunism [Williamson 1971, 1975]. Either of two results follows:

(i) *P* reports the theft to the authorities and the absconder is promptly apprehended and adequately punished so that the absconding, in the end, does not pay; this strong outside enforcement undergirds the textbook principal-agent contract.

(ii) *P* reports the theft but the authorities are too weak to apprehend and/or properly punish the absconder. This is an example of North's [1990] weak third-party enforcement.

In (ii), the temptation to abscond is real and the incentives-compatible contract must reflect this. In other words, the contract must factor in a homemade property rights protection. Here, we explore "bribe" as a mechanism.

Where third-party property-rights protection is weak, agents have had to craft "second party" devices or "private ordering" as Williamson [1983] called it. The Maghribi system [Greif 1993] served as the enforcement infrastructure for Mediterranean trade in the 12th and 13th centuries. Merchant law and community responsibility system supported cross-border trade in Medieval Europe [Greif, Milgrom, and Weingast 1994; Greif 2001]. They were costly but served the purpose of supporting long-distance exchange. In East Asia, the guanxi (relational contracting) supported trade under difficult circumstances [The Economist 2001]. Williamson [1983] proposed the "hostage model" as a "private ordering".

The literature on contracts appears largely silent on weak TPE issue apart from Laffont and Mortimart [2002] and Laffont and Meleu [2000]. Typically, the obeisance to the governance issue takes the form of the initial assumption that an outside court will properly and promptly prosecute and punish contract violators and, thus, violations do pay not ex ante [Laffont and Mortimart 2002]. This leads to the insistence that contracts be based on verifiable obligations that allow the contracts to be enforceable by an outside arbitrator or court of law

[Laffont and Mortimart 2002; Macho-Stadler and Perez-Castrillo 1997; Stolle 1998]. The implication is that the environment is characterized by a strong TPE, which deters possible reneging by the principal. The "incentives compatibility constraint" (ICC) addresses reneging by the agent.

When TPE is absent as in the Shapiro and Stiglitz [1984] agency game, the supergame version with a trigger mechanism sustains an exchange contract. This is the usual bilateral private ordering mechanism. Cooter [1996] suggests other private ordering mechanisms apart from repeat transactions, such as reputation or contract design.

Laffont and Mortimart [2002] and Laffont and Meleu [2000] broke new ground by studying the contract design implication of possible agent reneging in an adverse selection principal-agent model. Since agent misrepresentation in their single-period situation is the target of an incentives-compatibility constraint, the problem arises only when a negative payoff (which cannot be fully enforced) is called for in case of a bad outcome. In lieu of the usual ICC, they introduce a strategy-proofness constraint, which makes enforcement quality irrelevant. We think that possible agent reneging impact on contract design is best studied in the Shapiro-Stiglitz two-period environment.

This paper investigates the impact of weak TPE on the design of the optimal cash-in-advance contract. In section 2, we formulate the decision problem facing the principal, on the one hand, and the agent on the other, and formally define the weak TPE environment. We introduce the augmented participation constraint (APC), which features a "bribe". We show that the ordinary principal-agent framework is a nested subcase of the present model when TPE is strong. This forces the contract to incorporate a property-rights protection feature. In contrast to Williamson's [1983] model where the opportunism-advantaged party (agent) offers a hostage as credible commitment, here, the opportunism-disadvantaged party (principal) offers a bribe. In section 3, we characterize the exogenous cash-in-advance contract and show how, without additional instruments, the agent is strictly better off and the principal worse off under weak than under strong TPE, and that an improvement in the governance efficiency worsens the position of the agent even as it improves that of P. In section 3, we endogenize the cash-advance commitment and show that a flight from more productive contracts results from the erosion of TPE. In section 4, we give a summary.

2. The model

2.1. The exogenous cash-in-advance contracts

Consider an activity where a good or a service results from a production contract between a principal P and an agent A. The contract requires some cash-in-advance $w_1 = bw, 0 \leq b \leq 1,$ as initial payment in period 1, where w is the agreed-upon wage. In period 2, A agrees to supply effort e (which we assume costlessly observable to isolate the governance problem from moral hazard), which produces the output also in period 2. P sells the output in the competitive market and makes the final payment $w_2 = (1-b)w$ to A. A, however, may decide to abscond with the initial payment and instead supply e to his next-best alternative activity to realize U^0, his outsider reservation utility. This ex post opportunism makes for a peculiarity in the contractual relation. We deal first with *exogenous cash-in-advance contracts*, i.e., where the fraction b of the wage paid in advance is fixed.

2.2. The decision problem for A

A enters the contract if his reservation utility U^0 is met, i.e., if the sum of utility $u(w_1)$ of the advance payment w_1 and the utility $u(w_2)$ of the subsequent payment w_2 allows him to better or equal his reservation U^0. We will call U^0 his outsider reservation utility. We assume $u(w)$ to be strictly concave and twice differentiable. Eschewing discounting for simplicity, we have

$$u(w_1) + u(w_2) - v(e) \geq U^0. \tag{1}$$

This is the participation constraint (PC). However, due to the cash advance w_1, A has the derivative option as an insider to abscond, i.e., supply $e = 0$ to the project and still realize U^0 in his next best alternative. This is ex post opportunism on the part of A and violates P's claim over the cash advance.

Let Q be the probability that A is caught and punished if he absconds. Let L be the penalty attached to being caught. A's period 2 expected utility is $U^0 - QL$. This is in addition to $u(w_1)$ that A enjoys in period 1. Q is the index of third party enforcement efficiency. Thus, A will abide (deliver e) for as long as the contract $C(w,e)$ satisfies:

$$u(w_1) + u(w_2) - v(e) \geq U^0 + u(w_1) - QL. \tag{2}$$

We call "*b*" the cash advance commitment of the contract. Inequality (2) insures that *A* delivers contract obligation *e* in period 2 after receiving w_1 in period 1. Notice that if $\left[u(w_1)-QL\right]>0$, then satisfying (2) also satisfies the PC in (1). This inequality (2) must always hold if ex post opportunism is to be deterred in the environment of interest to us, i.e., where TPE is weak or *QL* is low. If *QL* is large enough, the inequality reverses, and satisfying PC in (1) also satisfies (2) which becomes irrelevant. The model nests the standard two-period P-A model.

Definition 1: We call (2) the augmented participation constraint.

Definition 2: A weak TPE environment exists if $u(w_1)-QL>0$. It is strong otherwise.

One can also view $\left(u(w_1)-QL\right)$ as a "bribe" required to keep *A* in line. The APC can be thought of as a cross between a PC and an ICC in that it ensures fidelity to the contract ex post and ex ante. It is like an ICC because, if satisfied, *A* has no incentive to lie in period 1 about his true course of action in period 2. Likewise, it ensures participation by *A*. This is related to the strategy-proofness constraint of Laffont and Mortimart [2002]. APC is more direct and apropos. It is understood that *A* is opportunistic, i.e., despite the promise to abide in period 1, *A* will abscond in period 2 in a weak TPE environment unless materially induced via the APC not to. Table 1 gives the possible payoffs for *A*.

Definition 3: We call $U^{00}=U^0+\left[u(w_1)-QL\right]$ the agent's insider and U^0 the outsider reservation utility.

Table 1. Possible payoffs for A

	Payoffs		
Action	*Period 1*	*Period 2*	*Total payoff*
(a) Abides	$u(w_1)$	$u(w_2)-v(e)$	$u(w_1)+u(w_2)-v(e)$
(b) Absconds	$u(w_1)$	U^0-QL	$U^0+u(w_1)-QL$

Our interest is clearly the weak TPE environment. The principal offers a contract $C(w,e)$ which satisfies APC in (2), which means that *A* delivers *e* once enlisted. This optimal contract factors in property rights protection for *P*. Note that the APC (2) can be rewritten as

$$u(w_2)-v(e)\geq U^0-QL. \tag{3}$$

2.3. The decision problem of P

P offers a contract $C(w,e)$ that maximizes the profit function $\pi = pF(e) - w$, subject to either (3) in the weak TPE or (1) in the strong TPE case. $F(e)$ is twice differentiable and strictly concave in effort e. The Lagrangean function in a weak TPE environment,

$$L = pF(e) - w + \lambda\left[u(w_2) - v(e) - U^0 + QL\right], \tag{4}$$

is maximized with respect to w and e. Let contract $C\left(w^*, e^*\right)$ solve this program. The first order conditions (FOCs) necessary for an interior solution under weak TPE are

$$\begin{aligned} &(i) \quad -1 + \lambda\left[u'(w_2)(1-b)\right] = 0 \\ &(ii) \quad pF' - \lambda v' = 0 \end{aligned} \tag{5}$$

where u', F', and v' are first derivatives with respect to respective arguments. (5.i) implies that

$$\lambda = \left[u'(1-b)\right]^{-1} > 0, \tag{6}$$

which means that the APC binds strictly, i.e., (3) holds as an equality. The corresponding FOCs under strong TPE when the PC(1) is satisfied are

$$\begin{aligned} &(i) \quad -1 + \lambda\left[u'(w_1) + u'(w_2)(1-b)\right] = 0 \\ &(ii) \quad pF' - \lambda v' = 0. \end{aligned} \tag{7}$$

Likewise, $\lambda > 0$ here, so the PC (1) binds strictly. Equations (7) and (1) together generate the optimal strong-TPE contract $C\left(w^{**}, e^{**}\right)$.

2.4. The optimal contract

We now characterize the incentives-compatible exogenous cash-in-advance contract $C\left(w^*, e^*\right)$ implied by (7) and the binding DCC (2), under weak TPE and opportunistic agent. We have:

Claim 1: In an optimal exogenous cash-in-advance contract $C\left(w^*, e^*\right)$ under weak TPE, the opportunistic agent, A, receives exactly his insider reservation utility U^{00}, which exceeds the outsider reservation utility U^0 he would receive under strong TPE, i.e.,

$$u\left(w_1^*\right) + u\left(w_2^*\right) - v\left(e^*\right) = U^{00} > U^0$$

Proof: Since $\lambda > 0$, the ICC binds in the weak TPE case, i.e., at optimum:

$$u\left(w_1^*\right)+u\left(w_2^*\right)-v\left(e^*\right)=U^0+u\left(w_1^*\right)-QL=U^{00}.$$

But, by Definition 2, in a weak TPE environment we have

$$u\left(w_1^*\right)-QL>0.$$

Since the PC binds in the strong TPE, A receives there $U^0 > U^{00}$. Q.E.D.

From (5.ii) and (6), we have:

$$pF'=v'\left[u'\left(w_2\right)\left(1-b\right)\right]^{-1}. \tag{8}$$

Equations (8) and (3) as equality are solved simultaneously for w^* and e^* to get the optimal contract $C\left(w^*,e^*\right)$. We have the following:

Corollary 1: For the same optimal effort level $e^{**}=e^*$, the wage rate w^* of the optimal cash-in-advance contract under weak TPE exceeds the wage rate w^{**} under strong TPE, and vice versa.

How the optimal contract structure under weak TPE responds to changes in Q and statutory penalty L is of interest. We have:

Claim 2: In an optimal exogenous cash-in-advance contract in a weak TPE environment, a rise either in TPE efficiency Q, or statutory penalty L lowers w^* and raises e^*.

Proof: See Appendix.

What this says, in effect, is that the stronger TPE, the cheaper the fixed cash-in-advance contract from the viewpoint of the principal. By the same token, the stingier it is from the perspective of the agent. In contrast, the weaker governance is, the better for the agent if the activity takes place and he gets contracted. Excessively weak TPE can lead to the principal backing out and to the disappearance of such a contract altogether.

We now investigate the effect on the contract of the intensity of the cash-in-advance feature of the contract. That is, we ask how a rise in "b", the fixed proportion of the wage paid in advance in period one, impacts on $C\left(w^*,e^*\right)$. We have the following:

Claim 3: Let the environment be TPE-weak. Let A's utility exhibit an Arrow-Pratt relative risk aversion $R\left(w\right)$ of less than one. A rise in the cash advance commitment "b", ceteris paribus, (i) lowers e^* and (ii) raises w^*.

Proof: See Appendix.

It is instructive to contrast Claim 3 with the effect of a change in b under strong TPE:

Claim 4: Let the environment be TPE-strong. Let *A*'s utility exhibit an Arrow-Pratt relative risk aversion $R(w)$ less than one and monotonic, i.e., $R(w_1) > R(w_2)$ if $w_1 > w_2$.

(i) If $b > 0.5$ initially, w^{**} rises and e^{**} falls with a rise in b.

(ii) If $b < 0.5$, w^{**} falls and e^{**} rises with a rise in b.

(iii) If $b = 0$ initially, w^{**} and e^{**} are invariant to changes in b.

Proof: Appendix B.

In this strong TPE environment, raising b allocates more of *A*'s pay from period 2 to period 1. Since the discount rate is 1 (present and future flows are valued equally), *A*'s utility is maximum at $b = 0.5$. If $b < 0.5$, moving toward 0.5 improves utility and to keep U^0, w^{**} falls and e^{**} rises. If $b > 0.5$, moving away from 0.5 lowers utility, which falls below U^0. Thus, the opposite happens. The movement of b affects the optimal contract only insofar as *A*'s utility (or preference) is affected through the participation constraint.

This contrasts with Claim 3 where a rise in b affects not only the preference of *A* but also his propensity to abscond. The second consideration overwhelms the first and makes w^* and e^* responses monotonic and invariant with respect to the initial value of b.

All else being equal, the more important the cash-in-advance feature of the contract in a weak TPE environment, the costlier it is for the principal and the better it is for the agent. Thus, the principal will want to avoid cash-in-advance-intensive contracts in a weak TPE environment. In this particular case, it is the agent who has an interest in maintaining a poor TPE if he is sure to land a contract.

Claim 2 shows a lower e^* with a fall in Q in weak TPE environment. If e^* is being supplied by a collective of agents in some fixed proportion who then divide w^*, this means that the contract will involve less e^* and thus more underemployment or lower employment.

The principal's payoff is $\Pi^* = pF(e^*) - w^*$. From Claim 2, $d\Pi^*/dQ > 0$ since e^* rises and w^* falls with a rise in Q. Thus, when Q or L falls, Π^* falls and may, indeed, result in decontracting. Consider a case of n principals, all dependent on the same TPE and viable under the same Q, but with different Π^*. As Q falls, fewer and fewer of them remain viable and drop out. The impact of progressively weakening governance is that the market becomes less and less competitive. Thus the market structure may depend partly on the quality of governance. The economic activity shrinks and may cease altogether. Keeping the agent faithful to the contract via an ex ante devise becomes too costly for the principal. Thus weak TPE may result in missing market.

For the firms that survive, the incentive is to minimize the use of the factor that can abscond or renege in favor of those that cannot. Thus, capital will be favored over labor and labor absorption reduced. In least developed countries (LDCs) where labor is abundant but TPE-weak, factor intensity can be biased in favor of capital.

3. Endogenous cash-advance structure

Certain contracts exhibit the cash-in-advance feature more than others. This is tantamount to saying that "b" is larger. Ceteris paribus, the more important the cash-in-advance feature of a contract, the more costly it is for the principal. Again, those contracts may not be worth entering for the principal and thus will either be scarce or nonexistent. In a weak governance environment, the more intensive cash-in-advance contracts, however potentially productive, may disappear.

We now consider the case where P can vary the cash advance commitment, b, of the cash-in-advance contract. In this case, we explicitly recognize the rationale for the cash advance: the greater the cash advance feature of the contract, the more productive it is. This is in keeping with the Austrian School view relating productivity with roundaboutness. We formalize the commitment-productivity nexus in this way. Let $F(e) = KG(e)$ and $K = K(b), K' > 0, K'' < 0$. Thus the degree of cash advance commitment acts on the Hicks-neutral efficiency scalar K, which rises with b. The higher the b, the more productive the contract. In other words, contract commitment acts like a technical (more precisely, institutional) innovation that raises productivity. The firm may, for example, advance the cost of training of new hires. We now let the wage rate w be fixed.

The participation constraint for A is still (1) and the APC is still (3). The Lagrangean function

$$L = pKG(e) - w + \lambda\left[u(w_2) - v(e) - U^0 + QL\right] \tag{9}$$

where λ is the Lagrange multiplier, is now maximized with respect to b and e. The first-order conditions for an interior maximum are

$$\begin{aligned} &(i) \quad pK'G(e) - \lambda u'w = 0 \\ &(ii) \quad pKG' - \lambda v' = 0 \end{aligned} \tag{10}$$

where $K' = (dK/db)$, $u' = (du/dw_2)$, $G' = (dG/de)$. (10)(i) gives $\lambda = (pK'G(e)/u'(w_2)w) > 0$. This means that the constraint (3) strictly binds. (10)(i) and (10)(ii) together gives

$$KG'u'w = K'Gv'. \tag{11}$$

Equations (3) and (11) can be solved for optimal b^* and e^* to generate the optimal contract $C(b^*, e^*)$. We now characterize the relation between contract structure and TPE.

The following spells out the response of contract structure to the quality of TPE.

Claim 5: The optimal cash advance commitment b (and, a priori, the productivity of the project) rises as either (i) TPE efficiency Q rises, or (ii) U^0 falls, or *(iii)* w rises with agent Arrow-Pratt relative risk aversion exceeding one ceteris paribus.

Proof: See Appendix.

A rise in outsider reservation, ceteris paribus, breaks the APC equality and motivates absconding. To restore the APC bound, b falls (w_1 falls), leading to a rise in $w_2 = (1-b)w$, making the second period payoff more attractive. A rise in the fixed wage has the opposite effect.

Thus the deterioration of TPE induces a flight from contracts with greater cash-advance commitment and toward less productive contracts for the same wage rate. As Q falls, the ICC bound breaks and absconding becomes attractive. To counter this, b falls, thus, rendering w_2 more attractive. The contract progressively becomes "spot". The overall impact on economic activity is similar to that discussed in section 2.

4. Summary

This paper investigates the nature of incentives-compatible contracts that arise when the cash-in-advance feature is prominent in a weak third-party enforcement environment. Cash-in-advance means that the principal advances part of the payment to the agent before the actual production period starts. A pure loan contract is a cash-in-advance arrangement with the cash advance being the loan itself. Weak TPE in this context is formally defined as the expected penalty of absconding being less than the benefit of absconding due either to the small (exogenous) probability of being punished or the light punishment meted absconders when caught, or both.

In the cash-in-advance context, there arises a distinction between the outsider reservation utility and the insider reservation utility (i.e., post-receipt of the cash advance). The latter is larger if TPE is weak. The option to abscond empowers the agent only if the governance is weak. To forestall absconding, the principal, if he takes the initiative, is forced to sweeten the deal for the agent (i.e., higher wage and/or lower effort) by satisfying the augmented participation constraint. The APC explicitly factors in the quality of third-party enforcement and carries a reward for fidelity, the counterpart of Williamson's hostage. When the latter is strong, the APC becomes an ordinary PC. The paper then shows that a rise in TPE efficiency or in the penalty lowers the wage and raises the effort in the incentives-compatible exogenous cash advance contract. The higher is the cash-in-advance commitment b of the contract, the lower is the effort and the higher is the wage rate if the relative risk aversion of the agent is less than one. This contrasts with the response of the wage rate and effort under strong TPE where the sign depends on whether the initial $b >< 0.5$. The overall effect is that the weaker TPE and/or the more pronounced the cash-in-advance feature, the costlier the contract for the principal and the economic activity covered may cease to exist.

Where the cash-advance commitment is endogenous, weaker TPE leads to a flight away from higher commitment and more productive contracts. The importance of proper and efficient institutions for economic progress is reinforced. In this case, weak TPE fails to protect the principal's property right over the cash advance.

It is generally believed that weak TPE (i.e., weak institutions for the enforcement of contracts) leads to poor economic performance [Dixit 2003; North 1994]. Where governance is weak—or equivalently, costly—economic activity tends to favor contracts that require less and less third-party enforcement [Fafchamps and Minten 1999]. This means that more productive activities involving time-mediated exchanges are foregone. The bridge between weak governance and poor economic performance runs through the scaffolding of the consequent scarcity of more productive, roundabout economic activity.

Since these exchanges are productive without opportunism, parties may resort to private remedies that do not depend on state-supplied TPE. A remedy that appears endemic in Third World countries is the private acquisition of what North [2002] calls "second-party enforcement". This, incidentally, is the thrust of the buoyant literature on weak or nonexistent rule of law.

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Appendix
A. Proofs of Claims 2 and 3

The two equations relevant for the proof of Claims 2 & 3 are equations (3) (as equality) and (8), i.e.,

$$(3')\; u(w_2)-v(e)=U^0-QL$$
$$(8')\; pF'\left[u'(w_2)(1-b)\right]-v'=0$$

Totally differentiating (3') and (8') focusing on w, e, Q and b, we have in matrix form:

$$\begin{bmatrix} H & -v' \\ M & D \end{bmatrix} \begin{bmatrix} dw \\ de \end{bmatrix} \begin{bmatrix} L \\ 0 \end{bmatrix} dQ + \begin{bmatrix} wu'(w_2) \\ N \end{bmatrix} db$$

where $H=u'(w_2)(1-b)>0$, $M=PF'u''(1-b)^2<0$, $D=\left[u'(w_2)(1-b)pF''-v''\right]<0$, $N=pF'u'(w_2)\left[1-R(w_2)\right]\leq 0$ if $\left[1-R(w_2)\right]\leq 0$, and $R(w_2)=\left[-u''(w_2)w_2/u'(w_2)\right]$. The determinant of the 2×2 matrix is $J=HD-M(-v')<0$. Solving for $(dw/dQ),(de/dQ)$ gives

$$(dw/dQ)=-u(L)D/J<0,$$
$$(de/dQ)=-M(-u(L))/J<0$$

which prove Claim 2. Solving for (dw/db) and (de/db) gives $(dw/db)=\left[u'(w_2)wD-N(-v')\right]/J>0$ if $\left[1-R(w_2)\right]\leq 0$ or $R(w_2)\leq 1$. $(de/db)=u'(w_2)\left[HpF'(1-R(w_2))-Mw\right]/J$.

The denominator can, by substituting for H and M, be rewritten as

$$(1-b)\left[u'(w_2)\right]^2 pF'\left\{(1-R(w_2))+R(w_2)\right\}=(1-b)u'(w_2)pF'>0.$$

Thus, $(de/db)<0$, which show Claim (3).

B. Proof of Claim 4

The relevant two equations are

$$U(w_1)+u(w_2)-v(e)=U^0$$

$$pF'\left[u'(w_1)-u'(w_2)b+u'(w_2)\right]-v'=0.$$

Totally differentiating and rendering in matrix form, we have

$$\begin{bmatrix} H^0 & -v' \\ M^0 & D^0 \end{bmatrix} \begin{bmatrix} dw \\ de \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix} db$$

where $H^0=\left[u'(w_1)b+u'(w_2)(1-b)\right]>0$,
$M^0=\left[u''(w_1)b^2+u''(w_2)(1-b)^2\right]<0$, $D^0=\left[pF''[.]-v''\right]<0$,
$X=-\left[u'(w_1)-u'(w_2)\right]w>0$ if $w_1 \geq w_2\,(b \geq 0.5)$ and vice versa,
$Y=-pF'\left[-u'(w_2)\right]\left[1-R(w_2)\right]+u'(w_1)\left[1-R(w_1)\right]>0$ if $w_1 \geq w_2\,(b \geq 0.5)$ since $u(w_2)>u'(w_1)$ and $R(w_2)<R'(w_1)<1$. The sign of Y reverses if $w_1<w_2$. The determinant of the 2×2 matrix is $J=H^0D^0+M^0v'<0$. Solving for $9dw/db$ and (de/db) gives

$$(dw/db)=\left[XD^0+Yv'\right]\big/J \geq 0 \text{ if } b \geq 0 \text{ if } b \geq 0.5$$
$$<0 \text{ if } b<0.5$$

$$(de/db)=\left[H^0Y+M^0X\right]\big/J \geq 0 \text{ if } b \geq 0 \text{ if } b \geq 0.5$$
$$<0 \text{ if } b<0.5$$

which proves Claim 3.

C. Proof of Claim 5

The proof of Claim 5 starts with the ICC (3), and (11). Totally differentiating these gives in matrix form:

$$\begin{bmatrix} -u'w & -v' \\ N & T \end{bmatrix} \begin{bmatrix} db \\ de \end{bmatrix} = \begin{bmatrix} -u(L) \\ 0 \end{bmatrix} dQ + \begin{bmatrix} -u'(1-b) \\ (KG'u')\left[R(w_w)+1\right] \end{bmatrix} dw + \begin{bmatrix} 1 \\ 0 \end{bmatrix} dU^0$$

where $N = G'r'wK' - KG'wu'' - Gv'K'' > 0$ and $T = Ku'wG'' - K'Gv'' - K'v'F' < 0$. Thus the determinant $J = -u'(w_1)T + Nv'$ of the 2×2 matrix is positive. Solving for db/dQ, db/dw and db/dU^0 we have

$$db/dQ = -u(L)T/J > 0$$

$$db/dw = \left[-u'(1-b)T - (v')\left(KG'u'(-R(w_2)) - 1\right)\right]/J > 0$$

$$\text{if } |R_r| \geq 1$$

$$\left(db/dU^0\right) = T/J < 0.$$

These prove Claims (5)(i), (5)(ii), and (5)(iii).