

MAXIMUM- SPEED DEVELOPMENT IN AN OPEN AGRARIAN ECONOMY WITH Z-ACTIVITIES

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It has long been recognized in the empirical literature that nonagricultural activities take up a significant portion of total labor time in agrarian areas. Some theoretical models have also appeared which examine rigorously certain implications of the existence of so-called Z-activities in the open agrarian economy. The interpretation of these models does not have to be restricted of course to purely agrarian economies; for example, the agricultural sector in a dualistic economy can be treated analytically in the same manner.

A static, partial equilibrium framework is used by Hymer and Resnik (1969) to analyze, among others, the supply responsiveness of agricultural producers to a once-for-all price change. A transformation curve between Z-goods and agricultural goods F is postulated and utility maximization by the peasant family is shown to imply some necessary relationships among the prices, levels of production and consumption of F- and Z-goods which do not make inevitable a positive response of marketable surplus to a change in the relative price of F-goods.

Z-goods in the formal Hymer-Resnick model serve only as substitute goods for industrial consumption goods, representing "a variety of processing, manufacturing, construction, transportation and service activities to satisfy the needs for food, clothing, shelter, entertainment and ceremony" (1969, p. 493). In fact, Z-activities also include capital-augmenting activities like "metal working, . . . , fence repairing, . . . , transport and distribution" (1969, p. 493n). The present writer has dynamized the Hymer-Resnick model by incorporating agricultural capital, which is taken to be the cumulated portion of Z-goods allocated in the past for investment. In "Dynamics of an Agrarian Model with Z-Goods" (Bautista, 1971), the pattern of development of the small agrarian economy open to trade is analyzed, assuming constant saving pro-

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pensity. The short-run and long-run effects of a change in the world price of F-goods relative to industrial consumption goods are also investigated and the results compared with those of the comparative static analysis used by Hymer and Resnick.

In "Intertemporal Optimization in an Agrarian Model with Z-Activities" (Bautista, 1974), an optimizing model of the agrarian economy is formulated under the assumption that the planning authority seeks to maximize the discounted sum of per capita utilities over time. Since there is only one target (objective) variable, only one policy (instrument) variable need be specified, in conformity to the well-known proposition in the Tinbergen-Theil theory of economic policy. The imposition of an import quota on industrial consumption goods is suggested to be both feasible and administratively least burdensome.

The question may properly be raised whether the objective function used in the study (Bautista, 1974) has immediate relevance to less developed economies. The planning authority in such countries may want to restrain increases in per capita consumption (utility) in the near future to make possible larger doses of investment with the view to speeding up the development process.¹ Such optimization program of course will not be carried out indefinitely for even in socialist countries the goal of higher consumption levels farther into the future presumably lies behind and represents the trade-off to, the austerity measures being forced upon the economy.

Fei and Chiang (1966) have developed an interesting one-commodity model of a "maximum-speed development" (MSD) economy in which the real wage rate serves as the policy instrument for the maximization of the growth rate of capital. A particular form of the "effort function" by a typical worker is postulated, the real wage rate appearing as argument, and specific conclusions are derived concerning the development path of the MSD economy resulting from the optimum pattern of the real wage rate.

In this paper the use of fiscal policy in influencing the speed of development in the agrarian economy with Z-activities is de-

¹For an elaboration of this argument and a systematic analysis of "constant per capita consumption growth," see Fei (1965).

monstrated. Given the socio-political structure of present-day societies, imposition of taxes is undeniably simpler and more realistic compared to the control of the real wage, which probably only the machinery of a thoroughgoing socialist economy may achieve in practice. The basic policy variable to be used is the tariff rate on imports of industrial consumption goods. The existence of an optimum pattern of tariff rate which maximizes the growth rate of agricultural capital is examined and the development path of the MSD economy characterized.

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Two types of production activities are distinguished in the present model — one producing the usual agricultural (consumption) goods F which may be sold to the world market in exchange for industrial consumption goods C and the other consisting of labor-using Z -activities which produce nontradable Z -goods that may be consumed as an inferior substitute for C -goods or used to augment agricultural capital.² For simplicity, production in both F and Z is assumed subject to constant returns to scale and unchanging technology.

Since labor is the only input in Z -production, we may write

$$(1) \quad x_z = \alpha l_z,$$

where:

x_z : per capita output of Z -goods

l_z : fraction of total engaged in Z -activities

α : (constant) average productivity of labor in Z -production.

Production in F , which requires inputs of labor and agricultural capital, is also characterized by positive but diminishing marginal returns. Thus,

$$(2) \quad x_F : l_F g_F(\bar{p}) \quad \text{where } g_F' > 0 \quad g_F'' < 0 \quad \text{for all } \bar{p} \text{ and}$$

²An analytical similarity may be noted with one-sector growth models which also assume a single commodity functioning either as a consumption good or as a capital good; once invested, however, it remains a capital good and cannot be used for consumption.

x_F : per capita output of F-goods

l_F : fraction of total labor engaged in F-production

$\bar{\rho}$: ratio of the stock of agricultural capital R to labor employed in F-production; a measure of capital intensity in producing F-goods.

Labor allocation in peasant agriculture may reasonably be assumed to equalize the value of marginal productivities in F- and Z-production,³ i.e.,

$$(3) \quad w = g_F - \bar{\rho}g'_F = \alpha p_Z,$$

where w and p_Z are the imputed (shadow) prices of labor and Z-goods, respectively – each expressed in terms of F-goods.

Since labor has positive marginal returns in either activity, it will be fully employed:

$$(4) \quad l_F + l_Z = 1.$$

Given the world price of industrial consumption goods in terms of F-goods \bar{p}_C , the tariff rate ξ determines uniquely the domestic price p_C :

$$(5) \quad p_C = (1 + \xi) \bar{p}_C.$$

Revenue from tariff is assumed to be redistributed uniformly; hence we may express per capita income y as follows:

$$(6) \quad y = w + \rho g'_F + \bar{p}_C \cdot \xi q_C,$$

where ρ is the ratio of the stock of capital to total labor and q_C is the amount of per capita consumption (importation) of industrial consumption goods. Noting the definition given earlier for $\bar{\rho}$, we may write

$$(7) \quad \rho = \bar{\rho} l_F.$$

³Professor Schultz (1964, Ch. 3) presents a strong case for the highly efficient manner in which resources are allocated even in traditional agriculture.

MAXIMUM-SPEED DEVELOPMENT

Equality of exports and imports in the agrarian economy requires

$$(8) \quad \bar{p}_C q_C = x_F - q_F,$$

where q_C and q_F are the per capita consumption of C- and F-goods, respectively.

The addition to agricultural capital is represented by the difference between total output of Z-goods and Z-consumption q_Z . The problem facing the planning authority is to determine the optimum pattern of tariff rate $\zeta(t)$ which maximizes for every moment of time the rate of growth of capital $\psi(t)$ given by

$$(9) \quad \psi(t) = \frac{I}{\rho(t)} [x_Z(t) - q_Z(t)],$$

subject to the conditions (1) - (8) and the following consumption functions:

$$(10) \quad \begin{aligned} q_F &= q_F(y) \\ q_Z &= q_Z(y, p_Z, p_C) \\ q_C &= q_C(y, p_Z, p_C). \end{aligned}$$

Per capita consumption of F-goods is assumed to depend only on per capita income while that of Z- and C-goods, because they are substitutable, depend as well on the imputed price of Z-goods, and the domestic price of industrial consumption goods, each expressed in terms of F. The *a priori* signs of the partial derivatives are given by

$$(10a) \quad \frac{\partial q_F}{\partial y} > 0, \quad \frac{\partial q_C}{\partial y} > 0, \quad \frac{\partial q_Z}{\partial y} \leq 0 \quad \begin{array}{l} \text{according as Z-} \\ \text{goods are inferior} \\ \text{or non-inferior} \end{array}$$

$$\frac{\partial q_i}{\partial p_i} < 0, \quad \frac{\partial q_i}{\partial p_j} > 0, \quad (i, j = Z, C).$$

It is necessary to show first that the equilibrium position at each moment of time (i.e., given ρ) is determined by the policy variable ξ . Sufficient conditions are derived in this section for the uniqueness of static (momentary, short-run) equilibrium.

The static system (1) – (8) and (10) in ρ , ξ and the endogenous variables y , \bar{p} , w , x_F , l_F , l_Z , etc., simplified to one involving only ρ , ξ , y and \bar{p} . From (3) and (5), respectively,

$$(11) \quad dp_Z = \frac{-\bar{p}g_F''}{\alpha} d\bar{p} = \frac{dw}{\alpha} \quad \text{and} \quad dp_C = \bar{p}_C d\xi$$

Taking total differentials in (6) and substituting (11), we obtain

$$(12) \quad \left(1 - \bar{p}_C \xi \frac{\partial q_C}{\partial y}\right) dy + \frac{\bar{p}g_F''}{\alpha} \left(\alpha l_Z + \bar{p}_C \xi \frac{\partial q_C}{\partial y}\right) d\bar{p} \\ = g_F' d\rho + \bar{p}_C \left(q_C + \bar{p}_C \xi \frac{\partial q_C}{\partial p_C}\right) d\xi$$

A second relationship among the total differentials in (10) may be derived by eliminating l_F , l_Z , x_F , x_Z , and p_C in (1), (2), (4) (5), (7) and (8), from which

$$(13) \quad \left(\frac{\partial q_F}{\partial y} + \bar{p}_C \frac{\partial q_C}{\partial y}\right) dy + \left[\frac{w l_F}{\bar{p}} - \frac{\bar{p}g_F'' \bar{p}_C}{\alpha} \frac{\partial q_C}{\partial p_Z}\right] d\bar{p} \\ = \frac{g_F}{\bar{p}} d\rho - \bar{p}_C \frac{\partial q_C}{\partial p_C} d\xi.$$

The coefficient of dy in (12) may be written, noting (5), $\left(1 - \frac{\xi p_C}{1 + \xi} \cdot \frac{\partial q_C}{\partial y}\right)$ which is positive if the reasonable assumption is made that the marginal propensity to consume C-goods is less than one (a sufficient but not necessary condition). It follows that for ρ , $\xi = \text{constant}$ in (12) an increase in the capital intensity in F-production must lead to an increase in total income, i.e.,

$\frac{dy}{d\bar{p}} \Big|_{\rho, \xi = \text{const. in (12)}} > 0$. In (13) the opposite is true:

$\frac{dy}{d\bar{p}} \Big|_{\rho, \xi = \text{const. in (13)}} < 0$. These results are depicted in Figure 1.

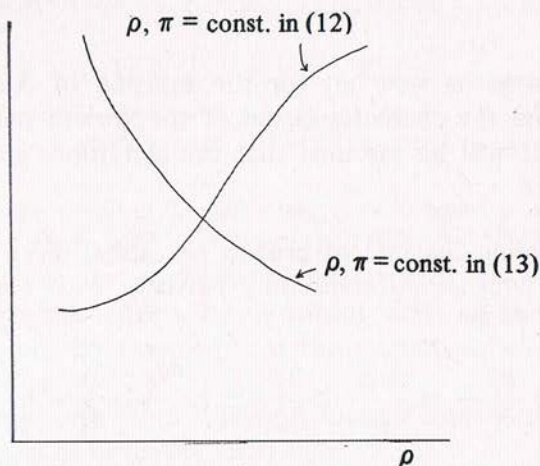


Figure 1

Only at the point of intersection of the two curves will (12) and (13) be satisfied simultaneously. Therefore, at any time period (i.e., given ρ), y and \bar{p} are determined uniquely by ξ . From (5), the domestic price of industrial consumption goods is also a function of the tariff rate alone. It follows from the other equations that the short-run values of the remaining variables are determined once the tariff is fixed.

Equations (12) and (13) also imply that⁴

$$(14) \quad \frac{\partial y}{\partial \xi} > 0 \text{ and } \frac{\partial \bar{p}}{\partial \xi} > 0 \quad \text{if} \quad \left| \frac{p_C}{q_C} \cdot \frac{\partial q_C}{\partial p_C} \right| < \frac{1 + \xi}{\xi},$$

i.e., the price elasticity of demand for C-goods does not exceed $\frac{1 + \xi}{\rho}$. This would seem a reasonable assumption for low

levels of the tariff rate. Moreover,

$$(15) \quad \frac{\partial y}{\partial \rho} > 0 \text{ and } \frac{\partial \bar{p}}{\partial \xi} > 0 \quad \text{if} \quad \left(1 - \frac{\partial q_F}{\partial y} - \bar{p}_C \frac{\partial q_C}{\partial y} \right) > 0,$$

⁴See Appendix for the derivation of (14) and (15).

i.e., the sum of the marginal propensities to consume F- and C-goods is less than unity. If Z-goods are assumed inferior, this condition is equivalent to having the marginal propensity to save exceed the absolute value of the marginal propensity to consume Z-goods.⁵

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The stage is now set for the solution of the maximization problem and the characterization of the growth path of the MSD economy. It will be assumed that the conditions given in (14) and (15) are met.

Expressing the rate of growth of capital given in (9) in terms of ρ , ζ , y and \bar{p} , differentiating partially with respect to ζ , and using (11), we get

$$(16) \quad \rho \frac{\partial \psi}{\partial \zeta} = \frac{\alpha \rho}{\bar{p}^2} \cdot \frac{\partial \bar{p}}{\partial \zeta} - \left[\frac{\partial q_Z}{\partial y} \frac{\partial y}{\partial \zeta} + \frac{\partial q_Z}{\partial p_Z} \left(\frac{-\bar{p} g_F''}{\alpha} \cdot \frac{\partial \rho}{\partial \zeta} \right) + \frac{\partial q_Z}{\partial p_C} \cdot \bar{p}_C \right]$$

The short-run impact of a change in the tariff rate resolves into the effect on Z-output, represented by the first term in the RHS of (16), and the effect on the consumption of Z-goods, represented by the bracketed expression. Since $\frac{\partial \rho}{\partial \zeta} > 0$, the level of Z-production is seen to increase with the tariff rate. The effect on current consumption of Z of a change in the tariff rate resolves further into the income and price effects, where, using previous results,

$$(16a) \quad \frac{\partial q_Z}{\partial y} \cdot \frac{\partial y}{\partial \zeta} \leq 0 \quad \text{according as Z-goods are inferior or non-inferior;}$$

$$\frac{q_Z}{p_Z} \left(\frac{-\bar{p} g_F''}{\alpha} \right) \frac{\partial \bar{p}}{\partial \zeta} < 0 \quad \text{the pure substitution (own-price) effect; and}$$

⁵The same condition is used in Bautista (1971) to assure uniqueness of short-run equilibrium.

$$\frac{\partial q_Z}{\partial p_C} \cdot \bar{p}_C > 0, \text{ the cross-price effect.}$$

The first order condition for a maximum rate of growth of capital to be achieved at each period is that an optimum tariff rate ξ be chosen such that

$$(17) \quad \left. \frac{\partial}{\partial \xi} \psi(\rho, \xi; \bar{p}(\rho, \xi), \gamma(\rho, \xi)) \right|_{\xi = \xi^*} = 0$$

where, as may be recalled, \bar{p} and γ are functions of ρ and ξ . Equation (17) states that at ξ^* the marginal effect on the growth rate of capital of a change in the tariff rate is equal to zero (equivalently, the marginal effects on Z-output and on Z-consumption are equal). It will be assumed that the second order condition is

fulfilled, i.e., $\frac{\partial^2 \psi}{\partial \xi^2} < 0$.⁶ Hence, for any capital-labor ratio ρ there corresponds an optimum tariff rate

$$(18) \quad \xi^* = \Phi(\rho), \text{ determined from (17).}$$

Substituting from (18) into (9) yields the function $\psi(\rho, \Phi(\rho))$, which expresses the rate of growth of capital corresponding to the optimum tariff rate in terms solely of the capital-labor ratio. Equat-

$$\text{ing} \quad \left. \frac{d\psi}{d\rho} \right|_{\xi^*} = \frac{\partial \psi}{\partial \rho} \quad \text{to zero gives}$$

$$(19) \quad \frac{\alpha}{\bar{p}} + \psi = \frac{\alpha \rho}{\bar{p}^2} \frac{\partial \bar{p}}{\partial \rho} - \frac{\partial q_Z}{\partial y} \frac{\partial \gamma}{\partial \rho} - \frac{\partial q_Z}{\partial \rho_Z} \left(\frac{-\bar{p} \rho_G''}{\alpha} \frac{\partial \bar{p}}{\partial \rho} \right).$$

Recalling previous results, inferiority of Z-goods is sufficient to make the RHS of (19) greater than zero.

If the second order condition is fulfilled, then in the neighborhood of $\rho = \bar{\rho}$ where $\bar{\rho}$ satisfies (19), the $\psi(\rho, \Phi(\rho))$ curve is concave from below, as represented in Figure 2. (It is not possible to determine the global maximum at the present level of generality.)

⁶For this to be verified, more specific assumptions concerning the forms of the production and demand functions are necessary.

That ψ is a single-valued function of ρ follows from Figure 1: given ρ the optimum tariff rate is determined from (18) and hence $\bar{\rho}$ and γ , as well as the other variables (including ψ), are uniquely determined.

The dynamic path of the agrarian economy pursuing maximum speed development through an optimum tariff policy may be characterized using the construction of Figure 2. Notice that in addition to the ψ curve the horizontal line labelled η is drawn, representing the sum of the depreciation rate and population growth rate. Maximum growth of capital takes place at the capital-labor ratio $\bar{\rho}$. Two positions of dynamic equilibrium are possible — one at $\rho = \rho^*$ and the other at $\rho = \rho^{**}$. As may be discerned from the direction of the arrowheads, the system is unstable at ρ^{**} .

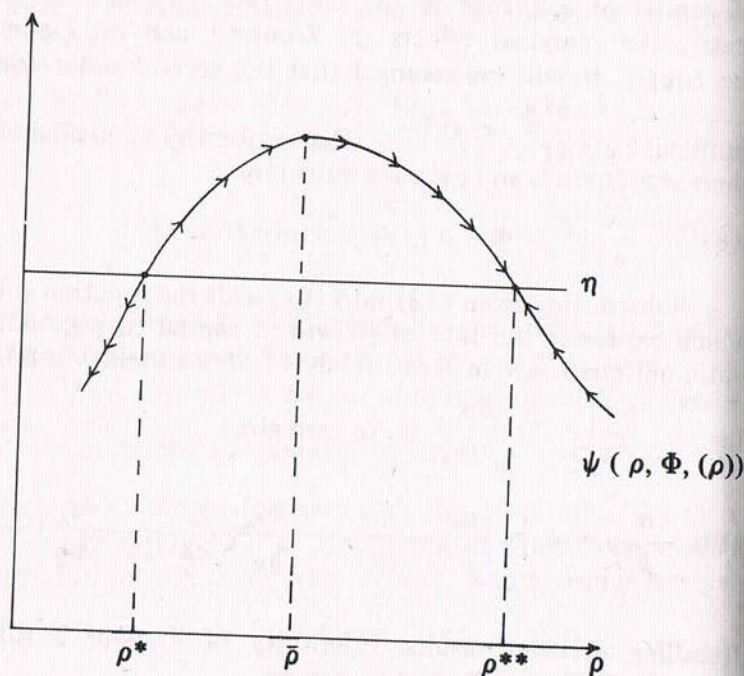


Figure 2

If initially the economy has a capital-labor ratio $\rho < \rho^*$, the dynamic consequence of an optimum tariff policy is retrogression: in trying to maximize the rate of growth of capital at each short-run period, the capital-labor ratio actually suffers because of the high value of η , i.e., the economy is caught in the Malthusian trap. At an initial ρ greater than ρ^* but less than $\bar{\rho}$, the economy will experience an increasing capital-labor ratio and growth rate of capital as well

until it reaches $\bar{\rho}$, after which the rate of growth of capital will diminish but with the capital-labor ratio still increasing until the stationary level ρ^{**} is reached. Finally, if the MSD economy starts at an initial capital-labor ratio exceeding ρ^{**} , the optimum pattern of tariff rate will result in an increasing growth rate of capital accompanied by a decreasing capital-labor ratio until its value is reduced to ρ^{**} , the position of stable equilibrium. The implication for policy of such characterization of the growth process seems clear: Before maximum-speed development using the tariff rate on industrial goods as instrument is embarked upon, the planning authority should make sure that the capital-labor ratio is greater than the critical level ρ^* . This may be achieved by reducing the rate of population growth, hence bringing down the horizontal line in Figure 2, and /or influencing the price and income elasticities such that the ψ curve is raised upward. Otherwise, the MSD policy becomes self-defeating.

The dynamic behavior of the other variables as the agrarian economy develops at a maximal rate will similarly depend on the initial position, as well as to the form of the function $\Phi(\rho)$. For

example, since $\frac{d\zeta}{d\rho} = \Phi'(\rho)$,

$$(20) \quad \frac{d\bar{\rho}}{d\rho} = \frac{\partial \bar{\rho}}{\partial \zeta} \Phi'(\rho) + \frac{\partial \bar{\rho}}{\partial \rho}$$

$$\frac{dy}{d\rho} = \frac{\partial y}{\partial \zeta} \Phi'(\rho) + \frac{\partial y}{\partial \rho}$$

The evaluation of $\Phi'(\rho)$, however, cannot be done without making additional specific assumptions. It will depend, among other things, on the third derivative of the F-production function and the second derivatives of the demand functions for F-, Z- and C-goods.

APPENDIX

Proof of (14) and (15):

Setting $d\rho = 0$ in (12) and (13) and solving for $\frac{\partial y}{\partial \xi}$ and $\frac{\partial \bar{p}}{\partial \xi}$ by Cramer's rule, we get

$$(i) \quad D \frac{\partial y}{\partial \xi} = \bar{p}_C (q_C + \bar{p}_C \xi \frac{\partial q_C}{\partial p_C}) \left(\frac{wl_F}{\bar{p}} - \frac{\bar{\rho} g_F''}{\alpha} \cdot \bar{p}_C \frac{\partial q_C}{\partial p_Z} \right) + \frac{\bar{\rho} g_F''}{\alpha} (\alpha l_Z + \bar{p}_C \xi \frac{\partial q_C}{\partial p_Z}) \cdot \bar{p}_C \frac{\partial q_C}{\partial p_C},$$

$$(ii) \quad D \frac{\partial \bar{p}}{\partial \xi} = (1 - \bar{p}_C \xi \frac{\partial q_C}{\partial y}) \left(-\bar{p}_C \frac{\partial q_C}{\partial p_C} \right) + \bar{p}_C (q_C + \bar{p}_C \xi \frac{\partial q_C}{\partial p_C}) \left(\frac{\partial q_F}{\partial y} + \bar{p}_C \frac{\partial q_C}{\partial y} \right),$$

where

$$D = (1 - \bar{p}_C \xi \frac{\partial q_C}{\partial y}) \left(\frac{wl_F}{\bar{p}} - \frac{\bar{\rho} g_F''}{\alpha} \cdot \bar{p}_C \frac{\partial q_C}{\partial p_Z} \right) - \frac{\bar{\rho} g_F''}{\alpha} (\alpha l_Z + \bar{p}_C \xi \frac{\partial q_C}{\partial p_Z}),$$

(iii)

$$\left(\frac{\partial q_F}{\partial y} + \bar{p}_C \frac{\partial q_C}{\partial y} \right) > 0, \text{ noting (10a).}$$

It follows from (i) and (ii) that a sufficient (but not necessary) condition for $\frac{\partial y}{\partial \xi} \cdot \frac{\partial \bar{p}}{\partial \xi} > 0$ is that $(q_C + \bar{p}_C \xi \frac{\partial q_C}{\partial p_C}) > 0$.

This expression may be written, using (2),

$$q_C \left(1 + \frac{\xi}{1+\xi} \frac{p_C}{q_C} \frac{\partial q_C}{\partial p_C} \right) \text{ which in turn is equivalent to}$$

$$\frac{q_C \xi}{(1+\xi)} \left(\frac{1+\xi}{\xi} + \frac{p_C}{q_C} \frac{\partial q_C}{\partial p_C} \right) \text{ from which (14) follows.}$$

To prove (15), set $d\xi = 0$ in (12) and (13) and solve for $\frac{\partial y}{\partial \rho}$ and $\frac{\partial \bar{\rho}}{\partial \rho}$:

$$(iv) \quad D \frac{\partial y}{\partial \rho} = g'_F \left(\frac{wl_F}{\bar{\rho}} - \frac{\bar{\rho} g''_F}{\alpha} \cdot \bar{p}_C \frac{\partial q_C}{\partial p_Z} \right) - \frac{g_F}{\bar{\rho}} \cdot \frac{\bar{\rho} g''_F}{\alpha} (\alpha l_Z + \bar{p}_C \xi \frac{\partial q_C}{\partial p_Z})$$

$$(v) \quad D \frac{\partial \bar{\rho}}{\partial \rho} = \frac{g_F}{\bar{\rho}} (1 - \bar{p}_C \xi \frac{\partial q_C}{\partial y}) - g'_F \left(\frac{\partial q_F}{\partial y} + \bar{p}_C \frac{\partial q_C}{\partial y} \right),$$

where $D > 0$ is as defined in (iii).

Notice from (iv) that $\frac{\partial y}{\partial \rho} > 0$ unambiguously. This verifies the first relation in (15). In (v), $\frac{\partial \bar{\rho}}{\partial \rho} > 0$ if and only if

$$(vi) \quad g_F (1 - \bar{p}_C \xi \frac{\partial q_C}{\partial y}) - \bar{\rho} g'_F \left(\frac{\partial q_F}{\partial y} + \bar{p}_C \frac{\partial q_C}{\partial y} \right) > 0$$

Noting (3), a sufficient condition for the inequality (vi) to hold is that

$$(vii) \quad 1 - \bar{p}_C \xi \frac{\partial q_C}{\partial y} > \frac{\partial q_F}{\partial y} + \bar{p}_C \frac{\partial q_C}{\partial y}$$

which may be written

$$(viii) \quad 1 - \frac{\partial q_F}{\partial y} - (1 + \xi) \bar{p}_C \frac{\partial q_C}{\partial y} > 0$$

and hence, in view of (5), the second relation in (15) is proved.

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