

## A PITFALL IN THE WELFARE COST OF INFLATIONARY FINANCE

By Eli M. Remolona\*

### 1. Introduction

In his celebrated indictment of inflationary finance, Bailey (1956) delineated an area under the demand curve for real money balances as the deadweight loss or the excess burden from inflation. Tower (1971) later derived this measure of deadweight loss in marginal terms to make it applicable to the determination of the optimal inflation rate. If inflation is a tax on the holding of real money balances, the idea is then to set this tax so as to equate its marginal welfare cost with the marginal welfare cost of alternative taxes.

However, to go from deadweight loss to welfare cost, following Bailey and Tower, one needs a measure of tax revenue. In other words, the specification of the marginal welfare cost of a tax has two components: (a) the marginal deadweight loss of the tax, and (b) the marginal tax revenue. The marginal welfare cost is then simply (a) divided by (b). The pitfall I wish to point out lies in (b). Bailey, Tower, and the others<sup>1</sup> failed to see that the appropriate form for (b) is already implied by the form of (a). As it turns out, the form of revenue they used for the inflation tax is inconsistent with the form they used for the deadweight loss. In my view, their measure of deadweight loss is correct. It is their measure of tax revenue that must be changed.

To establish the relationship between deadweight loss and tax revenue, I first return, in Section 2, to Hotelling (1938), who made the notion of a deadweight loss rigorous for the first time. Using Hotelling's derivation, I then show, in Section 3, the pitfall in the usual specification of the welfare cost of inflationary finance. In Section 4, I then try to justify what I consider to be the proper specification. In Section 5, I conclude by illustrating what difference the proper specification makes to the evaluation of the optimal inflation tax.

---

\*Assistant Professor of Economics, University of the Philippines. I wish to thank Dan Canlas, Paul Evans, Philip Medalla, and Butch Montes for comments and encouragement at various stages of this paper.

<sup>1</sup>See, for example, Frenkel (1976) and Aghevli and Khan (1977).

## 2. Hotelling's Deadweight Loss

Hotelling's classic derivation of the deadweight loss from a tax is based on the notion of consumer's surplus and producer's surplus. Consumer's surplus is the maximum amount consumers would be willing to pay for a given quantity of a good or service minus the amount they actually pay. Correspondingly, producer's surplus is the amount of revenue producers actually get minus the minimum amount they would accept. In Figure 1, let  $DD$  be the demand curve for a commodity and  $SS$  be the supply curve. Then if  $OA$  were the equilibrium price, and  $OC$  the equilibrium quantity, consumer's surplus would be the area  $DBA$  and producer's surplus would be the area  $SBA$ .

Now suppose an excise tax were imposed on this particular commodity. The tax would drive a wedge between the price paid by the consumer and the price received by the producer. If the unit tax were equal to  $LN$ , the consumer price would go up to  $K$ , and there would then be a loss in consumer's surplus measured by the area  $KLBA$ . The producer price would fall to  $M$ , and the loss in producer's surplus would be  $ABNM$ . The loss to consumer and producer together would then be  $KLBNM$ .

At the same time, there would, of course, be a gain in tax revenue to the government. This tax gain would be measured by the area  $KLNM$ . This gain would be a transfer within the economy and should therefore not be counted as part of what would be lost to the economy. What would be lost to the economy would thus be the loss in consumer's and producer's surplus minus the tax revenue. This would be the shaded area  $LBN$ , and this is what Hotelling would call the deadweight loss.<sup>3</sup> It would measure the loss to the economy due to the tax distortion. Note that to derive this measure of deadweight loss, one has to introduce a measure of tax revenue.

All distortionary taxes entail deadweight losses. If the government must raise revenue by means of such taxes, then the efficient way to do it is to choose the taxes so as to minimize the sum of the deadweight losses. Inflation, as Bailey pointed out, is a distortionary tax, and it should therefore be included in that minimization.

---

<sup>2</sup>The long controversy surrounding the use of consumer's surplus for welfare analysis was recently settled by Willig (1976), making Harberger's (1971) famous postulates justifying its use unnecessary. The notion of a deadweight loss Hotelling attributed to the French engineer Jules Dupuit.

<sup>3</sup>For a modern derivation, see Diamond and McFadden (1974).

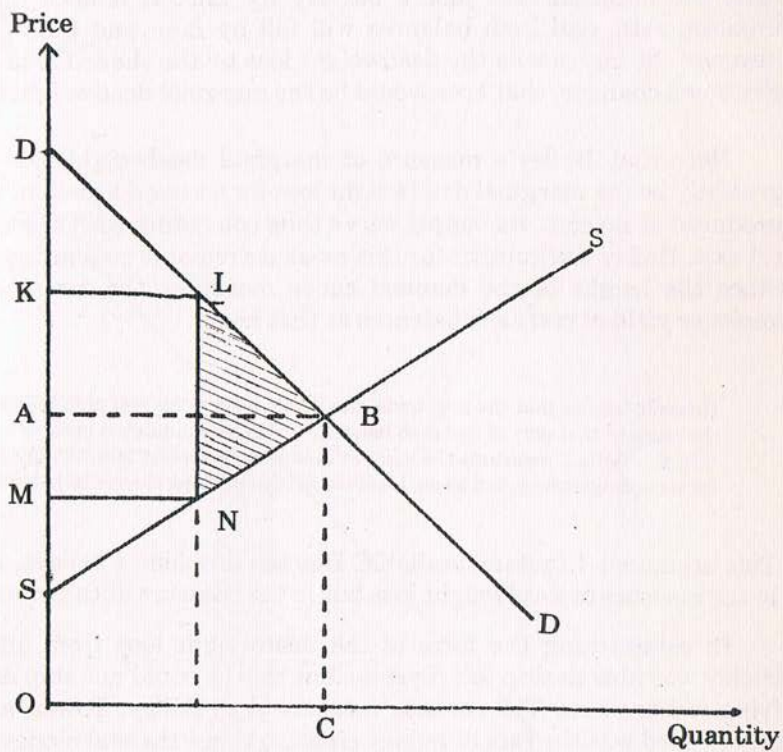


Figure 1

## 3. Bailey's Welfare Cost

In this section, we follow Tower's refinement of Bailey's work, which is to cast the notion of welfare cost in marginal terms. We assume, for convenience, a stationary economy with a given real interest rate  $r$ . The demand for real cash balances is a function of the nominal interest rate and is represented by the curve  $LL$  in Figure 2. The initial equilibrium is a nominal interest rate of  $OT$  and real holdings of cash of  $OZ$ . The government is to decide on whether it should raise the inflation rate just a bit, say by  $\Delta\pi$ . If it does raise the inflation rate, real cash balances will fall by  $\Delta m$ , and Bailey would measure the increase in the deadweight loss by the shaded area  $SVZY$ . For small changes, that area would be the marginal deadweight loss.

Note that Bailey's measure of marginal deadweight loss would precisely be the marginal deadweight loss for a tax on a good or service produced at no cost, its supply curve thus coinciding with the horizontal axis. Bailey's argument for this measure remains convincing to me. Since the height of the demand curve measures the marginal convenience yield of real cash balances at that level:

Hence it follows that the area under the demand curve for real cash balances, over the range of that part of real cash balances which is relinquished because of a given rate of inflation, measures the costs in loss of convenience, increasingly awkward barter arrangements, and so on, involved in relinquishing those real balances.

This argument I find unassailable. But the problem, I believe, lies not in the measure of deadweight loss but in the measure of tax revenue.

In establishing the form of the deadweight loss from inflation, Bailey was able to skip tax revenue, but this he could not skip in specifying welfare cost. The revenue measure that Bailey, Tower, and the others used was the rate of money creation times the real money stock. In a stationary economy, this would be the same as specifying revenue as

$$(1) \quad R = \pi m$$

where  $\pi$  is the inflation rate and  $m$  is the real money stock. In terms of Figure 2, the original tax revenue would be the area  $TVXr$ , and the revenue for the slightly higher inflation rate would be  $RSWr$ . The increment in revenue would therefore be  $RSUT$  minus  $UVXW$ . Bailey's marginal welfare cost would then be simply the marginal deadweight loss divided by the marginal tax revenue. In the diagram, this would be  $SVZY$  divided by the difference between  $RSUT$  and  $UVXW$ .

# INFLATIONARY FINANCE

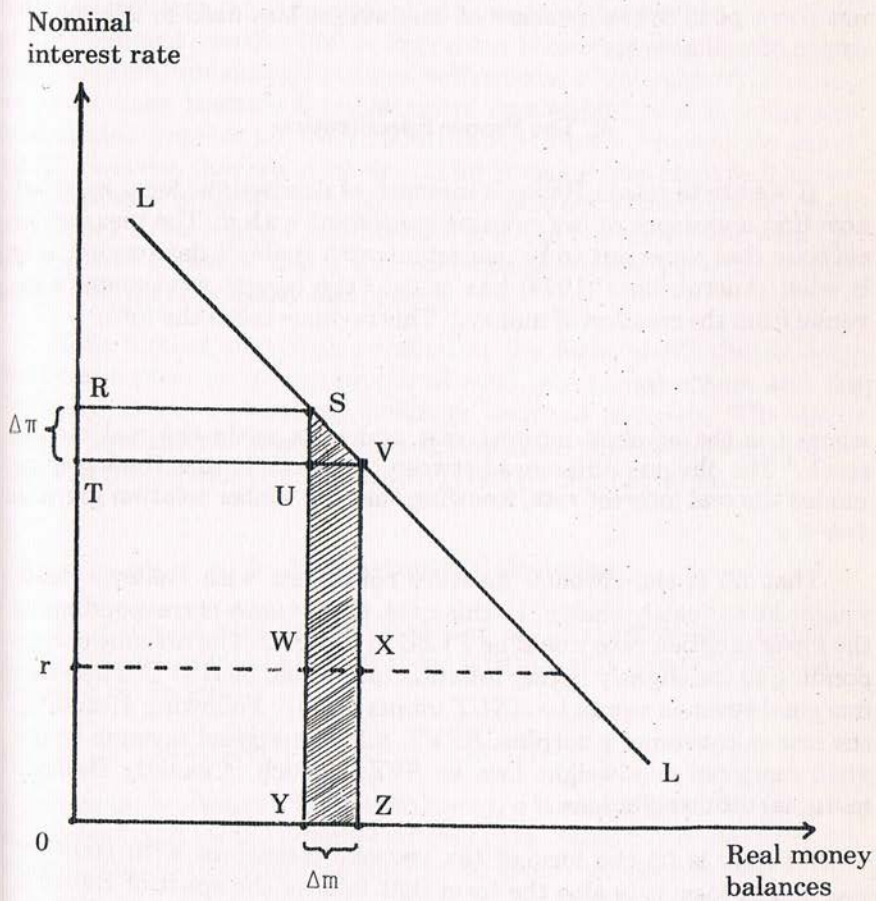


Figure 2

The problem with the above measure of the marginal revenue from the inflation tax is that it does not yield the measure of marginal deadweight loss with which it is used. To show that, note that the loss in consumer's surplus from the higher inflation rate would be the area *RSVT*. Following Hotelling, the marginal deadweight loss would then be the loss in consumer's surplus minus the gain in tax revenue, which in the diagram would clearly be the area *SVXW*. But this derived measure would be short of Bailey's marginal deadweight loss, specifically by the area *WXZY*. In other words, the measure of tax revenue does not correspond to the measure of deadweight loss used in the specification of welfare cost.

#### 4. The Proper Specification

If we are to retain Bailey's measure of deadweight loss, we must now find a measure of tax revenue consistent with it. The measure of revenue that turns out to be consistent with Bailey's deadweight loss is what Auernheimer (1974) has called "the honest government's revenue from the creation of money." This revenue takes the form

$$(2) \quad S = im$$

where  $i$  is the nominal interest rate and  $m$  is again the real money stock.<sup>4</sup> The obvious difference between (1) and (2) is that the latter includes the real interest rate, knowing that the Fisher relation gives us  $i = r + \pi$ .

That (2) is the revenue measure consistent with Bailey's deadweight loss is easily shown. In this case, the revenue corresponding to the lower inflation rate would be *TVZO* in Figure 2. The revenue corresponding to the slightly higher inflation rate would be *RSYO*. Therefore marginal revenue would be *RSUT* minus *UVZY*. Following Hotelling, the loss in consumer's surplus, *RSVT*, minus marginal revenue would yield marginal deadweight loss as *SVZY*, which is exactly Bailey's marginal deadweight loss.

Not only is (2) the form of tax revenue consistent with Bailey's deadweight loss, it is also the form that follows the spirit of Bailey's work. Bailey wanted to focus on an aspect of inflationary finance he considered fundamental, in his words, "because it cannot be avoided by sliding-scale arrangements or by precise foreknowledge of individual prices." Auernheimer showed that specifying revenue as (1) in-

<sup>4</sup>This form of seigniorage revenue has also been used by Phelps (1973) and Marty (1978) although without using the notion of deadweight loss. Instead, in determining the optimal inflation tax, they use the Ramsey (1927) approach of choosing taxes to maximize household utility subject to raising a given amount of tax revenue.

volves the assumption that money holders are caught completely by surprise by an increase in the inflation rate, so that, in the process, they unexpectedly suffer capital losses as the adjustment to lower real money balances is accommodated by a once-and-for-all jump in the price level. It is (2) that assumes no unexpected capital losses to money holders and is therefore the form of revenue consistent with agents having foreknowledge of prices.<sup>5</sup>

Auernheimer's analysis justifies (2) as the measure of revenue from the inflation tax independent of a notion of deadweight loss. Such an independent justification is important because it means that if we want a consistent measure of the welfare cost of inflationary finance, we must keep Bailey's formulation of deadweight loss, it being the formulation implied by Auernheimer's no-surprise revenue. In other words, it is not that we must use (2) for revenue just because it is the one consistent with Bailey's deadweight loss, but rather that we must use Bailey's deadweight loss because it is the one consistent with (2), the revenue measure that really corresponds to the perfectly anticipated inflation tax Bailey had in mind.

Note further that with revenue in the form of (2), the analogy between a good or service produced with zero marginal cost and the liquidity services of money holdings becomes complete. The deadweight loss and the revenue from the inflation tax are the deadweight loss and the revenue from a tax on a good produced at no cost.

### 5. An Illustrative Calculation

To illustrate how our reformulation of the revenue from inflation affects the calculation of the optimal inflation rate, consider a demand function of the form

$$(3) \quad m = \alpha e^{-\beta i}$$

where  $m$  is the demand for real balances,  $\alpha$  is a constant,  $\beta$  is Cagan's coefficient, and  $i$  is the nominal interest rate. Note that we have  $i = r + \pi$  where  $\pi$  is the inflation rate and  $r$  is the given real interest rate.

With this demand function, Bailey's marginal deadweight loss can be shown to be  $\beta im$ . Marginal revenue, using  $R = \pi m$ , would be

<sup>5</sup>Perhaps even more compelling is the argument that if the government treated (1) as the revenue from the inflation tax and if the public had perfect foresight, the government would be led to time inconsistency, that is, it would always cause higher inflation rates than it would itself deem optimal, whatever its objective function. On the other hand, if it took (2) as the revenue, it would be led to time consistency. This line of argument is too technical to be fully stated here, but the reader may refer to Calvo (1978) and Remolona (1982).

$(1-\beta\pi)m$ . Hence marginal welfare cost, as it has been inconsistently calculated, would be

$$(4) \quad Z^R = \frac{\beta i}{1-\beta\pi}$$

In contrast, if instead  $S = im$  were used, marginal revenue would be  $(1-\beta i)m$ , and marginal welfare cost would be

$$(5) \quad Z^S = \frac{\beta i}{1-\beta i}$$

Already it can be seen that for a given inflation rate,  $Z^R < Z^S$  so long as  $1 - \beta i > 0$ . If it is the marginal welfare cost we take as given, then it is clear that  $Z^S$  will imply a lower inflation rate than will  $Z^R$ .

To be more specific, Aghevli, Khan, Narvekar, and Short (1976) have estimated  $\beta$  for the Philippines to be 3.5 in the long run. Now let  $r$  be 5 per cent and let the marginal welfare cost of other taxes be given as 3.0. Then the optimal inflation rate according to  $Z^R$  would be 20.2 per cent while according to  $Z^S$  it would be just 16.4 per cent. Hence the consistent specification of welfare cost yields a lower optimal inflation rate.

## REFERENCES

- Aghevli, B. and Khan, M. (1977), "Inflationary Finance and the Dynamics of Inflation in Indonesia, 1951-72," *American Economic Review*, 67 (June): 390-403.
- Aghevli, B., Khan, M., Narvekar, P.R. and Short, B. (1976), "Monetary Policy in Selected Asian Countries," *IMF Staff Papers*, 26 (December): 775-824.
- Auernheimer, L. (1974), "The Honest Government's Guide to the Revenue from the Creation of Money," *Journal of Political Economy*, 82 (May/June): 589-606.
- Bailey, M.J. (1956), "The Welfare Cost of Inflationary Finance," *Journal of Political Economy*, 64 (April): 93-110.
- Calvo, G. (1978), "On the Time Consistency of Optimal Policy in a Monetary Economy," *Econometrica*, 46 (November): 1411-1428.
- Diamond, P.A. and McFadden, D.L. (1974), "Some Uses of the Expenditure Function in Public Finance," *Journal of Political Economy*, 3 (January): 3-21.
- Frenkel, J.A. (1976), "Some Dynamic Aspects of the Welfare Cost of Inflationary Finance," in *Money and Finance in Economic Development*, ed. R.I. McKinnon (New York: Marcel Dekker), pp. 177-195.



- Harberger, A.C. (1971), "Three Basic Postulates for Applied Welfare Economics: An Interpretive Essay" *Journal of Economic Literature*, 9 (September): 785-797.
- Hotelling, H. (1938), "The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates," *Econometrica*, 6 (July): 242-269.
- Marty, A.L. (1978), "Inflation, Taxes, and the Public Debt," *Journal of Money, Credit, Banking*, 10 (November): 437-452.
- Phelps, E.S. (1973), "Inflation in the Theory of Public Finance," *Swedish Journal of Economy*, 75 (March): 67-82.
- Ramsey, F.P. (1927), "A Contribution to the Theory of Taxation," *Economic Journal*, 37 (March): 47-61.
- Remolona, E.M. (1982), "Inflation, Public Debt, and the Reserve Tax: A Theory of Optimal Deficit Finance," unpublished Ph.D. dissertation, Stanford University, pp. 14-43.
- Tower, E.S. (1971), "More on the Welfare Cost of Inflationary Finance," *Journal of Money, Credit, Banking*, 3 (November): 850-860.
- Willig, R.D. (1976), "Consumer's Surplus Without Apology," *American Economic Review*, 66 (September): 589-597.