ON THE USE OF LINEAR PROGRAMMING FOR FAMILY PLANNING RESOURCE ALLOCATION

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1. Introduction

The scarcity of family planning resources in developing countries has led researchers to suggest the use of operations research techniques to allocate these resources in order to optimize the objectives of a family planning program. The objective could be that of minimizing births (or maximizing averted births) given the resources or of minimizing costs given target birth rates. Essentially, a family planning program can be regarded as a production problem with its various activities as inputs and averted births as output. This makes it easy to formulate the family planning program as a resource allocation problem. Consequently, the mathematical programming model has been suggested as the appropriate model to use.

Although nonlinear programming models have been proposed (e.g., Gould and Magazine (1971), the resource allocation models that have been developed for existing family planning programs are linear. Resource modelling for family planning is fairly recent (Reinke suggested it in 1970) and initial attempts at modelling usually begin with the linear case. Also, linear programming has been successful in solving other resource allocation problems.

This paper looks at linear programming (LP) models that have been developed for specific family planning programs and suggests modifications that could (a) give averted births more accurately and (b) reduce the size of the model. An accurate determination of

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averted births is important for policy making while model size reduction could mean savings in computational expenses.

2. Review of Existing LP Models for Family Planning Resource Allocation

Early resource allocation modelling for family planning were mostly theoretical discussions on the applicability of the mathematical programming model to the family planning program. Reinke (1970) presents a simple hypothetical static model which illustrates the formulation of a two-contraceptive method family planning program into a linear programming problem. His model maximizes averted births subject to cost and maximum acceptance constraints. Correa and Beasley (1971) give a basic static model whose objective is to maximize prevented pregnancies subject to a cost constraint. Several modifications of this basic model are presented to show how nondemographic objectives can be incorporated in the model. Both models do not include physical resource (manpower, materials) constraints and do not discuss the estimation of the parameters of the model.

Going beyond these theoretical considerations, Lawrence, Mundigo, and ReVelle (1972) developed a linear programming model of resource allocation specifically adapted to the Honduras national family planning program. This model is simple in the sense that it involves only one contraceptive method (pills). But it has several important features that make it a workable model. First, it is a dynamic model which includes changes in demographic variables such as age and fertility of the women of reproductive age and program changes such as continuation and dropout. Second, it distinguishes between costs of a beginning contraceptor and the costs of a continuing contraceptor and allows these costs to change over time. Third, it includes among its constraints not only the budget constraints but also the physical recources available to the program over the planning horizon. Fourth, it also includes a crude birth rate constraint that indicates the desired crude birth rate at the end of the planning horizon. The complete model describes the entire program and minimizes total female births[1] over the planning horizon.

1. The use of female births as decision variables simplified the model. As the model is intended to be used for planning horizons that extend beyond fifteen years, the number of females that enter the reproductive group is an important variable in the model.

This model, therefore, can evaluate the feasibility of program targets given the projected resources and if these targets are feasible, the model gives the optimal strategy for achieving those targets.

Building on this model, Lawrence and Mundigo (1975) developed another model that incorporates the features of the Honduras model and includes two contraceptive methods (pills and IUDs), the parity of women of reproductive age, and switching to another method. This model was adapted to fit the national family planning program of the Dominican Republic.

Following these models, a similar model for the Philippine Family Planning Program was constructed (Pernia and Danao, 1978) that was specifically designed to evaluate the 5-year population program of the Population Commission. It has the essential features of the Dominican Republic model although it includes five contraceptive methods (pills, IUDs, rhythm, condom, sterilization). Because the planning horizon is less than 15 years, the females born during the planning period would not enter the reproductive age group; hence, they will not affect the fertility of the women under consideration. Consequently, the decision variables used were total births. A new feature of the model is the elimination of the crude birth rate constraint. Both the Honduran and the Dominican Republic model contain a crude birth rate constraint which states that the crude birth rate at the end of the planning horizon is less than or equal to a specified target crude birth rate. The presence of this constraint implies that if the target cannot be achieved with the given resources, the LP will simply give no feasible solution without any guide as to what can be achieved by those resources. On the other hand, the absence of the crude birth rate constraint leaves an LP that has an optimal solution from which the optimal crude birth rates achievable by the available resources can be computed. Hence, two things are accomplished: (a) the crude birth rates achievable by the given resources are determined and (b) these achievable crude birth rates can indicate to the decision-maker how far he is from his targets.

3. The Objective Function

Minimizing Births or Minimizing Costs?

The implementable models discussed in Section 2 minimize births over a planning horizon. This objective function was criticized by Haran (1979) who claims that "in the context of a family planning program in a developing country, it would seem more appropriate to minimize the total cost of the program subject to constraints on desired levels of birth rate so that the feasibility of achieving the targeted birth rate as well as the minimum cost required to achieve such a reduction in birth rate can be evaluated". Haran's choice of the objective function is a legitimate one but not appropriate for many developing countries. In many of these countries, birth rates are very high and financial resources are very low. The desired birth rates are so far from the existing birth rates that resources are exhausted before the desired birth rates are achieved. Consequently, one's objective is simply to maximize the output (fertility reduction) with the limited resources.

If the desired birth rate is not far from the existing birth rate and hence, is achievable, then cost minimization becomes an appropriate objective. This happens when the family planning program has already made substantial progress that the actual birth rate is not far from the desired birth rate. In fact, if the desired birth rate has been achieved, one would want to determine the minimum cost that would sustain it.

Minimizing Births or Maximizing Averted Births?

Conceptually, minimizing births is equivalent to maximizing averted births. The Honduras, Dominican Republic, and the Philippine models have birth minimization as the objective function. We suggest, however, that averted birth maximization has several advantages.

First, we note that when viewed as a production problem, the number of averted births is the output of the program. In fact, averted births per money unit (or the cost of an averted birth) has been used as an indicator of the cost-effectiveness of a family planning program. In the birth minimization model, this information is given by the dual problem. We claim, however, that the averted births per money unit given by the dual problem underestimates the actual value. This can be seen by means of a simple example.

Consider a one-period two-method model that minimizes births subject to a budget constraint:

Primal Problem: Minimize $\beta_0 N + \beta_1 X_1 + \beta_2 X_2$ (1)

$$\text{subject to} \quad c_1 X_1 + c_2 X_2 \leqq b \qquad (2)$$

$$N + X_1 + X_2 = w \qquad (3)$$

$$N, X_1, X_2 \leqq 0$$

where β_0 = proportion of noncontraceptors who bear a child in one period,

N = number of noncontraceptors,

β_m = proportion of method m contraceptors who bear a child in one period, m = 1, 2,

X_m = number of method m contraceptors, m = 1, 2,

c_m = cost of providing service to a method m contraceptor in one period, m = 1, 2,

b = budget for the period,

w = number of women under consideration.

Note that the equality constraint is an accounting identity which is included to preclude obtaining the trivial optimal solution $N = X_1 = X_2 = 0$.

The dual of the preceding problem follows:

Dual Problem: Maximize $bY_1 + wY2$ (4)

$$\text{subject to} \quad Y_2 \leqq \beta_0 \qquad (5)$$

$$c_1 Y_1 + Y_2 \leqq \beta_2 \qquad (6)$$

$$c_2 Y_1 + Y_2 \leqq \beta_2 \qquad (7)$$

$$Y_1 \leqq 0, \ Y_2 \text{ free.}$$

We note that the primal problem has a feasible solution N = w, $X_1 = X_2 = 0$ and since its objective function is bounded below by zero it has an optimal solution (N^*, X_1^*, X_2^*). It follows from the Duality Theorem (Bradley et al. [1977]) that the dual problem has an optimal solution (Y_1^*, Y_2^*) that satisfies

$$\beta_0 N^* + \beta_1 X_1^* + \beta_2 X_2^* = bY_1^* + wY_2^* . \quad (8)$$

Since the primal objective function denotes births, then each term on the right hand side of (8) denotes births. The variable Y_1^* must, therefore, be expressed in births per money unit, while Y_2^* must be expressed in births per woman. If we assume that there is at least one noncontraceptor, i.e., $N^* > 0$, then the first dual constraint is binding, i.e., $Y_2 = \beta_o$. Therefore, $wY_2^* = w\beta_0$, the number of births from all women under consideration in the absence of contraception. Hence,

$$\begin{aligned} bY_1^* &= -\beta_0 \, w + \beta_0 N^* + \beta_1 \, X_1^* + \beta_2 X_2^* \\ &= -\beta_0 \, (N^* + X_1^* + X_2^*) + \beta_0 N^* + \beta_1 X_1^* + \beta_2 X_2^* \\ &= (-\beta_0 + \beta_1) \, X_1^* + (-\beta_0 + \beta_2) X_2^* . \\ \therefore -bY_1^* &= + [(\beta_0 - \beta_1) \, X^*{}_1 + (\beta_0 - \beta_2) X_2^*] \end{aligned}$$

We have already noted that $Y_1{}^*$ represents births per money unit. Also, $Y_1{}^* \leqslant 0$. These suggest that $Y_1{}^*$ represents averted births per money unit. This is confirmed by the last equation above since the right hand side represents averted births. But note that the right hand side shows that the averted birth per method m contraceptor is $\beta_o - \beta_m$, i.e., the dual problem assumes that if the contraceptor were not contracepting, her fertility is that of the noncontraceptor. In reality, this is not the case. The contraceptors tend to have higher fertility than the noncontraceptors since they have a greater motivation to accept contraception.[2] This implies that the dual problem underestimates the number of averted births; hence, also the averted births per money unit.

4. Reducing Model Size

A dynamic linear programming model rapidly increases in size as the number of contraceptive methods is increased and the planning horizon is lengthened. Consider the LP whose objective is to minimize births and call it the MINBIRTH model. Its objective function can be written as

2. The Philippine data reveal that the pre-acceptance age-specific marital fertility rates of contraceptors are higher than the age-specific marital fertility rates of all married women of reproductive age. (Laing, 1977)

$$\sum_i \sum_t \beta^n_{it} N_{it} + \sum_i \sum_m \sum_t (\beta^b_{imt} X^b_{imt} + \beta^c_{imt} X^c_{imt}) \quad (9)$$

where β^n_{it} = yearly births per noncontraceptor of age i in year t,

N_{it} = number of noncontraceptors of age i in year t,

β^b_{imt} = yearly births per beginning contraceptor of age i using method m in year t,

X^b_{imt} = number of beginning contraceptors of age i using method m in year t,

β^c_{imt} = yearly births per continuing contraceptor of age i using method m in year t,

X^c_{imt} = number of continuing contraceptors of age i using method m in year t.

For the age range of 15-44, i.e., $15 \leqslant i \leqslant 54$, there are 30t noncontraceptor variables and 58mt contraceptor variables. If there are five contraceptive methods, i.e., m = 5, the total number of decision variables is 320t.

Let us now consider the *LP* whose objective is maximizing averted births and refer to this model as MAXAVERT. Its objective function can be written as

$$\sum_i \sum_m \sum_t \left[\alpha^b_{imt} X^b_{imt} + \alpha^c_{imt} X^c_{imt} \right] \quad (10)$$

where α^b_{imt} = number of averted births per beginning contraceptor of age i using method m in year t,

α^c_{imt} = number of averted births per continuing contraceptor of age i using method m in year t.

Note that MAXAVERT's decision variables do not include the noncontraceptor variables. Consequently, the decision variables are reduced by 30t.

The absence of noncontraceptor variables in MAXAVERT also has an implication in reducing the number of constraints. The equality constraint (3) in the Primal Problem of Section 3 is an accounting identity. This identity is incorporated in the MINBIRTH model in order to preclude obtaining the trivial optimal solution where all decision variables are assigned zero values. For each age i and time t, there is one accounting identity. Hence, there are 30t such accounting identities which are no longer necessary in the MAXAVERT model.

If sterilization is one of the contraceptive methods offered by the program, more simplifications can be made because of the exceptional characteristics of this method. It is the most cost-effective method in the long run since it has 100 percent continuation, 100 percent effectiveness, and no recurring costs. A dynamic model therefore, will choose to sterilize contraceptors as an optimal solution. Since this is not acceptable, maximum sterilization acceptances constraints have to be incorporated in the LP model. In this case, the model will first assign contraceptors to sterilization until the maximum limits are reached before assigning them to the other methods. However, as shown in Pernia and Danao (1978) no sterilizations are assigned on the last year of the planning horizon. This arises from the fact that on the last year of the planning horizon, the future effects of sterilization are not captured, making sterilization the least cost-effective method.

These observations suggest that sterilization should be treated separately. Administratively, the family planning decisionmaker determines the sterilization program over the planning horizon based on considerations like capacity of sterilization clinics and social acceptability. He then allocates the amount needed for this program. Computationally, given this program, a simple pre-LP routine can be made to calculate the yearly population of sterilization contraceptors together with the number of averted births.

The removal of the sterilization method from the LP model will further reduce the number of variables by 58t and the number of constraints by 30t.

5. The Dual Problem

The dual variable corresponding to a resource constraint can be interpreted as the shadow price of each unit of the resource. In the MINBIRTH model, the shadow price corresponding to the budget constraint turns out to be nonpositive (as shown in Section 2), which is not intuitively appealing. However, it was shown that this nonpositive dual variable represents averted births per money unit. But as pointed out earlier, the number of averted births determined by the dual problem underestimates the actual number of averted births; consequently, it also underestimates the shadow price. The MAXAVERT model, on the other hand, counts the actual number of averted births and the shadow price corresponding to the budget constraint is nonnegative whose interpretation as averted births per money unit is straightforward. Using the example in Section 3, its MAXAVERT version is written as follows:

Primal Problem: Maximize $\alpha_1 X_1 + \alpha_2 X_2$

subject to $c_1 X_1 + c_2 X_2 \leqslant b$

$$X_1, X_2 \geqslant 0$$

Dual Problem: Minimize bz

subject to $c_1 z \geqslant \alpha_1$

$$c_2 z \geqslant \alpha_2$$

$$z \geqslant 0.$$

It is easily seen that bz must represent averted births; hence, the dual variable z must represent averted births per money unit.

6. Conclusion

The objectives and activities of a family planning program nearly fit the mathematical programming model. The choice of the objective function depends on the immediate goals of the program as well as the extent of its resources. For programs that want to maintain a certain birth rate, minimization of costs is a proper objective. On the other hand, if the program has to reduce high birth rates with limited resources, the minimization of births or the maximization of averted births would be the objective. We have shown that in this case, maximization of averted births has the advantage of giving a more accurate estimate of the cost per averted birth which is directly obtained from the dual problem. Moreover, the maximization of averted births does not require the inclusion of noncontraceptors as decision variables. This reduces the number of decision variables as well as the number of constraints thereby effecting some savings in computer time.

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