COST ALLOCATIONS AND THE MANAGEMENT CONTROL PROBLEM

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1. Introduction

Management, among other users of financial data, often faces the question of whether to make some kind of cost allocations for internal reporting purposes. Accounting plays a major role in this issue, being directly concerned with a variety of cost allocation problems; namely, the apportionment of costs over time (e.g., depreciation), over products (e.g., factory overhead), or over departments ("responsibility accounting"). Essentially, there are two stages in this choice problem, as follows:

1.1 *The Purpose of Allocation:* Whether to make the cost allocation in the first place (questions like: Should central computer cost be charged to users? Should the cost of university administration be allocated over the different units or programs? Should company advertising expenses be spread over products?).

1.2 *Choice of an Allocation Method:* Once management has decided to allocate costs, which method should be used (e.g., through a lump sum departmental charge? per unit of output? or as a percentage of output?).

Horngren (1977) presents some possible answers to the first question:

"We noted earlier that at least four (overlapping) purposes may be sought by a given cost allocation:

1. *Predicting economic effects of decisions.* Estimate the impact of various actions on the total costs of the organization. Examples of such actions are decisions to add or discontinue products.

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2. *Determining income and asset valuations.* Examples are product costs used for computing cost of goods sold and inventory balances.

3. *Ascertaining a mutually agreeable price.* Examples are contracts based on costs instead of market price (as in "cost-plus" contracts).

4. *Obtaining desired motivation.* Achieve goal congruence and incentive when other means fail. An example is charging the operating division for central costs such as basic research even though cause-and-effect or fairness justifications are weak. On the other hand, some top managers will insist that such allocation practices get division managers to take a desired interest in say, central research activities" (pp. 501-502).

The first three reasons for cost allocation appear to be the least controversial since each is based on some form of cause-and-effect relationship between benefits and the cost items. In fact, these also form the rationale for traditional incremental cost analysis in managerial cost accounting. The fourth reason, the motivation aspect, could raise potential disagreements because here, the cost allocation is essentially arbitrary (i.e., the cost causality factor may not be present). Note that traditional economic analysis tends to view with suspicion such an evaluation process which ignores incremental costs and benefits. Further, arbitrary allocations seem counter to the notion in responsibility accounting that a manager should be accountable only for those costs which he can control.

This paper employs an economic, decision-theoretic framework to demonstrate the following:

a) that cost allocations could be effectively employed to address two specific, familiar problems in managerial motivation or control, i.e., an affirmative answer to (1.1); and
b) that specific cost allocation techniques will have different effects on the manager in different problem settings, i.e., the requirement in (1.2) is similar to a choice of a managerial control technique for a given situation.

2. Methodology

This theoretical analysis of cost allocation in relation to the problem of managerial motivation employs the following basic methodology:

1. Two alternative settings for a managerial control problem are set up. Model I is the familiar problem of a manager who overspends on perquisites or discretionary expenses (representation expenses, thick carpets, company car, etc.). Model II concerns the problem of a manager who is risk averse and therefore (see Leland, 1972; Sandmo, 1971) tends to produce less output than what the risk neutral of the firm wants.

2. The assumptions of each alternative economic setting are specified. These consist of:

   a) the outcome function, in both cases, the profits of the firm;

   b) the objective function of the manager, represented in his utility function;

   c) the manager's decision variables, i.e., the production level decision in Model II and both the production level and the amount of discretionary spending in Model I; and

   d) the cost allocation methods being studied. It shall be assumed that the cost allocation technique is unilaterally determined by the top management or owners of the firm, i.e., it is beyond the manager's control.

3. Inferences are derived from the models using the assumption of maximizing behavior by the manager under the various cost allocation regimes imposed by top management. Comparative static analysis is undertaken to ascertain how the manager responds to changes in the particular cost allocation policy.

The paper proceeds with a discussion on the representation of cost allocation techniques and then with the separate analysis of the two models. A summary section integrates the two results and presents the conclusions.

3. Cost Allocation Techniques

Three different cost allocation techniques are analyzed in this paper. For example, let the profit function, a function of output x $\pi(x)$ be represented as:

(1) $$\pi(x) = px - cx - F$$

where :
x : output level
p : output price
c : variable cost
f : fixed cost

Then a cost allocation charge represents a reduction in profits and could take three forms:

1. Lump Sum Cost Allocation

A lump sum cost, k, is added to fixed cost such that the new profit becomes:

(2) $$\pi(x) = px - cx - (F + k)$$

i.e., an arbitrary peso amount is charged against the manager's departmental profits.

2. Per Unit Cost Allocation

A per unit cost allocation is a charge based on output units or some surrogate like direct labor. This is a per unit charge, a, such that the manager's profit calculation becomes:

(3) $$\pi(x) = px - (c + a)\, x - F$$

i.e., an overhead charge per unit of output is imposed on departmental profits.

3. Proportional Cost Allocation

A third alternative is to reduce the manager's profit by a constant proportion, t, as follows:

(4) $$\pi(x) = (1 - t)(px - cx - F)$$

For example this is achieved in practice by allocation of costs to departmental profits, a favored surrogate for "ability to bear (common) costs."

4. Model I: Cost Allocations and Managerial Discretionary Expenses

Inherent in the managerial control problem is the claim that the manager maximizes his own well-being and not necessarily that of the owner/top management. It will be shown that this lack of congruence in owner-manager objectives results in inefficiencies when the manager could choose the level of expenditure on perquisites, defined here as factors of production or ("necessary expenses") for which he also derives incidental personal benefits. In turn, the results of my analysis shall indicate that top management could alleviate this discretionary spending problem by appropriate changes in certain cost allocation policies.

The model is described first.

4.1. Model Assumption

Consider the profit function for the firm with discretionary expenses, $\pi(x,s)$:

$$(5) \qquad \pi(x,s) = R(x,s) - c(x) - s$$

where:

$R(x,s)$: the revenue function which depends on output, x and discretionary expenses, s

$c(x)$: the cost function

It is assumed that the firm is a price taker, or the revenue function is linear in output $[\partial R(.)/ x > 0;\ \partial^2 R(.)/\partial x^2 = 0]$ and increasing at a decreasing rate in discretionary expenses $[\partial^2 R(.)/\partial s^2 > 0;\ \partial^2 R(.)/\partial s^2 < 0]$. The price-taking assumption implies nonsubstitution in the inputs x and s. It is convenient to view s as "representation expenses" which increase the revenues of the firm but with diminishing returns beyond some point. The cost technology is assumed to be strictly increasing in output $[\partial c(x)/\partial x > 0;\ \partial^2 c(x)/ x^2 > 0]$.

The manager's preference function, $U(\pi, s)$, is defined over profits and representation expenses:

$$U(\pi\,.\,s)=\alpha\, s^{\beta_1}.\,\pi^{\beta_2}$$

where: α, β_1 and β_2 are constants, such that

$$\alpha > 0;\ 0 < \beta_1 < 1;\ 0 < \beta_2 < 1$$

$$\pi \equiv \pi\ (x,s),\ \text{defined in Equation (5)}$$

Note that the relative magnitude of the constants in U(.) determines the manager's relative preference for profits and for discretionary expenses. This is the familiar Cobb-Douglas formulation yielding the plausible behavioral hypothesis of positive but diminishing marginal utility from both profits and expenses [$\partial U(\cdot)/\partial \pi > 0$, $\partial^2 U(\cdot)/\partial \pi^2 < 0$, $\partial U(\cdot)/\partial s > 0$, $\partial^2 U(\cdot)/\partial s^2 < 0$].

The next step is to verify that the owner has a "control problem" with respect to the manager described in Equation (6).

4. 2 Implications of Managerial Behavior: The Control Problem

In the absence of further restrictions on the manager's choice of output, x and spending, s, he shall maximize his utility in equation (6) given that he faces a demand function and cost technology in equation (5). Substituting (5) into (6) and taking the derivative with respect to his choice variables x and s yield the following first order condition for a utility maximum:

$$\text{(7i)} \quad \beta_1 \alpha s^{\beta_1 - 1} \pi^{\beta_2} + \beta_2 \alpha s^{\beta_1} \pi^{\beta_2 - 1} (R_s - 1) = 0$$

[7]

$$\text{(7ii)} \quad \beta_2 \alpha s^{\beta_1} \pi^{\beta_2 - 1} (R_x - C_x) = 0$$

where the subscripts on R and C are partials with respect to the indicated choice variables.

To be assured that the solution to (7), an optimum output-representation expense *pair* (x^*, s^*), is a maximum, the matrix of second partials of $U(\cdot)$ or the Hessian matrix, D, must satisfy:

$$\text{(8)} \qquad D = \begin{vmatrix} Uxx & Uxs \\ Usx & Uss \end{vmatrix} > 0$$

i.e., for this unconstrained maximization problem, the Hessian must be positive definite.

It could be readily verified that the model assumptions ensure that (8) holds and so (x^*, s^*) gives the manager his maximum utility.

The following implications of managerial choice behavior could be derived from (7).

a) From (7ii), the manager will follow the *correct* output choice rule: produce x until the managerial cost of production, Cx, is equal to its marginal benefits or price, Rx. Stated another way, his output decision is not affected by his discretionary expense decision.

b) However, the manager will overspend on discretionary expenses, i.e., beyond its marginal benefits. To see this, rearrange (7i) in the form:

$$R_s = \left[1 - {}^{U_s}/_{U_\pi}\right] < 1 \qquad (9)$$

i.e., since $U_s = \partial U(\cdot)/\partial s > 0$, the optimal discretionary expenses, s^* shall always be chosen according to (9) or such that the marginal return from the expense is less than the marginal expense itself.

Figure 1 shows the choice of the optimal discretionary expense.

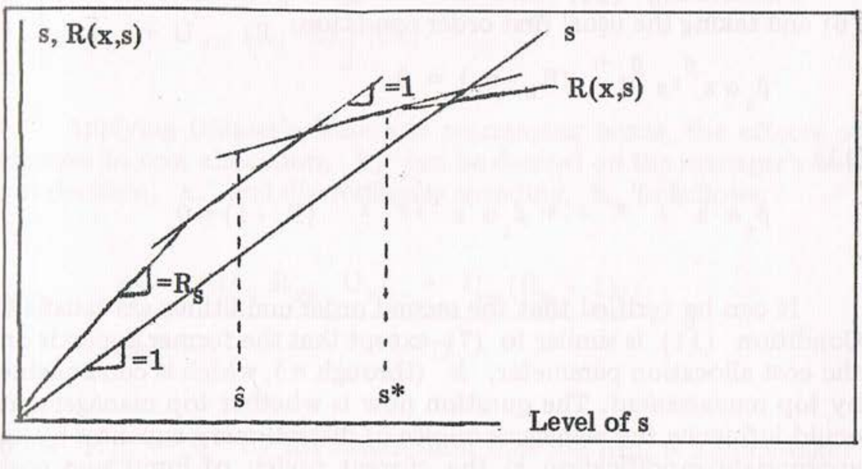

Figure 1 — Optimal s vs. Desired $\hat{s}$ (by owner)

Notes:
Manager chooses s^* according to (9):
$s = \{s > 0 : R_s < 1\}$

Top management wants manager to choose s:
$s = \{s > 0 : R_s = 1\}$

Clearly there are incentives for top management to expend effort to control discretionary expenses or from Figure 1, bring s^* closer to $\hat{s}$.

The next section explores top management use of various cost allocation policies in response to this problem and the manager's behavioral response in each case.

4.3 Specific Cost Allocation Policies

The analysis of the different cost allocation policies shall be pursued in somewhat more detail in the case of lump-sum cost allocation. Similar analysis will be done for the other types of allocation, but the calculations are relegated to the Appendix to improve readability. In each case, the result is summarized in a theoretical proposition.

For lump-sum cost allocation, k, the profit function becomes:

$$\pi(x,s;k) = R(x,s) - c(x) - s - k \tag{10}$$

Substituting (10) into the manager's preference function in (6) and taking the usual first order condition:

$$\beta_2 \alpha s^{\beta_1} \pi^{\beta_2 - 1} (R_x - c_x) = 0$$

(11)

$$\beta_1 \alpha s^{\beta_1 - 1} \pi^{\beta_2} + \beta_2 \alpha s^{\beta_1} \pi^{\beta_2 - 1} (R_s - 1) = 0$$

It can be verified that the second order conditions are satisfied. Condition (11) is similar to (7) except that the former depends on the cost allocation parameter, k (through π), which is controllable by top management. The question now is whether top management could influence the manager's choice of discretionary expenses by an appropriate modification in the current policy of lump sum cost allocation, k. There are however, two potential effects on the manager of changes in k, namely:

a) He might choose a different output level (compared to x^* from (7), which is always to the detriment of the owner; and/or

b) He might choose a different level of discretionary spending (compared to s^* from (7)), which could be to the benefit of the owner or not depending on whether it decreases or increases s, respectively.

Observe that top management's problem is how to modify its policy such that the manager's consumption of perquisites is reduced but *at the same time*, he is not induced to deviate from the optimum output choice rule.

Now consider changes in the top management lump sum cost allocation policy. Totally differentiating the manager's first order conditions in (11) with respect to k yields:

$$\begin{bmatrix} U_{\pi\pi} C_{xx} & U_{\pi s}(R_x - c_x) + U_\pi R_{xs} \\ U_{\pi s}(R_x - c_x) + U_\pi R_{xs} & U_{ss} + U_{\pi s}(R_s - 1) + U_\pi(R_{ss}) \end{bmatrix} \begin{bmatrix} dx \\ ds \end{bmatrix} =$$

$$(12) \quad \begin{bmatrix} 0 \\ U_{\pi s} + U_{\pi\pi}(R_s - 1) \end{bmatrix} [dk]$$

Applying Cramer's Rule and rearranging terms, the effects of changes in cost allocation, k, can be derived on the manager's output decision, x, and discretionary spending, s, as follows:

$$(13) \quad \frac{dx}{dk} = \frac{[-U_\pi R_{xs} \; U_{\pi s} + U_{\pi\pi}(R_s - 1)]}{D} = 0$$

since $R_{xs} = 0$ by the assumption that the firm is a price taker and

$$(14) \quad \frac{ds}{dk} = \frac{U_\pi C_{xx}[U_{\pi s} + U_{\pi\pi}(R_s - 1)]}{D} < 0$$

since the firm faces an increasing cost technology ($C_{ss} > 0$). The manager trades profits for perquisites ($U_{\pi s} > 0$) and overspends on perquisites for all output levels ($R_s - 1 < 0$ in (9) and D is positive from the second order condition in (8).

The results in equation (13) and (14) can be summarized in a formal theoretical proposition.

Proposition 4.1

For the model of the firm and of managerial behavior described in Section 4.1, a *lump sum* cost allocation will tend to reduce the manager's overspending on discretionary expense without affecting his already optimal output decision.

Figure 2 illustrates the effect of a lump sum cost allocation on managerial discretionary spending.

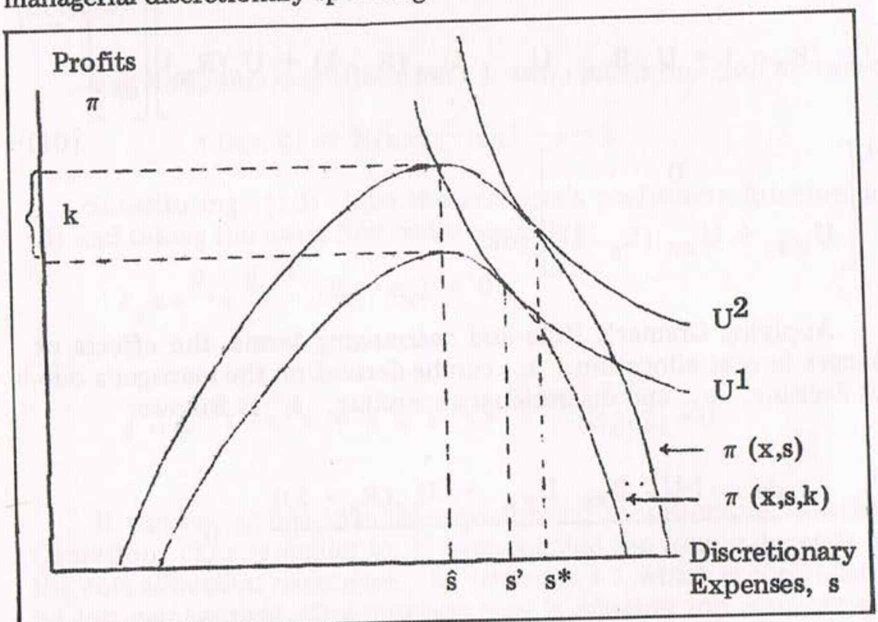

Figure 2

This observation, called the "lump sum tax effect", was also made by Zimmerman (1979) but was not rigorously derived as in this paper. To interpret the result in equation (14), observe that the lump sum allocation reduces the manager's opportunity set from $\pi(s)$ (without allocation) to $\pi(s; k)$ in Figure 2. Consequently, the level of discretionary spending goes down from s^* to s'. Equation (14) predicts the precise degree of adjustment in s and depends on the curvature of the manager's preference function. Of course, top management cannot impose an arbitrarily large lump sum cost because the manager must be satisfied a minimum utility level, say U^1 in Figure 2; otherwise he leaves the firm. The further result that the manager will not deviate from the correct output choice is an important one because it implies that this policy is "costless" (i.e., free of undesirable side effects) from the viewpoint of top management.

Similar analyses were conducted for the two other cost allocation methods.

Only the conclusions are presented here.

Proposition 4.2

For the model described in Section 4.1, a *per unit* cost allocation will tend to reduce the manager's overspending on discretionary expense but will also distort his perception of the marginal relationships such that actual output shall be below the optimum level.

Proof:

See Appendix A.

Proposition 4.3

For the model described in Section 4.1, the effect of *proportional* cost allocation on managerial discretionary spending is ambiguous although the output decision is unaffected.

Proof:

See Appendix A.

While a per unit cost allocation effectively addresses the overspending problem, top management could expect the manager to view the cost allocated as just another variable cost, resulting in underproduction, e.g., below x^* in equation (7), say x'. In this case, top management's use of per unit cost allocation policy will depend on whether the savings on discretionary expenses, $s^* - s'$, exceeds the profits foregone on the lower production, $\pi(x^*,s') - \pi(x',s')$.

The problem of "undesirable side effects" on the output decision is not present in proportional cost allocation. However, for a general utility function of the form $U(\pi, s)$, an increase in this cost allocation may increase or decrease the manager's discretionary spending. The analysis in Appendix A traces the reason for this ambiguity through a formulation similar to a Slutsky decomposition in consumer theory. In words, a proportional cost allocation has two separable effects. First, proportional cost allocation has the effect of reducing the "income" or opportunity set of the manager. From Proposition 4.1, we know that this results in a reduction in discretionary spending. Second, this cost allocation also has the simultaneous effect of serving as a "tax" on profits and of altering the manager's perception of the relationship between "taxed" profits and the "untaxed" commodity, discretionary expenses. Thus the manager increases his consumption of the "cheaper", untaxed good, perquisites, to maximize his utility. Whether the first effect on perquisites dominates the second opposing effect depends on the manager's unique preferences (i.e., the coefficients α, β_1, β_2).

Cost allocation is next discussed in a somewhat different model.

5. Model II: Cost Allocations and Managerial Output Decision Under Uncertainty

In the previous model, it was assumed that the manager knows the demand and cost parameters of the firm with perfect certainty, allowing him to derive x^* which maximizes profit. In a world of uncertainty, the output decision is more complex since considerations of managerial attitude toward risk shall be involved.

Sandmo (1971) and Leland (1972) generalized that the optimal output under uncertainty is less than the firm's output under certainty if the decision maker is risk averse. It follows that the *risk neutral* (or less risk averse) owner/top management would like to see the *risk averse* manager increase his output in an environment of uncertainty.

In this section, I abstract from the discretionary spending problem to focus on the firm's output decision problem under uncertainty. My objective is to show that certain cost allocation policies can be employed to induce a risk averse manager to increase his output.

The model is described next.

5.1 Model Assumption

Consider the firm with a single product which faces an uncertain price for the product. The firm's profit function, $\pi(x,\tilde{p})$, is defined:

(15) $$\tilde{\pi}(x, \tilde{p}) = \tilde{p}x - cx - F$$

where:

x : output
$\tilde{p}$: random price
C, F : variable and fixed costs

It is assumed that the manager "likes" expected profit, π, and "dislikes" variance in profit, σ^2_π. His utility functions:

(16) $$G(\bar{\pi}, \sigma^2_{\pi}) = b\bar{\pi} - c\bar{\pi}^2 - c\sigma^2_{\pi}$$

where:
$\bar{\pi} = \bar{p}x - cx - F$, expected profit (computed from (15))

$\sigma^2_\pi \equiv x^2$, the variance of profit (computed from (15)) expected price
b,c $\equiv$ positive constants
$b\pi - c\pi^2 > 0$, (he likes expected profit).

Observe that the manager has a positive but diminishing marginal utility for profits $(\partial G(\cdot)/\partial \bar{\pi} > 0, \partial^2 G(\cdot)/\partial \bar{\pi} \sigma^2 < 0$ and a constant disutility for variance of profits $(\partial G(\cdot)/\partial \sigma^2_\pi = C)$. It is convenient to define a "price of risk", m, as follows:

(17) $$m = \frac{(\bar{p} - c)}{\sigma^2_{\tilde{p}}}$$

in order to rewrite the expected profit, π, as a function of the variance of profit, σ^2_π:

(18) $$\bar{\pi}(\sigma^2_\pi) = m\sigma^2_\pi - F$$

The next section discusses the effects of cost allocations on the output decision.

5.2 Cost Allocation Policies and the Output Decision

As before, the effects of cost allocation shall be covered in some detail for one specific form. In this section, only proportional and lump sum cost allocations are analyzed.

Consider the proportional cost allocation, t, in (4). Then the manager's problem is to maximize his utility in (16) subject to an expected profit derived from (15). Formally, this could be set up as a constrained maximization problem in the Lagrangean form:

$$(19)\quad L = \{b(1-t)\bar{\pi} - c[(1-t)\bar{\pi}]^2 - c\sigma_{\pi}^2 - \lambda(\bar{\pi} - \sigma_{\pi}^2 m + F)$$

Taking the derivative of (19) with respect to the manager's choice variables, σ_{π}^2 and $\bar{\pi}$ and the multiplier λ yields the first order condition:

$$(20)\quad \begin{aligned} -c + \lambda m &= 0 \\ b(1-t) - 2c(1-t)\bar{\pi} - \lambda &= 0 \\ \bar{\pi} - m\sigma_{\pi}^2 + F &= 0 \end{aligned}$$

For a well-defined maximization in (20), it is sufficient that the determinant of the matrix of second derivatives in the constrained problem, or the bordered Hessian, be negative definite, i.e.,

$$(21)\quad D = \begin{bmatrix} L_{\sigma^2\sigma^2} & L_{\bar{\pi}}^2 & L\sigma_{\pi}^2 \\ L_{\pi\sigma^2} & L_{\bar{\pi}\bar{\pi}} & L_{\bar{\pi}\lambda} \\ L_{\lambda\sigma^2} & L_{\lambda\bar{\pi}} & L_{\lambda\lambda} \end{bmatrix} = \begin{bmatrix} 0 & 0 & m \\ 0 & -2c & -1 \\ m & -1 & 0 \end{bmatrix} = 2cm^2 > 0$$

where the subscripts are the partials of the Lagrangean with respect to the indicated arguments.

The owner's cost allocation policy affects the manager's output decision in (20). As before, totally differentiating equation (20) with respect to the cost allocation parameter, t, gives the effect of an increase in t on the output choice, x^*:

$$(22)\quad \frac{dx^*}{dt} = \frac{C/(1-t)}{D} > 0$$

This proves the next theoretical proposition.

Proposition 5.1

For the model of the firm and manager under uncertainty described in 5.1, an increase in *proportional* cost allocation will result in a decision by the risk averse manager to increase output.

In contrast, the reverse result holds for the lump sum cost allocation policy.

Proposition 5.2

For the model of the firm and manager setting under uncertainty described in 5.1, an increase in *lump sum* cost allocation will result in a decision by the risk averse manager to reduce output.

Proof:

See Appendix A

The intuition behind the first result is that the proportional cost allocation works like a "tax" which increases the "price of risk," m, which is equivalent to increasing the reward for risk-taking (producing more output). The lump sum cost allocation result is similar to the "fixed cost effect" in cost volume profit analysis (Adar, Barnea and Lev, 1977).

Conclusions

The paper evaluated the usefulness of certain cost allocation methods in addressing problems of managerial control. In general the output decision under uncertainty and profit-perquisites models used here appear to be appropriate settings for the cost allocation problem since each results in conditions where, due to the agent's maximizing behavior, the owner/top management is not necessarily satisfied with the situation.

One could make a key observation in the profits-perquisites model as to what drives the theoretical results. In this setting, the manager trades off two goods — profits and discretionary expenses — such that the principal exercises his prerogative of control by altering the "relative prices" of these goods through cost allocations. A similar observation is applicable to the model of the firm under uncertainty. For the scenario, top management could use cost allocations to modify the rewards to risk-taking that the manager faces.

A number of benefits could be gained from the analysis in this paper, particularly towards an understanding of the management control process and theory, among which are:

1. There could be some underlying rational economic explanations for some actual, observed management control practices which at first glance appear to be without theoretical basis. "Arbitrary" cost allocation may be one of them.

2. One should consider both the cost and benefits of a given management control technique. In the profit-perquisites model, it was not enough that a cost allocation technique reduced discretionary spending — there should be an equal consideration of its effects on other decisions by the manager, in this case, the output decision.

3. The conclusions to be derived from management control situations are in the end, case-to-case results. Alternatively stated, it is incorrect to say that cost allocation is a valid control technique for *all* situations. For example, more lump sum cost allocation works for the discretionary expense problem but not for the output-decision-under-uncertainty problem. Likewise, some effects, like the proportional cost allocation in Model I, are manager-specific.

APPENDIX

Model I: Cost Allocations and Managerial Discretionary Expenses

A. FIXED, LUMP-SUM ALLOCATION

$$U\,[s,\Pi\,(x,s,k)] = \alpha s^{\beta_1}\,[R(x\,s) - c(x) - s - k]^{\beta_2}$$

First Order Condition

$$U_x :\ \beta_2\,\alpha\,S^{\beta_1}\Pi^{\beta_2-1}\,(R_x - C_x) = 0$$

$$U_s :\ \beta_1\,\alpha\,S^{\beta_1}\Pi^{\beta_2} + \beta_2\,\alpha\,S^{\beta_1}\Pi^{\beta_2-1}\,(R_s - 1) = 0$$

Second Order Condition:

$$|D| = \begin{vmatrix} U_{xx} & U_{xs} \\ U_{sx} & U_{ss} \end{vmatrix} > 0$$

where:

$$U_{xx} = \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (Rxx - Cxx)$$

$$U_{ss} = \beta_1(\beta_1 - 1)\alpha S^{\beta_1 - 2} \Pi^{\beta_2} + 2\beta_1\beta_2 \alpha S^{\beta_1 - 1}\Pi^{\beta_2 - 1}(Rs - 1) + \beta_2 S^{\beta_1}\Pi^{\beta_2 - 1} Rss$$

$$U_{sx} = u_{xs} = \beta_2 \alpha S^{\beta_1}\Pi^{\beta_2 - 1} R_{xs} + \beta_1\beta_2\alpha S^{\beta_1 - 1}\Pi^{\beta_2 - 1}(Rx - Cx)$$

Total Differentials:

$$\begin{bmatrix} \beta_2^{\beta_2 - 1} \alpha S^{\beta_1}\Pi^{\beta_2 - 2}(Rxx - Cxx) & \beta_1\beta_2\alpha S^{\beta_1 - 1}\Pi^{\beta_2 - 1}(Rx - Cx) + \beta_2 \alpha S^{\beta_1}\Pi^{\beta_2 - 1}(Rxs) \\ \beta_2\beta_1 \alpha S^{\beta_1 - 1}\Pi^{\beta_2 - 1}(Rx - Cx) & \beta_1(\beta_1 - 1)\alpha S^{\beta_1 - 2}\Pi^{\beta_2} + \beta_1\beta_2\alpha S^{\beta_1 - 1}\Pi^{\beta_2 - 1}(Rs - 1) \\ + \beta_2\alpha S^{\beta_1}\Pi^{\beta_2 - 1}(Rxs) & + \beta_2\alpha S^{\beta_1}\Pi^{\beta_2 - 1}(Rss) \end{bmatrix} \begin{bmatrix} dX \\ dS \end{bmatrix} =$$

$$\begin{bmatrix} 0 \\ \beta_2\beta_1\alpha S^{\beta_1 - 1}\Pi^{\beta_2 - 1} + \beta_2(\beta_2 - 1)\alpha S^{\beta_1}\Pi^{\beta_2 - 2}(Rs - 1) \end{bmatrix} \begin{bmatrix} dk \end{bmatrix}$$

$$\frac{dX}{dk} = \frac{[-\beta_2 \alpha S^{\beta_1}\Pi^{\beta_2 - 1} R_{xs} \quad \beta_2\beta_1\alpha S^{\beta_1 - 1}\Pi^{\beta_2 - 1} + \beta_2(\beta_2 - 1)\alpha S^{\beta_1}\Pi^{\beta_2 - 2}(Rs - 1)]}{|D|}$$

$$= \frac{U_{\pi} R_{xs} \, [U_{\pi s} + U_{\pi\pi} \, (R_s - 1)]}{|D|} < 0 \rightarrow = 0 \text{ for } R_{xs} = 0 \text{ assumption}$$

$$\frac{ds}{dk} = \frac{\beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (R_{xx} - C_{ss}) [\beta_2 \beta_1 \alpha S^{\beta_1 - 1} \Pi^{\beta_2 - 1} + \beta_2 (\beta_2 - 1) \alpha S^{\beta_1} \Pi^{\beta_2 - 2} (R_s - 1)]}{|D|}$$

$$= \frac{U_{\pi} (R_{xx} - C_{xx}) \, [U_{\pi s} + U_{\pi\pi} (R_s - 1)]}{|D|} < 0$$

B. PROPORTIONAL ALLOCATION

$$U[s, \Pi(x,s,t)] = \alpha S^{\beta_1} [(1-t)(R(x,s)) - c(x) - S]^{\beta_2}$$

where t = allocation rate

First Order Condition:

$$U_s : \beta_1 \alpha S^{\beta_1 - 1} \Pi^{\beta_2} + \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (1-t)(R_s - 1) = 0$$

$$U_x : \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (1-t)(R_x - C_x) = 0$$

Second Order Condition:

$$|D| = \begin{vmatrix} U_{xx} & U_{xs} \\ U_{sx} & U_{ss} \end{vmatrix} < 0$$

$$U_{ss} = \beta_1 (\beta_1 - 1) \alpha S^{\beta_1 - 2} \Pi^{\beta_2} + \beta_2 \beta_1 \alpha S^{\beta_1 - 1} \Pi^{\beta_2 - 1} (1-t) \text{ Rss}$$

$$U_{xx} = \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (1-t)(R_{xx} - C_{xx}) + \beta_2(\beta_2 - 1)\alpha S^{\beta_1} \Pi^{\beta_2 - 2} (1+t)(Rx - Cx)$$

$$U_{sx} = U_{xs} = \beta_1 \beta_2 \alpha S^{\beta_1 - 1} \Pi^{\beta_2 - 1} (1-t)(Rx - Cs) + (\beta_2 - 1)\ (\beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 2} (1-t)(Rxs)$$

Total Differentials:

$$\begin{bmatrix} \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (1-t)(R_{xx} - C_{ss} & \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (1-t)(R_{xs}) \\ \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (1-t)(R_{xs}) & \beta_1(\beta_1 - 1)\alpha S^{\beta_1 - 2} \Pi^{\beta_2} + 2\beta_2 \beta_1 \alpha S^{\beta_1 - 1} \Pi^{\beta_2 - 1} (1-t) \\ & (R_s - 1) + \beta_2(\beta_2 - 1)\alpha S^{\beta_1} \Pi^{\beta_2 - 2} (1-t)(R_{ss}) \end{bmatrix} \begin{bmatrix} dX \\ dS \end{bmatrix} =$$

$$\begin{bmatrix} \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (R_x - C_x) \\ \beta_2 \beta_1 \alpha S^{\beta_1 - 1} \Pi^{\beta_2 - 1} (R-C-S) \\ + \ \beta_2(\beta_2 - 1)\alpha S^{\beta_1} \Pi^{\beta_2 - 2} (1-t)(R_s - 1)(R-C-S) \\ + \ \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (R_s - 1) \end{bmatrix} \begin{bmatrix} dt \end{bmatrix}$$

$$\frac{dX}{dt} = \frac{-\beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (1-t)\ R_{xs} [\beta_2 \beta_1 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (R-C-S) + \beta_2(\beta_2 - 1)\alpha S^{\beta_1} \Pi^{\beta_2 - 2} (1-t)(R_s - 1)(R-C-S) + \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (R_s - 1)]}{|D|}$$

$$= \frac{U_\pi (1-t)\ R_{xs} [U_{\pi S} \cdot \Pi + U_{\pi\pi} (1-t)(R_s - 1)\Pi + U_\pi (R_s - 1)]}{|D|} = 0 \quad \text{since} \quad R_{xs} = 0$$

$$\frac{dS}{dt} = \frac{1}{|D|} \cdot \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (1-t)(R_{xx} - C_{xx}) [\beta_2 \beta_1 \alpha S^{\beta_1 - 1} \Pi^{\beta_2 - 1} (R-C-S) +$$
$$\beta_2 (\beta_2 - 1) \alpha S^{\beta_1} \Pi^{\beta_2 - 2} (1-t)(R_s - 1)(R-C-S) +$$
$$\beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (R_s - 1)]$$

$$= \frac{[U_\pi (1-t)(R_{xx} - C_{xx})][U_{\pi s} + U_{\pi\pi}(1-t)(R_s - 1)]\Pi}{|D|} + \frac{U_\pi (1-t)(R_{xx} - C_{xx})[U_\pi (R_s - 1)]}{|D|}$$

$$= (1-t)\Pi . \frac{dS}{dT} + \frac{[U_\pi (1-t)(R_{xx} - C_{xx})][U_\pi (R_s - 1)]}{|D|} \lessgtr 0$$

Per Unit Allocation:

$$U[s_1 \pi(x,s,a)] = \alpha S^{\beta_1} [R(x,s) - c(x) - ax - s]^{\beta_2}$$

where a = UNIT ALLOCATION

First Order Condition:

$$\beta_1 \alpha S^{\beta_1 - 1} \Pi^{\beta_2} + \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (R_s - 1) = 0$$

$$\beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (R_x - C_x - a) = 0$$

Total Differential:

$$\begin{bmatrix} \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} (R_{xx} - C_{xx}) & \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} R_{xs} \\ \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} R_{xs} & \beta_1 (\beta_1 - 1) \alpha S^{\beta_1 - 2} \Pi^{\beta_2} + 2\, \beta_2 \beta_1 \alpha S^{\beta_1 - 1} \Pi^{\beta_2 - 1} (R_x - 1) \\ & + \beta_2 \alpha S^{\beta_1} \Pi^{\beta_2 - 1} R_{ss} \end{bmatrix} \begin{bmatrix} dX \\ dS \end{bmatrix} =$$

$$\begin{bmatrix} \beta_2 \alpha S^{\beta_1 \Pi^{\beta_2 - 1}} \\ X(\beta_1 \beta_2 \alpha S^{\beta_1 - \alpha} \Pi^{\beta_2 - 1}) + X(\beta_2(\beta_2 - 1)) \alpha S^{\beta_1 \Pi^{\beta_2 - 2}} (R_s - 1) \end{bmatrix} \begin{bmatrix} da \end{bmatrix}$$

$$\frac{dX}{da} = \frac{1}{|D|} \cdot \{\beta_1(\beta_1 - 1)\alpha S^{\beta_1 - 2} \Pi^{\beta_2} + 2\beta_1 \beta_2 \alpha S^{\beta_1 - 1} \Pi^{\beta_2 - 1} (R_s - 1) + \beta_2 \alpha S^{\beta_1 \Pi^{\beta_2 - 1}} (R_{ss})$$

$$[\beta_2 \alpha S^{\beta_1 \Pi^{\beta_2 - 1}}] + [-\beta_2 \alpha S^{\beta_1 \Pi^{\beta_2}} R_{xs}][\beta_1 \beta_2 \alpha S^{\beta_1 - 1} \Pi^{\beta_2 - 1} +$$

$$\beta_2 (\beta_2 - 1) \alpha S^{\beta_1 \Pi^{\beta_2 - 1}} (R_s - 1)] X$$

$$= \frac{1}{|D|} [U_{ss} + 2U_{s\pi}(R_{s-1}) + U_\pi R_{ss}] U_\pi + [-U_\pi R_{xs}][U_{\pi s} + U_{\pi\pi}(R_{ss} - 1) X]$$

or

$$\frac{dX}{da} = \frac{1}{|D|} U_\pi (U_{ss} + 2U_{s\pi}(R_{s-1}) + U_\pi R_{ss}) + X \cdot \frac{dX}{dT}$$

Note: The proofs for the result in Model II proceed as above and are available on request from the author.

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