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TWO EMPIRICAL NOTES ON THE URBAN ECONOMY

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I. Land Rents and Monocentricity

This note presents a simple test of the hypothesis of monocentricity common in urban location models. This assumption of a single center for a city is extremely convenient for analyzing the behavior of such critical variables as land rents, housing prices, and the location of various economic activities.

Theoretical Framework

Consider a city with a single center on a homogeneous plain of land. Households in the city consume only two goods: a private composite good and residential land. The members of each household make a fixed number of trips to the center (to work or shop) per unit time and incur transportation costs, the magnitude of which depends on the distance of their residence from the center. The expenditure function of a representative household may then be expressed as:

(1) $E(1, R(X), U) = Y - T(X)$

where :
- 1 : the normalized price of the composite good
- R(X) : rent for land located at distance X from the center of the city
- Y : the household's given income
- U : an index of household utility
- T(X) : transportation cost

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In long-run equilibrium, the size of the city, the number of households and the utility index for each household are fixed exogenously. Assume that household income and the price of the composite good are invariant with distance from the center. Thus equation (1) implicitly specifies land rent at any distance from the center of the city.

Differentiating (1) with respect to X, rearranging and dividing by R yields

(2) $R'/R = -T'/E_R R$

where :

R'/R : rent gradient

T' : marginal transportation cost

E_R : partial derivative of the expenditure function, giving the household demand for residential land.

All variables are functions of distance X. We now assume unitary price elasticity of demand for residential land and constant marginal transportation costs. These assumptions imply R'/R is constant. Hence,

(3) $R = R_o \exp (bX)$

where R_o is land rent at the center of the city and the constant rent gradient is $b = -T'/E_R R$.

Suppose, however, that the city has more than one center, say three centers. Maintaining the assumptions of unitary price elasticity and constant marginal transportation costs to each center, we have

(4) $R = R_o \exp (b_1 X_1 + b_2 X_2 + b_3 X_3)$

where X_i is distance from the *ith* center and $b_i = -T_{x_i}/E_R R$.

Data and Estimates

The observations are on 54 randomly selected enumeration districts in Metropolitan Manila. Land rents are derived from the median assessed land values within each district and are expressed in pesos per square meter. Distances are measured from the centroid of each district to the centroids of the three major commercial centers in Metropolitan Manila (Escolta, Cubao, and Makati Commercial Center) and are expressed in kilometers.

We assume multiplicative error terms which are log-normally distributed. The regression estimates of equations (3) and (4) are:

$$(5)\quad R^* = 5.84 \underset{(7.14)}{-0.15X_1} \qquad (R^2 = 0.48) \quad (SSR = 11.545)$$

$$(6)\quad R^* = 5.74 \underset{(7.20)}{-0.16X_1} \underset{(1.46)}{+0.03X_2} \underset{(0.59)}{-0.01X_3} \qquad (R^2 = 0.49) \quad (SSR = 11.056)$$

where R^* is land rent in logarithmic form. The coefficients of determination are adjusted for degrees of freedom. The numbers in parentheses below the coefficients are t-ratios. Only the coefficient of X_1 is statistically significant (at 5 per cent). It shows that land rent declines at the rate of 15 per cent per kilometer from the first center in Metropolitan Manila. The plotting of residuals reveals no strong evidence of heteroscedasticity. The nature of the data is also such that serious correlations between error terms are unlikely.

Test of Hypothesis

The null hypothesis is that only the distance to one center in a city matters in the aggregate behavior of land rents. The relevant test is a test on a single linear constraint that sets b_2 and b_3 both equal to zero. Equation (5) results from estimation subject to such a constraint, and equation (6), from unconstrained estimation. Knowing that the likelihood ratio reduces to a power of the ratio of the sum of squared residuals (SSR) of the two equations, we can derive the proper F statistic as

$$F(1, N-K) = (SSR_1 - SSR_2)\,(N-K)/SSR_2$$

where $N-K$ is 50; SSR_1 refers to equation (5) and SSR_2 to equation (6). The computed F we get is 2.211 which is not statistically significant (at 5 per cent). Hence, we fail to reject the null hypothesis.

Conclusion

Metropolitan Manila is as large as cities come. Yet even in this case, monocentricity does not appear to be a very restrictive assumption. Urban economic theorists need not apologize each time they assume a single center.

II. An Estimate of the Income Elasticity of Urban Transportation Costs

The income elasticity of urban transportation cost is an important parameter in urban economics. It is relevant in such applications as the evaluation of public transportation projects, as well as in settling such issues as why the rich tend to live farther away from the center of the city than the poor do.

This note attempts to get at the value of this income elasticity by an indirect but simple procedure. The procedure is derived from the standard long-run equilibrium condition for household location in a city. The data required are land rents, land per household and location in terms of distance from the center. Income data are not required if an independent estimate of the income elasticity of the demand for land is available.

Using data for Metropolitan Manila, the income elasticity of the cost of urban commuting is estimated to be roughly half of the income elasticity of demand for residential land. This result is consistent with the stylized fact of the rich living farther away from the center of the city than the poor.

Theoretical Framework

We assume here that urban households consume only two goods: a numeraire composite good Z and residential land L. The members of each household make a fixed number of trips to the center per unit of time and incur transportation costs which include the value of travel time. Following Muth (1969), members of a household behave to maximize the Lagrangean expression:

$$\mathcal{L} = U(Z,L) + \lambda\,(Y - Z - R(K)\,L - T(K,Y))$$

where :

- Y : household income including the value of leisure and travel time
- $R(K)$: rent per unit of land at distance K from the center of the city
- $T(K,Y)$: transportation cost as a function of both distance and income.

The two first-order conditions with respect to the consumption of Z and L are familiar: $U_Z = \lambda$ and $U_L = \lambda R(K)$. The third condition with respect to location is:

$$-\lambda\,(R'(K)L + T_K(K,Y)) = 0.$$

The household chooses the residential location where the increase in transportation cost from a small movement away from the center of the city is just equal to the decrease in total land rent the household will have to pay. This third condition reduces to

$$-R'(K) = T_K(K,Y)/L.$$

Suppose now that marginal transportation costs are constant for a given household income. Specifically, let $T(K,Y) = AY^{\alpha}K$ where α is the income elasticity of transportation costs. Then equation (1) gives us the estimating equation

$$(-R')^* = b_o + b_1 Y^* + b_2 L^* \qquad (2)$$

where the asterisks indicate logarithms of the variables and $b_o = A^*$, $b_1 = \alpha$ and $b_2 = -1$. If we had data on household income, equation (2) can be estimated directly to give us an estimate of α. However, since income data are currently unavailable, we will have to exploit what we know about the functional relationship between Y and L to get at the value of α.

The whole procedure involves three steps. First, values of the rent gradient $R'(K)$ are generated by estimating an envelope bid-rent function for the entire city. Then, equation (2) is estimated by least squares but leaving out the variable Y^*. This yields a biased estimate of b_2. Finally, knowing the extent of the bias and knowing the income elasticity of the demand for residential land allows us to derive an estimate for the income elasticity of transportation cost.

Data and Estimates

The data consist of observations on a random sample of 89 census enumeration districts in Metropolitan Manila. Land rents are taken from median assessed land values. Distances are measured in kilometers from the centroid of each enumeration district to the predominant commercial center of the city. Demand for residential land is derived by dividing the area of the enumeration district by the number of households in the district.

Several functional forms for the envelope bid-rent function were estimated. The gamma function appears to give the best fit for least squares:

$$R^* = 6.005 + 0.019\,K - 0.690\,K^* \qquad (3)$$
$$(67.16)\quad (1.02)\qquad (-6.14)$$

$$R^2 = 0.57 \qquad SSR = 11.99 \qquad F = 57.79$$

where the asterisks again indicate logarithms and the numbers in parentheses are t-values. As expected, this estimated bid-rent function is downward-sloping and convex.

Using the absolute values of the rent gradients predicted by equation (3), we estimate equation (2) omitting Y^* because of the lack of data. The result is

$$(-R')^* = 4.905 - 0.529\,L^* \qquad (4)$$
$$(15.61)\quad (-5.33)$$

$$R^2 = 0.24 \qquad SSR = 113.84 \qquad F = 28.38$$

As expected, the estimated coefficient of L^* is significantly different from -1 because of the omission of the relevant variable Y^*. Let b' be the vector of coefficients for the estimated equation. Then, econometric theory tells us that $E(b') = Pb$ where P is the matrix of regressions of the relevant variables on the ones included and b is the vector of true coefficients. Hence, if n is the income elasticity of demand for residential land and b_2 is the estimated coefficient for L^*, we have

$$E(b_2') = b_2 + b_1/n = -1 + \alpha/n. \qquad (5)$$

The income elasticity of the demand for housing has been estimated elsewhere (Muth, 1971) to be close to unity. If residential land is treated properly as an input in the production of housing and if such production is characterized by constant returns to scale, then the long-run derived demand for residential land should have an income elasticity of close to unity as well. There is no obvious reason why this estimate of income elasticity should be different for Metropolitan Manila. Substituting this value for n and the estimate of b_2' from equation (4) into equation (5) gives us an estimate for α of 0.47. Whatever the true value of n is, the income elasticity of transportation cost is estimated here to be 47 per cent of it. With this result, the rich will indeed tend to live farther away from the center than will the poor.

Conclusion

We have estimated the income elasticity of urban transportation cost by deriving it from the long-run equilibrium condition for household location in a city. We assumed constant marginal transportation costs for given income and used a gamma function to fit the envelope bid-rent function. It remains to be seen how robust our estimate will be for a different set of data or for modified assumptions.

REFERENCES

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