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This paper presents a test of multi-period asset pricing models using quarterly Philippine data. Using a consumption-based asset-pricing model, the study finds the rate of time preference to be 5.20 percent (on an annual basis). The estimated risk aversion coefficient of 0.043 seems to be on the low side when compared with estimates for other countries. Hansen's J-test finds favorable evidence for the C-CAPM as the overidentifying restrictions are not rejected.

1. Introduction

This study conducts a test of the consumption-based capital as set pricing model (C-CAPM) using Philippine data. The test generates estimates of the subjective rate of time preference and the risk aver sion parameter using the power utility function. The 91-day Trea sury bill is the asset used in the test.

The C-CAPM has been tested extensively for other countries with mixed results. For example, Hansen and Singleton (1982), using US data, rejects the model in most of their tests. For some countries however, the model fits the data quite well [See for example, Hamor (1992) and Lund and Engsted (1996)]. All these empirical investigations were conducted using the generalized method of moments (GMM estimation procedure due to Hansen (1982). This study also make use of this estimation procedure and compares the results with those conducted for these countries.

Also known as the constant relative risk aversion (CRRA) utility function.

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The paper is organized as follows. The next section discusses the consumption-based asset-pricing model. The third section describes the GMM estimation. The fourth section describes the data, presents the results, and compares them with the results of other studies. The last section gives the areas for further studies.

2. The C-CAPM

The C-CAPM basically relates asset prices to the economic agent's consumption and portfolio decisions over time.² The intertemporal nature of the problem is best expressed as a constrained maximization process where the representative economic agent possesses an expected utility function:

(1)
$$E_i \left[\sum_{k=0}^{\infty} \beta^k u \left(c_{i+k} \right) \right]$$

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The intertemporal budget constraint is given by:

(2)
$$w_{t+1} = (w_t - c_t) \sum_{i=0}^{N} x_i R_{it+1}$$

 β is the time discount factor that is assumed constant; c_t is real consumption; w_t is real wealth; x_i is the share of asset i in the individual's wealth and $\sum\limits_{i=0}^{N} x_i = 1$. R_{it} is the real gross return of asset i at time t. E_t is an expectations operator conditioned on information available at time t.

The solution, using dynamic programming techniques, is a set of Euler equations of the form:

(3)
$$1 = E_i \left(m_{i+1} R_{ii+1} \right)$$
 ; $i = 0, 1, ..., N$

² The pioneering article on multi-period asset pricing along these lines was written by Lucas (1978).

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where $m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$ is the stochastic discount factor for this model. In this model, it is also the intertemporal marginal rate of substitution.

To see how asset prices relate to consumption/investment decisions, take the unconditional expectation of (3) and lag one period to get the unconditional version:

$$(4) 1 = E\left(m_{i}R_{ii}\right)$$

Using the definition of the covariance, Exy = ExEy + cov(x, y), one can rewrite (4) as:

$$1 = Em_{i}ER_{ii} + \operatorname{cov}(m_{i}, R_{ii})$$
(5)
$$ER_{ii} = \frac{1}{Em_{i}} \left[1 - \operatorname{cov}(m_{i}, R_{ii}) \right]$$

Next, note that for the risk-free rate, $cov(m_t, R_0) = 0$. Therefore,

$$ER_0 = \frac{1}{Em_i}$$

Substituting this in (5), one gets:

(6)
$$E\left(R_{ii}-R_{0}\right)=-ER_{0}\operatorname{cov}\left(m_{i},R_{ii}\right).$$

For the risk premium to be positive, the covariance term in (6) has to be negative or that the asset return has to be positively correlated with consumption. Note that, for a given risk-free rate, a low return for asset i that implies a higher m_i , implies a low consumption level. This asset fails to deliver wealth when it is most needed and therefore is, in this sense, a risky asset that requires a higher premium.

As mentioned, this study makes use of the power utility function:

(7)
$$u\left(c_{i}\right) = \frac{c_{i}^{1-\alpha} - 1}{1-\alpha}$$

where $\alpha \neq 1$ is the coefficient of relative risk aversion.³ Computing for the marginal utility and substituting in (3) yields:

(8)
$$1 = E_i \left[\beta \left(\frac{c_{i+1}}{c_i} \right)^{-\alpha} R_{ii+1} \right] ; \quad i = 0, 1, ..., N$$

Equation (8) is the central asset pricing equation that has been the basis of extensive empirical examination. Asset pricing issues are examined using this framework by estimating the parameters β and α. This is done for the Philippines using quarterly data from 1981 to 1997.

3. GMM estimation

The generalized method of moments (GMM) can be viewed as an extension of instrumental variables regression. GMM estimators coincide with 3SLS estimators when errors are serially independent and the same instruments are used for every equation.⁴

Hansen and Singleton (1982) has demonstrated that the parameters of dynamic nonlinear rational expectations models can be estimated directly from the first order conditions like those in Equation 3 (or 8) using GMM. These Euler equations, which must be satisfied at equilibrium, imply a set of population orthogonality conditions whose parameters may be estimated by making sample orthogonality conditions as close to zero as possible.

³ It is well known that the inverse of α is the elasticity of intertemporal sub-

⁴ A good introduction to GMM estimation can be seen in Ogaki (1992). A comparison with maximum likelihood estimation is given in the Appendix of Campbell, et al. (1997).

Let $e_t = e_t(x_{t+1}, \theta)$ be the disturbance that arises from the first order conditions in Equation 8. x_t contains c_t and R_{it} while $\theta = (\alpha, \beta)$ is the parameter vector. The implication of the model is that $E(e_t|I_t) = 0$ where I_t is the agent's information set.

Thus, for any vector of instruments, z_l , in the agent's information set, it follows that:

$$(9) \quad E(e_t z_t) = E[E(e_t(x_{t+1}, \, \theta)|I_t)|z_t] = 0.$$

Let $g_T(\theta) = \frac{1}{T} \sum_{i=1}^{T} e_i(x_{i+1}, \theta) z_i$ be the sample version of (9). The GMM estimator is obtained by choosing θ to minimize:

(10)
$$J_T(\theta) = g_T(\theta)' W_t g_T(\theta)$$

 W_t is a symmetric, positive definite weighting matrix that can depend on sample information. The optimal weighting matrix, W_t^* , is derived in two stages. First, initial estimates of the parameter vector θ_0 are obtained using a sub-optimal W. Next, these estimates are used to derive the optimal weighting matrix to arrive at the final estimates of the parameter vector.

In practice, there are more orthogonality conditions than parameters to be estimated. Thus, these models are said to be overidentified. A test of these overidentifying restrictions described in Hansen (1982) tries to determine how close the sample orthogonality conditions are to zero, and provides a measure of goodness of fit.

Hansen's J-test is a Lagrange multiplier test of overidentifying restrictions. It is equal to the minimized value of the objective function in Equation (10) times the number of observations, $J_T(\theta)T$, and is distributed as χ^2 with degrees of freedom equal to the number of orthogonality conditions minus the number of parameters to be estimated. Detailed discussion of GMM and recent developments can be seen in the references cited in footnote 4.

4. Data and Estimation Results

Data on the Philippine equities market exist but are not readily available in a form that is useful to researchers. While there is a stock market index, PSI, publicly made available on a regular basis by the Philippine stock exchange, there is no dividend series that accompanies the stock price index. Because of this, the study limits itself to the treasury bills market where the data are collected and published by the Bangko Sentral ng Pilipinas (BSP).

The results below were obtained using quarterly data on not seasonally adjusted real consumption and population from the National Statistical Coordinating Board. Data on the 91-day treasury bills rate were obtained from the BSP while the consumer price index (p_l) is from the National Statistics Office. Data on nominal annual yield (r) are converted into quarterly rates and the realized real interest rate is computed as:

$$\frac{p_{t-1}}{p_t}(1+r)^{1/4}$$
.

The data are for the first quarter of 1981 to the first quarter of 1997. Statistical properties of the data are described by the test for stationarity in Tables 1 and 2. Two tests for stationarity were used. The ADF test, shown on the first half of the Tables, has the unit root as the null (or that the variable follows a difference stationary process). The second half of each Table shows the KPSS tests that has either level or trend stationarity as the null. For most cases in Table 1, it seems that the null hypothesis of a unit root cannot be rejected for the log of real per capita consumption. This is reinforced by the rejection of trend stationarity as can be seen from the KPSS statistics. The opposite is true for the real interest rate. Trend stationarity cannot be rejected but the unit root hypothesis is.

 $^{^{5}}$ See Kwiatowski, et al. (1992) for a discussion of this test.

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Table 16 - Log of Quarterly Real Per Capita Consumption, 1981:1 - 1997:4 (68 observations)

	ADF			KPSS	
Lags	*	**	***	Level	Trend
0	0.29	-6.69	-9.71	2.235	0.1676
1	0.51	-3.08	-5.25	1.994	0.2173
2	0.49	-2.64	-5.49	1.586	0.2105
3	2.82	1.70	-1.62	1.409	0.2516
4	1.09	-0.01	-2.22	1.091	0.1776
5	1.02	-0.11	-2.43	0.940	0.1610
6	1.00	-0.19	-2.57	0.828	0.1477
7	0.83	-0.07	-3.45	0.761	0.1497
8	0.88	-0.64	-3.85	0.678	0.1316
Asymp	ototic Crit	ical Values			
1%	-2.56	-3.43	-3.96	0.739	0.216
5%	-1.94	-2.86	-3.41	0.463	0.146
10%	-1.62	-2.57	-3.13	0.347	0.119

^{* -} no constant, no trend

^{*** -} with constant and trend

^{** -} with constant, no trend

⁶ The ADF and KPSS statistics were computed using the GAUSS code due to Isaac and Rapach. The GAUSS code is available at http://gurukul.ucc.american.edu/econ/gaussres/COINT/COINTHTM.

Table 2 - Real Interest Rate, 1981:1-1997:4 (68 observations)

	ADF			KPSS	
Lags	* 100	**	***	Level	Trend
0	-0.10	-4.90	-4.87	0.1341	0.1053
1	0.06	-3.53	-3.49	0.0934	0.0735
2	-0.02	-4.30	-4.26	0.0749	0.0589
3	-0.17	-3.58	-3.58	0.0691	0.0544
4	-0.04	-3.68	-3.67	0.0667	0.0526
5	0.10	-4.19	-4.15	0.0678	0.0536
6	-0.08	-3.51	-3.50	0.0717	0.0568
7	-0.50	-3.22	-3.40	0.0782	0.0621
8	-0.12	-2.91	-2.99	0.0861	0.0687
Asymp	totic Crit	ical Values			
1%	-2.56	-3.43	-3.96	0.739	0.216
5%	-1.94	-2.86	-3.41	0.463	0.146
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Table 3 below presents the results of GMM estimation using the data described above. The instruments used in the estimation are the first to fourth lags of consumption and the real interest rate and a constant. The choice of lag length in the instruments was made based on the seasonality of consumption data. Likewise, the standard errors below were corrected for fourth-order moving average errors.

Table 3 - GMM Estimates of Equation 8

	Estimate	Standard Error
β	0.9874	0.0018
α	0.0430	0.0121
J(7)	5.5850	<i>P</i> -value = 0.589

As can be seen, both coefficient estimates are statistically significant. The implied rate of time preference is 5.20 percent (on an annual basis). Hansen's *J*-test finds favorable evidence for the C-CAPM as the overidentifying restrictions are not rejected.

The risk aversion coefficient of 0.043 seems to be on the low side when compared with other countries. Hamori (1992) derives a value of 0.242 for Japan. Test for overidentifying restrictions yielded good results for Japan. Lund and Engsted's (1996) country estimates for Germany, Denmark, Sweden, and UK show a range of values from -26.13 to 10.36 using four different sets of instruments for each country. The *J*-tests do not reject the model except for one in twenty estimates.

For the US, most studies reject the overidentifying restrictions. Because temporal aggregation and seasonality interact, several recent studies try to get around this problem by incorporating seasonal factors in model formulation as in Chan (1994). Chan obtains estimates of α that lie between 0.31 and 0.43. Hansen and Singleton (1982), using stock return data, provides estimates of α ranging from 0.36 to

0.99. However, overidentifying restrictions are strongly rejected in their case. Shome, Smith and Pinkerton (1988), on the other hand, obtain an estimate of 0.676 using nonlinear regression.

5. Areas for Further Research

This paper provides the first test of multi-period asset pricing models in the Philippines. The study computes initial estimates of important parameters, such as the rate of time preference, that are used in conducting economic cost-benefit analysis. This study was severely constrained by asset market data. A natural extension of the test is to use stock market data at the aggregate, as well as at a disaggregated, level. This can however be done only if reliable time series dividend yield data can be gathered. Thus, an important empirical undertaking is an assessment of equity market data and how it can be organized to generate these data series.

The C-CAPM using the power utility function is the basis of the literature on the equity premium puzzle. Another area of investigation is to determine if there is an equity premium and if the puzzle exists for the Philippines.

Recent developments in consumption-based asset pricing theory have largely been unexplored empirically. These studies have been conducted to mainly find solutions to the equity premium puzzle, as in Constantinides' (1990) habit formation model, or to break the link between the risk aversion coefficient and the elasticity of intertemporal substitution as in Epstein and Zin's (1991) model which generalizes the power utility function.

⁷ The puzzle is that, for US data, given (1) the equity premium of 6 percent and (2) the covariance between consumption growth and the return on risky asset, the risk aversion parameter required by the data is exceedingly high to fit the model. See Mehra and Prescott (1985) for the original exposition of the puzzle and Kocherlakota (1996) for an updated review.

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