

## ON MULTIPLE OBJECTIVES IN THE FIRM AND ARROW'S THEOREM

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An increasing amount of literature on the theory of the firm has argued for acceptance of the fact that the firm will typically have several objectives. Section I traces some points in this development and is not meant to be a survey of the very extensive literature. In section II conditions for the celebrated impossibility theorem of Professor Arrow are restated in an obvious manner for the context of the firm's decision making when there are several objectives. The argument of section III is that one of the conditions for the Arrow theorem could be dropped with good reason. If the conclusion is cogent, it would help to nullify what seems to have been a discouraging effect of the theorem on the construction of hypotheses regarding the firm's behavior under multiple objectives.

### I

Professor Hicks seems to have been the first to question the universal validity of the profit maximization assumption when he remarked that "the best of all monopoly profits is a quiet life."<sup>1</sup> This statement was premised on the observation that since the monopolist is likely to be subject to "sharply rising subjective costs," it would probably be to his advantage not to try to achieve a maximum profit position. Subjective costs thus entering consideration, Professor Scitovsky then showed that if the entrepreneur is to maximize his satisfaction—taking the latter as depending on money income and leisure—by maximizing profit, he would have to have a special indifference map which cannot be expected to hold in general.<sup>2</sup> Scitovsky did not, however, suggest dispensing with profit maximization as an assumption for purposes of theory. According to him, the assumption is still to be considered useful. This evaluation is somewhat surprising, since on the basis of Scitovsky's own discussion, utility maximization is on the same logical footing as profit maximization, considering that his measure of profit is defined in terms of indifference curves.

<sup>1</sup> J. R. Hicks, "Annual Survey of Economic Theory: The Theory of Monopoly," *Econometrica*, 3 (1935), 1-20, p. 8.

<sup>2</sup> T. Scitovsky, "A Note on Profit Maximisation and Its Implications," *Review of Economic Studies*, 11 (1943-44), 57-60.

Subsequently Professor Reder, writing at the time of the Lester-Machlup controversy in the pages of the *American Economic Review*, suggested several hypotheses alternative to profit maximization.<sup>3</sup> Reder observed that if enough profit is made to satisfy stockholders, the management could pursue other objectives. He also suggested the possibility that the management may try to obtain some specified rate of return on net worth, beyond which efforts may be directed towards other objectives. Reder emphasized a constraint which is usually ignored in theoretical discussions, namely, that if the entrepreneur maximizes profit he does so provided that he retains control of the firm. When there is a likelihood of losing control, the entrepreneur would try to retain control rather than to maximize profit. The behavior of the firm would obviously be different under such circumstances. There are several interesting things to be seen in Reder's discussion. One is the emphasis on multiple objectives; another is the suggestion that when two objectives cannot be simultaneously satisfied, one or the other has priority in some sense. What would ordinarily be a constraint (*e.g.*, retaining control) becomes the dominant objective in determining behavior. There is also the point that after reaching a particular rate of return, management may be relatively unconcerned about earning more. If these observations are accurate, they will have to be explained by some future theory of the firm.

Other writers have questioned the profit maximization assumption. Professor Fellner discusses several motives of business behavior other than profit.<sup>4</sup> Professor Gordon points out that even if the ultimate objective were maximum profit, the business man would also have other subordinate objectives.<sup>5</sup> He suggests that "satisfactory" profit might be a more realistic concept than maximum profit. More recently, Professors Carter and Williams have discussed a variety of business motives which they have identified in the course of an empirical investigation of investment decisions.<sup>6</sup> Professor Chamberlain states that even when not made explicitly, share of the market as an objective can be identified in a firm's budgeting decisions.<sup>7</sup> He suggests that the evidence indicates that managements, rather than maximizing profit, attempt to realize some profit rate which is con-

<sup>3</sup> M. W. Reder, "A Reconsideration of the Marginal Productivity Theory," *Journal of Political Economy*, 55 (1947), 450-58.

<sup>4</sup> W. Fellner, *Competition Among the Few*, New York 1949, chs. 5 and 6.

<sup>5</sup> R. A. Gordon, "Short-Period Price Determination in Theory and Practice," *American Economic Review*, 38 (1948), 265-88.

<sup>6</sup> C. F. Carter and B. R. Williams, *Investment in Innovation*, London 1958, ch. 4.

<sup>7</sup> N. W. Chamberlain, *The Firm: Micro-economic Planning and Action*, New York 1962, p. 67.

sidered acceptable.<sup>8</sup> Most recently, Dr. Shubik has discussed the fact of multiple objectives in the firm and indicated some of the difficulties posed by the multiplicity of goals.<sup>9</sup>

All these simply emphasize the increasing recognition of the fact that the firm has multiple objectives. As one distinguished economist has put it, "the assumption that a firm pursues maximum profits is an extreme simplification. Indeed, it is a simplification to assume that any unified objective governs all the operations of a firm."<sup>10</sup>

The question raised by the fact of multiplicity is how incompatible objectives are to be expressed in a function reflecting the firm's preferences and decisions. It is clear that such a preference function, ordering the alternatives facing the firm, is necessary for a theory of the firm. But then we seem confronted with the Arrow impossibility theorem, according to which no such function is possible even under apparently reasonable conditions.

## II

Arrow's theorem was originally formulated in the context of welfare economics.<sup>11</sup> The premises of its construction, however, are quite general and capable of adaptation to various situations.<sup>12</sup> Soon after the first publication of Arrow on the subject, Professor Papandreou observed that if the preference function maximized by the chief coordinator in a firm reflects several lines of influence, then "if influence takes the form of authority, and if authority is simultaneously exercised by two or more interest groups in a contradictory manner, the peak coordinator will not be able to formulate a consistent preference system."<sup>13</sup> Papandreou seems to have had in mind here a direct translation of the Arrow result to the case of the firm, where the peak coordinator would express group choice as a function of the preferences of interest groups. While Papandreou's problem possesses some interest of its own, the question of aggregating several objectives to form a single overall preference function seems more tractable. Notice that if each interest group can be identified as pursuing one objective

<sup>8</sup> *Ibid.*, p. 76.

<sup>9</sup> M. Shubik, "Objective Functions and Models of Corporate Maximization," *Quarterly Journal of Economics*, 75 (1961), 345-75.

<sup>10</sup> J. M. Clark, *Competition as a Dynamic Process*, Washington 1961, p. 91.

<sup>11</sup> K. J. Arrow, "A Difficulty in the Concept of Social Welfare," *Journal of Political Economy*, 58 (1950), 328-46; *idem*, *Social Choice and Individual Values*, New York 1951.

<sup>12</sup> See, e.g., K. O. May, "Intransitivity, Utility, and Aggregation," *Econometrica*, 22 (1954), 1-13, who applies the theorem to the case of an individual evaluating alternatives according to several criteria.

<sup>13</sup> A. G. Papandreou, "Some Basic Problems in the Theory of the Firm," in *A Survey of Contemporary Economics*, Vol. II, ed. B. F. Haley, Homewood, Ill. 1952, 183-219, p. 211.

to the exclusion of others, it suffices to consider only objectives as such. More important from the viewpoint of the economist formulating an explanation of the firm's behavior, the extent to which a particular objective is reached seems more observable. Accordingly we consider the problem of forming a preference function on the basis of several objectives.<sup>14</sup>

Let  $a, b, \dots$ , be the alternatives facing the firm. Each alternative  $x$  is a state of affairs which would result from particular decisions and policies which could be undertaken by the firm, having to do with inputs, outputs, prices, advertising expenditures, dividend payments, etc. Let the firm have  $n$  objectives, denoted 1, 2,  $\dots$ ,  $n$ . These would probably involve such items as profit, liquidity position, share of the market, dividend policy, sales revenue, growth rate of the firm, etc. We suppose that corresponding to each alternative  $x$  and each objective  $i$  there is a function  $f_i$  such that the value of the function indicates the valuation of the alternative in terms of the  $i$ -th objective. Thus,

$$u_i = f_i(x) \quad i = 1, \dots, n$$

with the property that  $x$  is superior to  $y$  according to the  $i$ -th objective, written  $xP_i y$ , if and only if  $f_i(x) > f_i(y)$ . The  $u_i$  then serve as preference indicators. For simplicity we ignore the possible case of indifference between alternatives. That is, we assume that given any two alternatives  $x$  and  $y$ , either  $xP_i y$  or  $yP_i x$ . Assuming also that if  $xP_i y$  and  $yP_i z$  then  $xP_i z$  (transitivity), the relation  $P_i$  thus constitutes an ordering of alternatives by the  $i$ -th objective. The alternatives can be imagined as being listed in vertical columns, one column for each objective, in such a way that one alternative is placed above another if its preference index  $u_i$  is higher. Accordingly there would be a profile of orderings  $(P_1, \dots, P_n)$ , and the Arrow problem is the existence of an overall ordering  $P$  which would be given by some function  $F$  of  $(P_1, \dots, P_n)$ . Put otherwise, the Arrow problem concerns the possibility of stating a relation of the form<sup>15</sup>

$$u = f(x) = F[f_1(x), \dots, f_n(x)]$$

such that  $xPy$  if and only if  $f(x) > f(y)$ , where the overall preference relation  $P$  is transitive. Under certain conditions, no  $F$  is possible. Accordingly we wish to examine whether such conditions<sup>16</sup> are reasonable in the context of the firm.

<sup>14</sup> See R. D. Luce and H. Raiffa, *Games and Decisions*, New York 1957, ch. 14, for a lucid presentation of the general problem.

<sup>15</sup> The  $f_i$  as well as  $x$  are arguments of the function  $F$ , and the  $f_i$  are not assumed fixed.

<sup>16</sup> The following statements of the conditions largely follow J. H. Blau, "The Existence of Social Welfare Functions," *Econometrica*, 25 (1957), 302-13, and Y. Murakami, "A Note on the General Possibility Theorem of the Social Welfare Function," *Econometrica*, 29 (1961), 244-46.

*Condition A (Unanimity Rule of Preference)*: For a given profile  $(P_1, \dots, P_n)$ , if  $xP_iy$  for all  $i$ , then  $xPy$ .

This condition required of  $F$  is obviously unexceptionable and calls for little comment. It would be an odd aggregating function which would permit the possibility of an overall preference for  $x$  against  $y$  when  $y$  is superior to  $x$  in terms of each objective.

*Condition B (Binary Choice)*: If two profiles are the same with regard to  $x$  and  $y$ , then the corresponding overall comparisons between  $x$  and  $y$  are identical.

That is, whether  $xPy$  or  $yPx$  is knowable by looking at how the objectives individually compare  $x$  and  $y$ . Other alternatives are irrelevant for this purpose, and it makes no difference whether, in a particular objective's ordering,  $x$  and  $y$  occupy top and bottom of the list or are next to each other. It is simply a matter of finding whether or not  $xP_iy$ ,  $i = 1, \dots, n$ , which information is sufficient to determine whether  $x$  is superior to  $y$  in the overall ordering. We shall return to this condition later.

*Condition C (Monotonicity)*: Let  $Q = (Q_1, \dots, Q_n)$  be a partial profile, listing only the individual comparisons between  $x$  and  $y$ . Suppose here that for some  $i$ ,  $yP_ix$ , and let this change to  $xP_iy$ , thus giving a new partial profile  $Q' = (Q'_1, \dots, Q'_n)$ . Then, if  $xPy$ ,  $xP'y$  also.

Given  $Q$ , the overall preference between  $x$  and  $y$  is known by virtue of condition B.  $Q'$  gives similar information. Condition C then requires that if  $xPy$  to begin with, and if in one or more objective orderings  $x$  has risen relative to  $y$ , in the new overall ordering we must still have  $xPy$ . In itself this condition seems innocuous and quite reasonable. It has an obvious interpretation in the preference function of the firm, if account is taken of the possibility that new information may affect the ordering of alternatives by some objective. For instance, it may be that with the information available at a given time,  $y$  is ranked higher than  $x$  according to the profit criterion. The possibility is not to be ruled out that with additional information,  $x$  is found superior to  $y$  in terms of profit. In cases like this, condition C requires that if  $x$  was originally superior to  $y$  in the overall ordering, and if other comparisons (i.e., by the other objectives) between  $x$  and  $y$  remain unchanged,  $xPy$ .

*Condition D (Nondominance)*: There is a set of three alternatives over which no objective's ordering is always that of the overall ordering, regardless of all conceivable orderings by other objectives.

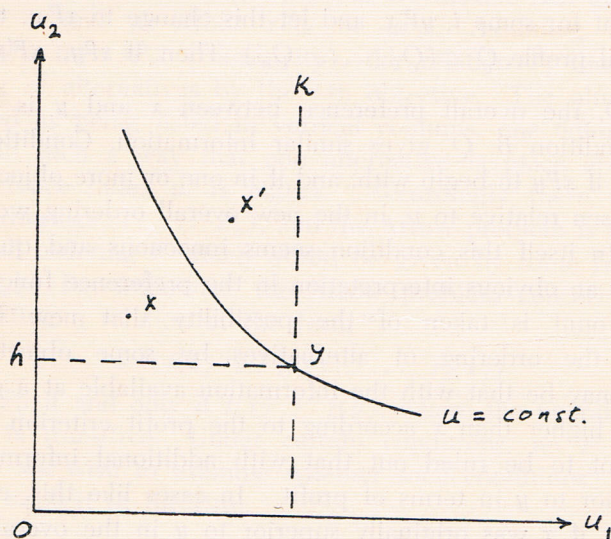
This is a relatively mild assumption about the set of all alternatives, requiring only that there be a set of three over which no objective completely dominates the ordering, and that the three alternatives could possibly occur in any order in each objective's preference list.

Now it turns out that no aggregating function  $F$  satisfies all four conditions A, B, C, and D.<sup>17</sup> And it would seem as if a preference function for the firm is not forthcoming, unless one or more of the conditions stated above are dropped because found overly restrictive.

### III

As more than one commentator has remarked, the binary choice condition is most suspect. Arrow's original argument for the reasonableness of this condition does not appear very compelling,<sup>18</sup> and it must be less so in the context of the firm where "natural" units are available for measuring the attainment of at least some objectives.

Consider the following diagram.



Let  $u_1$  be a measure of profit, say, and  $u_2$  a measure of, say, sales revenue. We have  $yP_1x$  and  $xP_2y$ . Suppose that some  $F$  would give

<sup>17</sup> See Theorem 2 in Murakami, *loc. cit.*

<sup>18</sup> Arrow, *op. cit.*, pp. 26-28.

$yPx$  here. Is it reasonable to require that since  $yP_1x'$  and  $x'P_2y$ ,  $yPx'$  also? Condition *B* tells us that the answer is 'yes'. On the other hand it would seem exceedingly reasonable to expect that through  $y$  would lie a (negatively sloped) curve where  $u$  is constant, equal of course to  $f(y)$ . Any point northeast of that curve would represent an alternative that would be considered superior to  $y$ . Yet the implication of the binary choice condition is that if (1)  $y$  is superior to some point inside the quadrant  $hyk$  northwest of  $y$ , then (2) all points in that quadrant must be inferior to  $y$ . This result of condition *B* can hardly make this condition a reasonable requirement to impose on  $F$ . Indeed, if now we make use of condition *A*, given the premise (1) above it is easily seen that for every  $z$  such that  $u_1(z) < u_1(y)$ ,  $yPz$ . That is, all points lying to the left to the vertical line through  $y$  are inferior to  $y$ .

Since the premise (1) is certain to be satisfied (by interchanging the axes  $u_1$  and  $u_2$  if necessary) and the point  $y$  was arbitrarily chosen, what the preceding argument shows is that the ordering of alternatives by  $F$  must necessarily be lexicographic<sup>19</sup>—if condition *B* is to be satisfied. In view of this result, which is traceable directly to condition *B*, one should find difficulty in accepting it as an a priori requirement on the aggregating function.

The obvious implication for a theory of the firm is heartening. Rejection of condition *B* leaves the way open for formulating hypotheses on multiple objectives preference functions without the somewhat inhibiting presence of the impossibility theorem.

<sup>19</sup> That is, like the ordering of words in a dictionary. In the present illustration, a lexicographic preference ordering  $xPy$  that  $u_1(x) > u_1(y)$  or else  $u_2(x) > u_2(y)$  when  $u_1(x) = u_1(y)$ . See B. L. van der Waerden, *Modern Algebra*, Vol. I, trans. F. Blum, New York 1949, p. 81.