PORTFOLIO MANAGEMENT AS A CONTINUOUS DECISION-MAKING PROCESS

BY

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The objective of this paper is to explore the possibilities of extending the essentially static theory of portfolio selection[1] by Harry Markowitz to cover decision-making in a dynamic setting. An attempt will be made to reconcile Markowitz' approach with a continuous decision-making process that includes the following features: (1) the formation of subgoals; (2) changes in information available to the decision-maker; and (3) the attempts to reduce risk via changes in portfolio composition. It should be immediately apparent that these concepts possess striking resemblance to the major relational concepts in Cyert's and March's behavioral theory of the firm.[2] Consequently, the extension from a static to a dynamic approach also entails a change from an optimizing to a suboptimizing or satisficing approach.

This paper is concerned with relating portfolio selection with portfolio management and performance. The succeeding discussion will be based on a premise that a portfolio has already been chosen, based on a

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[1] The theory has been so well discussed and written about that it will not be reviewed here. Cf. Harry Markowitz, *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley & Sons, Inc., 1959. A recent issue of the *Journal of Financial and Quantitative Analysis,* Vol. II, No. 2 (June 1967) was specially devoted to portfolio analysis.

[2] These major relational concepts used in their theory are: 1) quasi-relation of conflict; 2) uncertainty avoidance; 3) problemistic search; and 4) organizational learning. For a summary of this theory, see chap. 6 of R.M. Cyert and J.G. March, *A Behavioral Theory of the Firm* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963) pp. 114-27.

certain target return and accompanying risk for a given time period, through the use of Markowitz method or some other optimization method.[3]

I. *THE RELATIONSHIP OF SECURITY ANALYSIS TO PORTFOLIO SELECTION*

Most students who have gone through courses in investments analysis must at one time or another have heard the saying "portfolio analysis begins where security analysis ends." In this paper, that statement will be taken as a gross simplification of the relationships between these two areas.

In general, the problem of rational decision-making can be classified into three steps: 1) deciding upon an objective and the criteria for choosing among strategies; 2) filling out a payout matrix; and (3) choosing among strategies on the basis of this matrix and the criteria.[4] In real life, the second step: deciding on the size and composition of the payout matrix, measured by the number of columns representing relevant future occurrences and rows representing available strategies, and filling in the matrix with reasonable estimates of payouts and probabilities is by far the most difficult part of the decision-makers' job.[5] If we move from the general to a particular decision-making process as the selection of a portfolio, this second step also happens to be the responsibility of the security analyst. Since these estimates of payouts and probabilities are by nature subjective, the only test of a security analyst's performance rests with the eventual performance of a portfolio formed as a result of his judgments, for better or for worse.

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[3] A basic explanation of the optimization method used in selecting a portfolio is given in Harry Markowitz, "Portfolio Selection," *The Journal of Finance,* Vol. VII, No. 1 (March 1952), pp. 77-91.

For other methods, cf. Alberto Ilano, *Alternative Approaches to Risk in Portfolio Analysis,* (unpublished Ph.D. dissertation, University of California at Berkeley, 1969).

[4] Cf. H. Latane, "Criteria for Choice among Risky Ventures" *The Journal of Political Economy,* Vol. LXVII, No. 2 (April 1959), p. 144.

[5] Loc. cit.

In this paper, the relationships between security analysis and portfolio analysis will be cast in somewhat different light. Security analysis does not end with the supplying of input data to a portfolio selection problem. As will be explained later on in detail, security analysis also provides the information that could result in the reduction of risk in an existing portfolio. This in turn gives rise to decisions involving profit-taking and the possibility of portfolio revisions.

Since the intent here is to cast portfolio selection as a continuous process, let us assume that each subportfolio is formed as new funds accumulate to a certain arbitrary level. This assumption permits us to consider each subportfolio as a separate decision problem in its own light.[6] By using Markowitz' approach, this subportfolio can be formed on the basis of a subgoal, that is, the expected return and its accompanying variance. For the moment let us not be concerned about the relation of the subgoal to the overall objective. In any case, before this subportfolio can be formed, certain issues pertaining to security analysis will have to be resolved first.

II. *THE DETERMINATION OF THE NUMBER OF ASSETS TO BE CONSIDERED*

Security analysts are usually specialists in a certain number of stocks. The exact number depends of course on the relative span of attention for each analyst. And while the individual person has cognitive limitations if he is to be effective,[7] a firm faces economic constraints on the number of security analysts it can hire to keep track of the stock market. Hence, the number of stocks that are being considered for possible inclusion in a portfolio, which theo-

[6] No distinction will be made as to whether these funds have been generated internally or externally. This assumption will include the actual performance of the existing portfolio relative to its corresponding goal.

At any given time, a portfolio will consist of subportfolios that were formed at different points of time in the past.

[7] Cf. H. A. Simon, *Administrative Behavior,* New York: Collier-MacMillan Ltd., 2d ed., 1957, pp. 80-1, for a discussion of what he terms "the limits of rationality."

retically can encompass the totality of stocks traded publicly and over-the-counter, can be delimited by economic and rational factors at the practical level.

The security analyst can further narrow the number of assets to be considered by excluding those assets which in his judgment are temporaily overvalued. Needless to say, he must also provide some estimates of the nature and duration of this overvaluation.

To recapitulate, all these factors serve one purpose—they help in defining a manageable number of assets to be taken into consideration. If we want to relate this to the previous discussion on rational decision-making, these factors determine the eventual number of rows (i.e., the number of available alternatives) in the payout matrix referred to therein.

III. *THE FORMATION OF SUBGOALS*

In this section let us assume that a subportfolio has been formed using Markowitz' method. The amount of new funds available is denoted by W (0), where the bracketed term refers to time t=0. Assume further that the subportfolio was formed to earn a return equal to E for a period of T years. In other words, let us use an investment horizon of T years.

Without loss of generality, suppose that in this situation the Markowitz method yielded a portfolio of three assets, e.g., X= [x(1),x(2),x(3),] where (x(i) represents the percentage contribution of the *i*th asset to the subportfolio. The data used to arrive at a portfolio consisted of the expected returns of the assets, [(r(1), r(2), r(3)] and a matrix containing the variances and covariances [v(i,j), i,j = 1,2,3]. For convenience let $r(1) > r(2) > r(3)$ and v(1,1) > v(2,2) > v(33,3), that is, the assets have been numbered in an order of decreasing expected return and variance. Let us also assume that only asset 3 pays out dividends, to be denoted by d(3,t) for the duration t = 1, . . ., T.

In effect, a portfolio $X = [x(1), x(2), x(3)]$ has been formed from an amount $W(0)$. This relationship can expressed in the following ways[8]:

Eq. *1.1* $W(0) = x(1)W(0) + x(2)W(0) + x(3)W(0)$

1.2 $= w(1,0) + w(2,0) + w(3,0)$

1.3 $= n(1)P(1,0) + n(2)P(2,0) + n(3)P(3,0)$

where $w(i,0)$ — monetary amount invested in *i*th asset;

$n(i)$ — number of shares of *i*th asset; and

$P(i,0)$ — price per share of ith asset at $t = 0$, for $i = 1,2,3$.

As stated, the subgoal was for $W(0)$ to earn an expected return E for the investment period of T years, that is,

Eq. 2. $W(T) = W(0)\ (1 + E)^T$

where $W(T)$ represents the market value of the initial investment $W(0)$ if the desired expected return E is realize.[9]

Another way whereby E is realized occurs when all the three values $w(i,T)$ are realized, that is,

Eq. 2. $W(T) = W(0)\ (1 + E)^T$

if $w(i,T) = w(i,0)\ [1 + r(i)]^T$, for $i = 1,2,3$.

It must be pointed out that these three conditional equations involved only the initial and final values, i.e., each $w(i,0)$ and $w(i,T)$ and no conditions are imposed on whatever transpires in the interval between $t = 0$ and $t = T$.[10] Each of these three conditional equations also imply a final price $P(i,T)$ for which the equation will be satisfied.

At the time of the formation of a portfolio, therefore, two points of a random path have been set for each of the component assets:

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[8] We can assume that $W(0)$ is fully invested, i.e., cash balances are zero, if any one of $P(i,o)$ has a small unit price relative to the other two assets.

[9] For the moment, taxes and transaction costs will not be taken into account.

[10] The ideal case occurs when $w(i,0)$ earns a uniform return $r(i)$ for the periods between $t=0$ and $t=T$. Of course this is merely conceptual, since if this were the case, then there is no point in diversifying the portfolio.

$$P(1,0) \ldots\ldots\ldots\ldots P(1,T)$$
$$P(2,0) \ldots\ldots\ldots\ldots P(2,T)$$
$$P(3,0) \ldots\ldots\ldots\ldots P(3,T)$$

where

Eq. 4.1 $$P(i,T) = P(i,0)\ [(1 + r(i)]^T \text{ for } i = 1, 2 \text{ and}$$

Eq. 4.2 $$P(3,T) = P(3,0)\ [1 + r(3)]^T + \sum_{t=1}^{T-1} d(3,t)\ (1 + r)^{T-t}$$

where r denotes the riskless rate of interest.[11]

A great deal of difference exists between the two points in the random paths described. The initial values of price per share are actual values; the final values of price per share, however, are the expected values of their corresponding distributions of prices. Hence, these final prices represent desired or target values, and from now on, this distinction between actual and desired values will be set by affixing asterisks to the latter, e.g., the desired final price per share value of asset 3 in Eq. 4.2 will be $P^*(3,T)$.

Setting up the random paths for each asset can serve as a control device on the performance of each asset if we can compare them to some ideal standard. Let us denote this standard as an *ideal path*

$$P(i,0),\ P^*(i,1),\ P^*(i,2), \ldots,\ P^*(i,T)$$

where $P^*(i,t) = P(i,0)\ [(1 + r(i)]^t,\ t = 1, \ldots T; i = 1,2,3.$

Note that the ideal path is a theoretical construct that links the actual price at which an asset is bought to the desired value at the end of the investment periods on the assumption that the prices in the intervening periods will behave at a compounding rate equal to the expected return $r(i)$.

Once the subportfolio has been formed, some sort of progress report can be done on the performance of the component assets during the intervening periods. This can be in the nature of how the random paths compare with the ideal paths. For instance, at each period t the price pattern deviations can be denoted by

[11] This equation assumes that the dividend payments, $n(3)d(3,t)$ for each year t are invested at the riskless rate.

Eq. 5. $Z(i,t) = P(i,t) - P^*(i,t) \; i = 1,2,3; \; t = 1, \ldots, T\text{-}1.$

The sign of $Z(i,t)$ indicates whether or not the stock is performing to par.

IV. *THE INTERPRETATION OF RISK MEASURES*

The formulation of Eq. 5 above can also serve to provide a basis for distinguishing two kinds of variance. For our purpose here, let us label these as *ex ante* variance and *ex post* variance. The former is what the security analysts have to estimate and which become part of the data for portfolio analysis, e.g., $v(1,1)$ and $v(2,2)$ are the *ex ante* variances of assets 1 and 2, respectively. If, as in our example, $v(1,1) > v(2,2)$, then this inequality implies that the security analyst anticipated that the actual final value $P(1,T)$ could differ from the desired final value $P^*(1,T)$ by a greater probability relative to the chances of $P(2,T)$ differing from $P^*(2,T)$. It is in this sense that most investors would consider asset 1 "riskier" than asset 2. *Ex post* variance, on the other hand, has nothing to do with anticipations, since it is computed from actual or historical values. Inasmuch as an estimate of the *ex ante* variance was formed at $t = 0$ while the *ex post* variance can be determined only at $t = T$, it is only at the end of the investment period that these two values can be compared.

Now let us focus back the attention on Eq. 5 above. At any time t prior to T, one can set up the current liquidating value of the subportfolio as:

Eq. 6 $$L(t) = n(1)P(1,t) + n(2)P(2,t) + n(3)\left[P(3,t) + \sum_{S=1}^{t} d(3,S)\,(1+r)^{T\text{-}S}\right]$$

Note that the values of $P(i,t)$ in this equation refer to the prices at which the assets can be sold. Recalling now that variability per se was deemed undesirable only in the *ex ante* sense, a procedure will now be explained whereby variability in the *ex post* sense will be used to reduce the random path to a certain one.

For the most part, certain groups of stocks have prices that fluctuate much more than others, as the case, for instance of glamor stocks relative to utilities. In an *ex post* sense, such fluctuations are undesirable only if they are on the downside. In the above formulation, $Z(1,1) < 0$ and $Z(1,2) > 0$ signify an unfavorable performance by asset 1 in the first period and a favorable one in the second. The absolute values of $Z(i,t)$ can vary depending on where $P(i,t)$, lies in its range of values.[12] The point that needs to be stressed here is that it is possible for $Z(i,t)$ to have a positive value such that [13]

Eq. 7 $\quad n(i)P(i,t) \ (1+r)^{T-t} > w(i,T) \qquad \text{for } t = 1, \ldots, T\text{-}1.$

This inequality says that if the current price of the *i*th asset turns out to be very high compared to the desired price $P^*(i,t)$, selling this asset now and investing the proceeds at the riskless rate can result in an accumulated sum at least equal to, or even greater than the original target set for this asset when the subportfolio was first formed.

Assuming that there has been no change in the original subgoal to earn a total return of E for T years, the act of profit-taking when the above inequality occurs implies that the risk attached to a random path has been removed since the same accumulated value of $w(i,t)$ (at the least) can be obtained with a certain path from period t to T. This attains added significance if the above inequality occurs when there has been no change in the *ex ante* estimates of variance by the security analyst. Because if such were the case, then there is still some possibility that, if the *i*th asset is not sold at $P(i,t)$, its value at succeeding periods may fluctuate such that the above inequality might not be realized again. Here again, the security analyst assumes a significant role in the decision on whether to sell the asset or hold it to attain a higher return. The latter decision can

[12] It would be more interesting if newspapers show not only the daily quotations of stocks, but also the range of high and low values for the past twelve-month period.

[13] Since this inequality does not account for dividends, the subscripts i will refer to assets 1 and 2 in the given example.

be justified if certain changes, for instance new information, have come about so as to alter the security analyst's estimates of expected return and *ex ante* variance for the stock in question.[14]

V. *METHODS AND CRITERIA FOR RISK REDUCTION*

Let us now proceed to determine when this decision to sell or hold a particular stock arises. To be specific, let us suppose that we have formed a subportfolio three years ago and that our original investment period was set then at $T = 5$ years. Suppose further that the price movement of asset 1 has been favorable such that $Z(1,3) > 0$. This is a necessary but not sufficient condition to consider the sell-or-hold decision. Recall that the desired accumulated value at the end of the investment period can be stated in terms of the ideal path. In this particular case, we have obtained the following from Eq. 3 (for $i = 1$ and $t = 3$, $T = 5$):

$$n(1)P^*(1,3)\ [1 + r(1)]^{5\text{-}3} = w(1,5)$$

Substituting this into Eq. 7, we will have

$$n(1)P(1,3)\ (1 + r)^2 \geq n(1)\ P^*(1,3)\ [1 + r(1)]^2$$

or $$P(1,3) \geq P^*(1,3)\ [1 + r(1)]^2/(1 + r)^2$$

Subtracting $P^*(1,3)$ from both sides, we will get

$$Z(1,3) \geq P^*(1,3)\ [(1 + r(1)^2/(1 + r)^2 - 1]$$

Since $P^*\ (1,3)\ \geq 0$ and $r(1) > r$ both by definition, this last inequality necessarily implies that $Z(1,3) > 0$. Therefore, we can simplify matters by observing the following relationship for the duration of the investment period:

Eq. 8. $$P(i,t) \geq P^*(i,t)\ [1 + r(i)]/(1 + r)^{T\text{-}t}$$

The implications of this inequality will now be explained with

[14] For instance, if $T=5$, the security analyst will have more information during, say at $t=4$ than during $t=1$, regarding the validity of his estimates.

the aid of Fig. 1 in the following page. Let us suppose that we have the following data for asset 1:

$$T = 5 \text{ years} \qquad r = 6\%$$
$$P(1,0) = ₱100 \qquad r(1) = 20\%$$

The expected return of 20% can be realized if at the end of five years $P^*(1,5) = 100(1.2)^5 = ₱248.83$. Similarly the ideal path can be determined as

$$[P(1,0),\ P^*(1,1),\ P^*(1,2),\ P^*(1,3),\ P^*(1,4),\ P^*(1,5)]$$
$$= (100,\ 120,\ 144,\ 172.8,\ 207.36,\ 248.83)$$

and is plotted in Fig. 1 as a curve linking the two points $P(1,0)$ and $P^*(1,5)$.

The desired final value can also be attained if one invested a certain amount, say at $t = 0$ at the riskless rate. This amount can be found from the following

$$C(1,0) = P^*(1,5)/(1 + r)^5 = 248.83/1.340 = ₱185.70$$

Similarly, we can generate a curve by setting

Eq. 9. $\quad C(1,t) = C(1,0)\ (1 + r)^5 \qquad \text{for } t = 1,\ldots,5$

and this curve, which will be referred to as the *riskless path,* links the two points $C(1,0)$ and $P^*(1,5) = C(1,5)$.[15] This is depicted as the topmost curve in Fig. 1.

The series of jagged lines in Fig. 1 is supposed to typify what has been termed the random path; this can be plotted by following the actual price pattern after $t = 0$. Going back to our example, suppose the random path behaved as shown such that at $t = 3$, the prevailing price $P(1,3) = C(1,3)$. This turn of events provides an opportunity to transfer from the "risky" random path to the riskless path; this move can be accomplished by selling the shares at $P(1,3)$ and investing the proceeds at r.[16] In this way, the same desired end

[15] Again, the distinction must be made that P^* $(1,5)$ is the expected value of a frequency distribution of prices at $t=5$, whereas $C(1,5)$ is single-valued.

[16] Transaction costs have been ignored for the moment.

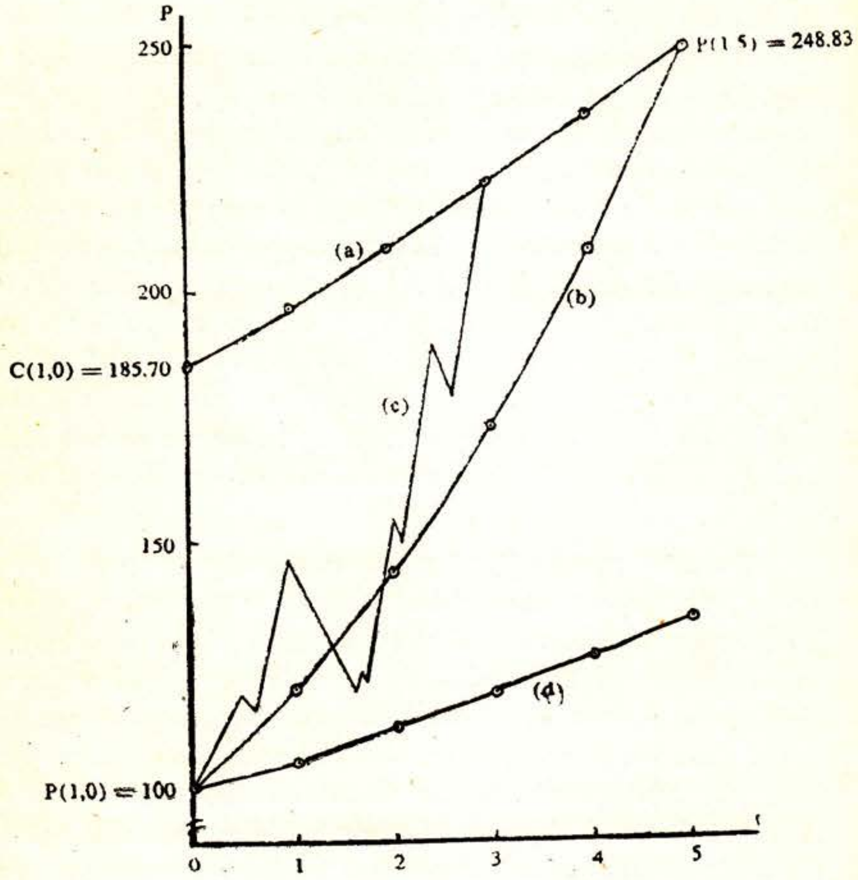
result can be obtained at $T = 5$ but with an important difference—whereas before there was present the risk of not attaining the subgoal $r(1)$, now the transfer to the riskless path at $t = 3$ completely does away with this aspect of risk.

If the decision to sell was made at $t = 3$, then the following can be inferred regarding the attitudes of the portfolio analyst—first, he sought to reduce uncertainty when the opportunity first arose and second, he considered the original subgoal $r(1)$ as being acceptable. The reason behind this second inference is that a decision not to sell at $t = 3$ can also imply a mere postponement in anticipation of a higher gain. This can happen if the random path goes over the riskless path at any time during the remainder of the investment period. Here again the security analyst has to be consulted as to whether the original estimates of $r(1)$ and $v(1,1)$ have materially changed. Because if they have not, then it is also possible for the random path to fluctuate below the riskless path, and the opportunity to transfer to the riskless path may not arise again.

The other curve in Fig. 1 represents the *opportunity cost path,* that is, the amounts the original available funds $w(1,0)$ would have accumulated to had it been invested at the riskless rate instead of asset 1. By the same token, it represents some kind of lower limit on the performance of the stock during the investment period. It can also serve as a control device if and when certain stocks perform consistently below this path. In such a situation, the causes for such behavior can be isolated, as to whether, for instance, they were due to events unforeseeable at $t = 0$ or whether the security analyst was just off in his original estimates.

VI. *PORTFOLIO MANAGEMENT AND RISK REDUCTION*

Thus far, the proceeding discussions on risk reduction have dealt with an individual asset. Let us now extend the boundaries of the procedure so as to apply to portfolios. Suppose, as before, that we have a three-asset portfolio with the composition $[x(1), x(2), x(3)]$.


P
250
P(1 5) = 248.83
(a)
200
(b)
C(1,0) = 185.70
(c)
150
(d)
P(1,0) = 100
0
1
2
3
4
5

The total realizable market value of this portfolio at any time during the interval $0 < t < T$ has been previously set up in Eq. 6 above. There the term $L(t)$ was used to denote the sum of the combined market value and the dividend stream coming, as previously assumed from asset 3.

To some extent, a similar decision on whether to sell or hold can now be applied to the component assets of a portfolio if [17]

Eq. *10.1* $$L(t)\ (1+r)^{T-t} \geq W^*(T) = W^*(t)\ (1+E)^{T-t}$$

or *10.2* $$L(t) \geq W^*(t)\,[(1+E)/(1+r)]^{T-t}$$

and $t = 1, \ldots, T\text{-}1$.

where $L(t)$ was obtained directly from Eq. 6.

Let us now proceed to formulate the equivalent expressions for the paths that were illustrated in Fig. 1. The first difference is that instead of dealing on a price per share basis, we now have to evaluate the decision directly in terms of the monetary flows at each point in time.

1. The ideal path for the portfolio can be established by setting the initial amount $W(0)$ to grow at the desired value of expected return E. Hence, it will contain the following points $[W(0), W^*(1), \ldots, W^*(T)]$ determined from the following equations:

$$W^*(t) = W(0)\ (1+E)^t \qquad \text{for } t = i, \ldots, T.$$

2. The riskless path can be set up by first determining the value of $C(0) = W^*(T)/(1+r)^T$; $C(0)$ is the monetary amount which, if invested at r for T years, will accumulate to the desired end result $W^*(T)$.[18] The other points in this path can be found by setting

$$C(t) = C(0)\ (1+r)^t \qquad \text{for } t = i, \ldots, T.$$

3. The random path $[L(0), \ldots, L(T)]$ can be evaluated at any

[17] The asterisk in $W^*(t)$ indicates that it is desired value belonging to a distribution.

[18] $C(0)$ is the present value of $W^*(T)$ at the given discount rate r.

moment during the investment period by substituting directly into Eq. 6.

4. The opportunity cost path can be found directly from

$$W(0)\ (1+r)^{t} \qquad \text{for } t = i,\ldots,T.$$

Before proceeding any further, let us first simplify the form of the inequality in 10.2 above. The entire term on the right-hand side can be simplified by performing the following substitutions:

$$\begin{aligned} W^*(t)\ [(1+E)/(1+r)]^{T-t} &= W^*(T)\ (1+r)^{T-t} \\ &= C(0)/(1+r)^{-t} = C(t). \end{aligned}$$

Hence, the decision to hold or to sell the portfolio boils down to checking whether $L(t) \geq C(t)$. Since $L(t)$ consists of several component parts, it becomes possible for $L(tc) \geq C(t)$ even if one or more of the component assets are not performing favorably. Evidently, this is the advantage to be derived in having a diversified portfolio.

While the criterion $L(t) \geq C(t)$ involves the sale of the entire subportfolio in order to attain a riskless path, it can also happen that although $L(t) < C(t)$, there may be one or more of the component assets that are performing favorably so that the relations $P(i,t) \geq C(i,t)$ apply to them. The decision problem now becomes one of determining whether to convert such component assets to their individual riskless paths while holding the remaining assets which are not currently performing favorably. Let us refer to this situation as a decision involving partial reduction of risk. In other words, whereas there was an opportunity for total risk reduction when $L(t) \geq C(t)$, there are also possible cases for partial risk reduction even when $L(t) < C(t)$.

Evidently there will be varied types of situations that can correspond to the criterion $L(t) < C(t)$. Hence, each particular case has to be decided in terms of the circumstances specific to it. To be less ambiguous, let us now discuss these circumstances as they can apply to the example given above involving a subportfolio of three assets at $t = 3$.

Suppose that in this example we have $P(1,3) > C(1,3)$ but both $P(2,3) < C(2,3)$ and $P(3,3) < C(3,3)$ such that for the subportfolio as a whole $L(3) < C(3)$. It is at this stage that the portfolio composition decision made at $t = 0$ emerges to play an important role. For example, depending on the compositions ($x(i) = n(i)\ P(i,0)/W(0)$ for $i = 1,2,3$) we might be able to find combinations such as

$$n(1)\ P(1,t) + n(2)\ P(2,t) \geq n(1)\ C(1,t) + n(2)\ C(2,t).$$

Here the partial reduction that can be made amounts to the sum of the component assets $[x(1) + x(2)]\%$. In effect, a more than favorable performance by one asset has been made to compensate for an unfavorable performance by another.

Another circumstance that can affect the decision to reduce risk partially involves the time remaining in the investment period, that is, T-t. For a given length of investment period, say $T = 5$ years, although it may not matter theoretically whether $L(t)$ becomes greater then or equal to $C(t)$ during $t = 1$ or $t = 3$,[19] from a practical standpoint, it matters a great deal that one can overcome uncertainty earlier. Later on, this advantage will be discussed in terms of the added flexibility afforded the portfolio analyst.

VII. *THE IMPLICATIONS ON MEASURING PORTFOLIO PERFORMANCE*

So far we have dealt only with cases where the subgoal could be fully or at least partially realized. Unfortunately, for the security analyst and the portfolio analyst who had to make the original decisions, it could happen that the subgoals have not been realized by the time the investment period would have elapsed. In terms of the analyses here, this implies that no opportunity arose for partial let

[19] As long as the end results are the same, there is little distinction that can be made between two riskless paths.

alone total risk reduction at any moment prior to the end of the investment period. If we treat this subportfolio as an independent investment, then for purpose of control and measuring performance, this subportfolio will just have to be recorded as having attained less than the intended goal. To make the most out of the situation, at least it can be said that something can be gained by analyzing the causes behind this turn of events. As a matter of fact, for purposes of measuring performance, the same can be said of subportfolios that succeeded in attaining their respective goals. There are after all many instances when a portfolio will be successful inspite of the judgments of the decision-makers.

If on the other hand we account for interdependencies among subportfolios, then the decision-making processes will have to be broadened since each subgoal will have to be formulated and then assessed in relation to an overall objective. As an example of this, if at the time a new subportfolio is being formed, the investment period for another previously formed subportfolio had just lapsed with the subgoal not attained, then such an event can influence the new subportfolio in terms, for example, of a higher return-higher risk subgoal.

At any given time, the overall portfolio can be regarded as being composed of several subportfolios formed at different periods in the past each with its own subgoals. Once formed, there can be a continuous assessment of their performances. Therefore, at any given point in time, some of these subportfolios have already been converted into their respective riskless paths, while there are also the others that have not fared as well.

Up to now the assumption has been made that all proceeds from the sale of assets as consequences of risk reduction would be invested at the riskless rate. To be sure, this convenient assumption has been resorted to merely as a conceptual device. For example, the sales proceeds arising from risk reduction opportunities can for all intents and purposes be considered as internally generated "new" funds available for forming new subportfolios. Since these funds have already earned better than the expected return, then reinvestment for the remaining (T—t) years to earn any rate greater than r would constitute additional return over and above the original goal. Needless to say, additional risks are also involved at the same time.[20]

The added flexibility mentioned earlier enters at this stage. Once a move to a riskless path has been made, there need not be made any new decisions, strictly speaking. However, the earlier this opportunity arises, the longer becomes the remaining time when new and better prospects can possibly come about.

It can even be argued that risk reduction as explained above can take the form of some accounting device. As an example, let us take a case when the following conditions occur simultaneously:

1. $L(t) > C(t)$ for a subportfolio;

2. The security analyst believes that the original estimates on expected return and variances have changed significantly such that the desired returns can now be shifted upwards.

In this situation the component assets need not be actually sold. For record-keeping purposes the original portfolio can be considered as having been "retired" since the original subgoal had already been realized. Instead of incurring actual transactions costs,[21] some method of transfer pricing can be established so that these component assets can be considered either for new subportfolios or be incorporated into existing subportfolios that have not yet been converted into their riskless paths.

NCLUSION

The treatment of portfolio selection as a continuous decision-making :ess has served to broaden the concept of diversification. Recall that ıe static Markowitz approach, diversification served to minimize total

[20] It must be pointed out that unless the reinvestment involves a higher riskn combination than the original, then the additional risk incurred can be to have been reduced in a relative sense.

[21] This procedure also manages to avoid the question of income taxes.

portfolio variance in an *ex ante* sense. Hence, the very selection of assets to form the portfolio has already corresponded to the minimum risk combination at given levels of expected portfolio return.

Since *ex ante* estimates may not be realized, the incorporation of the behavioral concepts as discussed in this paper has broadened the notion of diversification. Diversification also enables the portfolio manager to reduce risk through the use of new information, subgoal formation and revision of portfolio composition.