THE RISK FACTOR IN INVESTMENT DECISIONS*

BY

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Investment decisions play significant roles in all types of economies. In the case of developed economies, the level of aggregate investment demand affects the stability of the business cycle. For developing economies, the basic problems revolve around the need for capital formation.

The macroeconomic approach to the level of investment demand has been to assume that it is functionally dependent on the level of interest rates. For instance, low interest rates are necessary to induce high levels of net investments. Inasmuch as this macrorelationship consists of the totality of investment decisions occurring at individual firms, then its empirical validation can be made by asking managers to rank the factors affecting their investment decisions. Even in full-employment economies, however, empirical investigations have not borne out this inverse relationship between investments and the interest rate.[1] These studies either tended to provide evidence for the low interest elasticity of demand for investment or indicated that the interest rate became important only in the marginal (least profitable) projects.

When one considers investment decisions in developing economies, the situation becomes more complex, since there now exists a greater interdependence between the level of savings and the uncertainty, or risk factor.[2] The latter two factors are in turn interrelated, e.g., the

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[1]For a survey of these studies, see W. H. White, "Interest Inelasticity of Investment Demand," American Economic Review, 46: 565-87, 1956.

[2]The strict distinction between risk and uncertainty is that a condition under risk implies knowledge of the probability of occurrence of states of the world, whether subjective or objective, whereas a condition under uncertainty does not. In this paper, the term uncertainty will also be used loosely to apply to conditions under risk as well.

risk of incurring purchasing power loss favors present consumption at the expense of saving for the future. One can say that the level of saving depends upon the differential between the available lending rates and the perceived purchasing power loss. In other words, the supply of saving varies directly with the real, and not the nominal, rates of interest.

This paper will focus attention on the relationship between investment decisions and the risk factor at the micro, or individual firm, level. An investment decision can be defined as a commitment of funds for a particular project in anticipation of net future benefits to be derived over the duration of the project's life.[3] Two approaches can be made in analyzing this relationship. First, along normative lines, how should firms treat the uncertainty of projected future costs and revenues? Second, along descriptive lines, do firms utilize the available means of coping with such uncertainty? The succeeding sections of this paper will be limited to the first, or the normative, approach.

I. INVESTMENT DECISIONS UNDER CERTAINTY

For purposes of simplification, let us first formulate the model for investment decisions under certainty. This is not entirely without basis, since there are instances when a firm can attain such conditions, as in the case of an investment project involving a contract to sell a given quantity of a product at a prearranged price to a customer.

The elements of an investment decision under certainty are as follows:

(1) N – the lifetime of the project;

(2) R_i $(i = 1, ..., N)$, the revenue stream arising from the project;

(3) E_i $(i = 0, 1, ..., N)$, the expenditure stream arising from the project;

(4) r – the company's cost of capital; and

(5) S_n – the salvage value of the project at the N-th period.

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[3]Future net benefits are obtained as the difference between revenues and costs at corresponding future periods. Hence, the present commitment need not be limited to the present period.

Without loss of generality, we can make further simplifying assumptions that $R_i > E_i$ for $i = 1, ..., N$ and that $S_n = 0$. Let us denote $B_i = R_i - E_i$ as the net benefit stream. E_o is the amount of the initial investment to be amortized over the period of N years. The company's cost of capital r, can be obtained as the marginal cost of financing, given the firm's sources of borrowed and internally generated funds.[4]

Since money has time value, ₱100 to be received today should be preferred over ₱100 to be received, say, five years hence. This is true at any given rate of interest. For instance, if the ₱100 on hand can earn 10% interest for five years, then its *future compounded value* is ₱100 $(1 + .10)^5$ = ₱161.05. On the other hand, the ₱100 to be received five years hence has a *present discounted value* of only ₱100 $(1 + .10)^5$ = ₱62.10. Therefore, it is not valid to compare the present value of ₱100 on hand with the future value of ₱100 to be realized five years from now. What is valid would be to compare the ₱100 on hand with the present value of ₱62.10.

Since investment decisions involve the projection of cash flows into the future, it becomes necessary to discount all future streams back to the present, on a period-by-period basis. For instance, the present values of the revenue stream would be $R_1/(1 + r)^1, ..., R_i/(1 + r)^i, ..., R_n/(1 + r)^n$ where r, the company's cost of capital, is used as the discounting rate. This discounting process serves to render all streams comparable on the basis of their present values.

For given values of N, R_i, and E_i, the *Net Present Value (NPV hereafter)* of an investment project can be defined as a function of the discounting rate r, as follows:

$$NPV(r) = \sum_{i=1}^{n} \left[\frac{R_i - E_i}{(1 + r)^i}\right] - E_o \qquad \text{Eq. 1}$$

The project's *internal rate of return, r** (also commonly referred to as the *discounted cash flow rate of return)* is that particular value of the variable r, which satisfies the following equation:

[4]A detailed description of the manner of determining the cost of capital can be found in Ezra Solomon, "Measuring A Company's Cost of Capital," Journal of Business, October 1955.

$$NPV(r = r^*) = 0, \qquad \text{Eq. 2A}$$

$$\text{or} \quad \sum_{i=1}^{n} \frac{R_i - E_i}{(1 + r^*)^i} = E_o \qquad \text{Eq. 2B}$$

The internal rate of return, r^*, can be found by trial and error, as the rate that equates the sum of the discounted net benefits with the amount of the project's initial investment.[5]

In financial decision-making, the maximization of a firm's net present value usually serves as the predominant goal. It follows, therefore, that for investment decisions under certainty, the decision process is straightforward enough. If an individual project's net present value at the given firm's cost of capital,

$$NPV(r) = \sum_{i=1}^{n} \left[\frac{B_i}{(1 + r)^i} \right] - E_o > 0, \qquad \text{Eq. 3}$$

then the project should be adopted since it serves the predominant goal.

Another way of applying the decision rule would be as follows – if the project's internal rate of return, r^*, is higher than the cost of capital for the firm, the project should be adopted.

In some cases, projects have to be ranked in some order of preference. These situations occur if projects are mutually exclusive or if the firm is operating under a limited capital budget.[6] For investment decisions under certainty, the procedure will be to rank projects starting with the one having the highest value of internal rate of return – r^*.

ILLUSTRATIVE EXAMPLE

Assume that one has only ₱100 to invest and must decide between two mutually exclusive projects, A and C, each requiring an initial

[5]The value of r^* would be unique under the given simplifying assumptions. For cases involving multiple roots and their implications, see J. H. Lorie and L. J. Savage, "Three Problems in Rationing Capital," in E. Solomon (ed.), The Management of Corporate Capital, Collier-MacMillan Ltd., 1959, pp. 56-66.

[6]The term *mutually exclusive* implies that the adoption of one project automatically precludes the adoption of any other, e.g., in deciding to choose between two brands of a machine that performs similar functions.

investment of ₱100 and with the given net benefit streams for a period of 3 years:

Project	B_1	B_2	B_3	E_o
A	₱60	₱60	₱60	₱100
C	₱80	₱50	₱50	₱100

For Project A, the value of r^* can be obtained as follows:

1) By trial and error, at $r = 36\%$

$$\begin{aligned} NPV(.36) &= 60/1.36 + 60/(1.36)^2 + 60/(1.36)^3 - 100 \\ &= 100.41 - 100 = .41 \end{aligned}$$

This is close to zero. Testing at $r = 37\%$

$$\begin{aligned} NPV(.37) &= 60/(1.37) + 60/(1.37)^2 + 60/(1.37)^3 - 100 \\ &= 99.097 - 100 = -.093 \end{aligned}$$

By interpolation, $r^* = 36.8\%$ for Project A. With the same procedure, r^* for Project C = 40.76%. Hence, Project C should be preferred to Project A; and Project C should be adopted if the company's cost of capital happened to be less than 40.76%.

II. INVESTMENT DECISIONS UNDER RISK

Let us now proceed to the more interesting case of investment decisions under risk. Here, projections of revenues and costs are no longer considered certain, or at least, they are no longer single-valued estimates. Instead, future cash flows are considered in terms of their probability distributions. The existing procedures for investment decisions under certainty assume that some measure of the central tendency of each prospective cash flow is known, since a single forecast is required (i.e., "the most likely value"). It now remains to formulate a procedure to determine the measure of dispersion about this central value. The risk in investment decisions arises from the fact that the cash benefits to be realized eventually may turn out to be lower than the projected "most likely value".[7]

There are several ways of formulating the inclusion of the risk component in analyzing investment projects:

1. One would be to construct decision trees, such as

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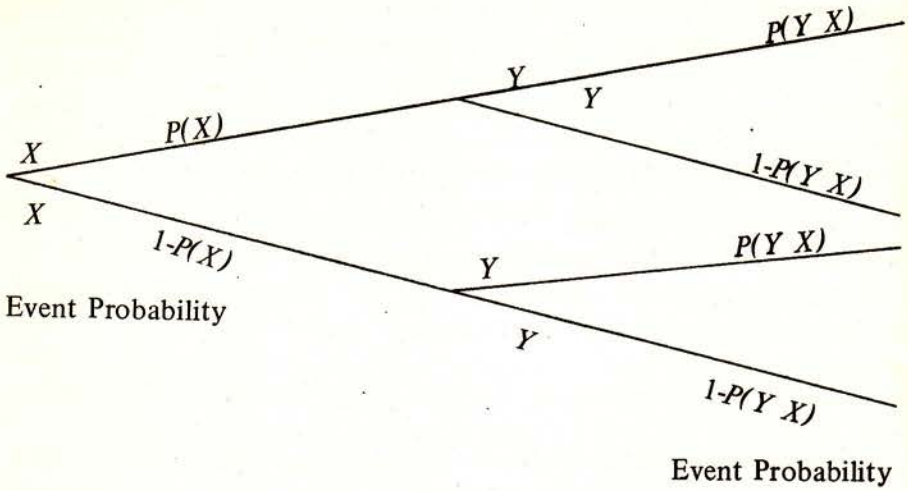
[7]This measure of risk has been termed the semi-variance in Chap. 9 of Harry Markowitz, Portfolio Selection, New York: John Wiley & Sons, Inc., 1959.



and arrive at discrete-valued distributions.

2. Another useful technique is sensitivity analysis. This involves revising uncertain estimates of prospective cash flows and investigating the sensitivity of the project's return to such revisions in the estimates. This serves to give some indication of the effect if one of the original estimates was either too optimistic or too pessimistic. Sensitivity analysis may be useful, but its conclusions tend to suffer from a lack of conciseness, precision, and comprehensiveness.

3. The theoretical approach would be to determine the "utility", or merit of each of the possible outcomes of an investment, and then obtain the expected utility of the project based on the decision-maker's attitude towards risk. Needless to say, this procedure is far too complex for the purposes of the practical decision-maker.

4. The last procedure would be to make simplifying assumptions about the probability distributions of the projected cash flows. Without these assumptions, weak probability statements can be made by using the Chebyshev's inequality. Stronger statements can be made by assuming for instance that the projected cash flows would be normally distributed with the "most likely value" as the mean and with a procedure for determining the standard deviation. In any event, the investment decision under risk incorporates another decision variable — the measure of risk.

Of the above-mentioned different methods of incorporating the measure of risk into the investment decision, the fourth case that involving the assumption of normality, appears to be the most useful. Even if the probability distribution of a future cash flow were not normal, it would seem that for many types of prospective cash flows one's best subjective probability distribution would be nearly a symmetrical distribution resembling the normal distribution.

Let us now analyze the risk factor's effect. Going back to Project C as discussed above, we can regard the given net benefits as the mean values, i.e., $E(B_1) = 80$, $E(B_2) = 50$, and $E(B_3) = 50$. However, the B_i are now treated as random variables.[8] To formulate their respective subjective probability distributions, the decision-maker must establish the answer to the following – what is your pessimistic estimate for say, B_i, with a one in a hundred chance of occurring? The answer to this question sets, for practical purposes, the lower limit of the distribution, and for normal distributions, would correspond to 2.58 standard deviations below the mean. If, for instance, this value was 54.2 corresponding to $E(B_1) = 80$, then

$$\text{Prob } \{+ 80 - 2.58\ \sigma_1 \leq B_1 \leq 80 + 2.58\ \sigma_1\} = .99$$

From $(80 - 2.58\ \sigma_1) = 54.2$, we get a tentative value of $\sigma_1 = 10$.

It still remains to test whether the following equations

$$\text{Prob } \{80 - \sigma_1 \leq B_1 \leq 80 + \sigma_1\} = .683$$

$$\text{Prob } \{80 - 2\ \sigma_1 \leq B_1 \leq 80 + 2\ \sigma_1\} = .954$$

conform with the decision-maker's subjective probability distribution of B_1. Recalling the example given above, suppose we have obtained the following additional information (arbitrarily obtained) for Projects A and C. We can then proceed to formulate the probability distribution of NPV (for convenience, NPV will be replaced by P in the notation).

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[8]As will be shown later, r^*, the internal rate of return, will also be a random variable with a probability distribution.

	PROJECT A		PROJECT C	
	Expected Value $E(B_i)$	Standard Deviation (σ_i)	Expected Value $E(B_i)$	Standard Deviation (σ_i)
B_1	60	3	80	10
B_2	60	4	50	6
B_3	60	5	50	6
E_o	100	0	100	0

Let us denote the expected value of the net present value of a project by P*, i.e.,

$$E\left\{ NPV(r) \right\} = E\left\{ P \right\} = P^* \qquad \text{Eq. (4.1)}$$

Taking the expected values of the prospective cash flows, in Eq. 1,[9]

$$P^* = \sum_{i=1}^{n} \frac{E(B_i)}{(1+r)^i} - E_o = \sum_{i=0}^{n} \frac{E(B_i)}{(1+r)^i} \qquad \text{Eq. (4.2)}$$

Assume initially that (E_o, B_1, . . ., B_n) are mutually independent. Therefore, it is well known that P would have a normal distribution, with mean given in Eq. 4B and with variance as follows:

$$\sigma^2_P = \sum_{i=0}^{n} \frac{\sigma_i^2}{(1+r)^{2i}} \qquad \text{Eq. 5}$$

Having thus obtained the probability distribution of P, the decision-maker can now evaluate the risk aspect of the investment decision. Consider Project C. At a discounting rate r = 35%, the procedure under certainty would favor adoption of the project. However, with additional information, σ_p = 8.46, the following additional analysis can be made. Using widely available tables for the standardized normal distribution, the decision-maker can note that the probability that $P<O$, so that the investment would not pay, is .204[10]. Hence, this value of Prob $\{P<O \mid r\}$ provides the amount of

[9] For convenience, let $E_o = - E(B_o)$ in anticipation of Eq. 5.

[10] This value can be obtained from tables of normal distributions as follows, the mean of the distribution of P at 35% is 7.02. Dividing (0-7.02) by the standard deviation, o_p = 8.46, we get –.83. Hence Prob $\{P < O \ / \ r\}$ = .204.

risk *exposure* of the firm in adopting the project. Note that Prob $\{P<O \mid r\}$ = Prob $\{r^* < r\}$[11]. The "risk-free" component would be

$$\text{Prob}\ \{ P \geq O \mid r \} = 1 - \text{Prob}\ \{ P < O \mid r \}. \qquad \text{Eq. 6}$$

The corresponding graphs of Equation 6 can be plotted for Projects A and C for different values of r (see Figure 1.)

To interpret Figure 1, let us first take Project A. Recall that its internal rate of return r^*_a = 36.8% as obtained under certainty. Note that is the value at Prob $\{P \geq O \mid r = 36.8\}$ = .5 following the normality assumption. Hence, under risk, r^* is a random variable normally distributed with mean $E(r^*)$ = 36.8%.

From $O \leq r \leq .27$, Project A has almost negligible risk exposure. The chances are 99% that Project A would yield at least 27%. As one assigns higher values of the discounting rate, the value of Prob $\{P \geq O \mid r\}$ diminishes. Hence, the risk exposure increases with r so that at around r = 45%, the Prob $\{P \geq O \mid r\}$ approaches zero for Project A and maximum risk exposure is attained.

When one compares the graphs of the two projects, it can be noted that the two curves intersect at around r = 33.4%. To the left at this intersection point, Project A has a smaller risk exposure compared to Project C. To the right of this point, the relationship becomes the other way around.

Therefore, if one had to choose between the two projects, Project C will be preferred to Project A *only if* the discounting rate used is greater than 33.4%. Otherwise, Project A will be preferred at $r <$ 33.4%, because Project C becomes a riskier investment at this range of r. Recall that under certainty, Project C was unconditionally preferred over Project A.

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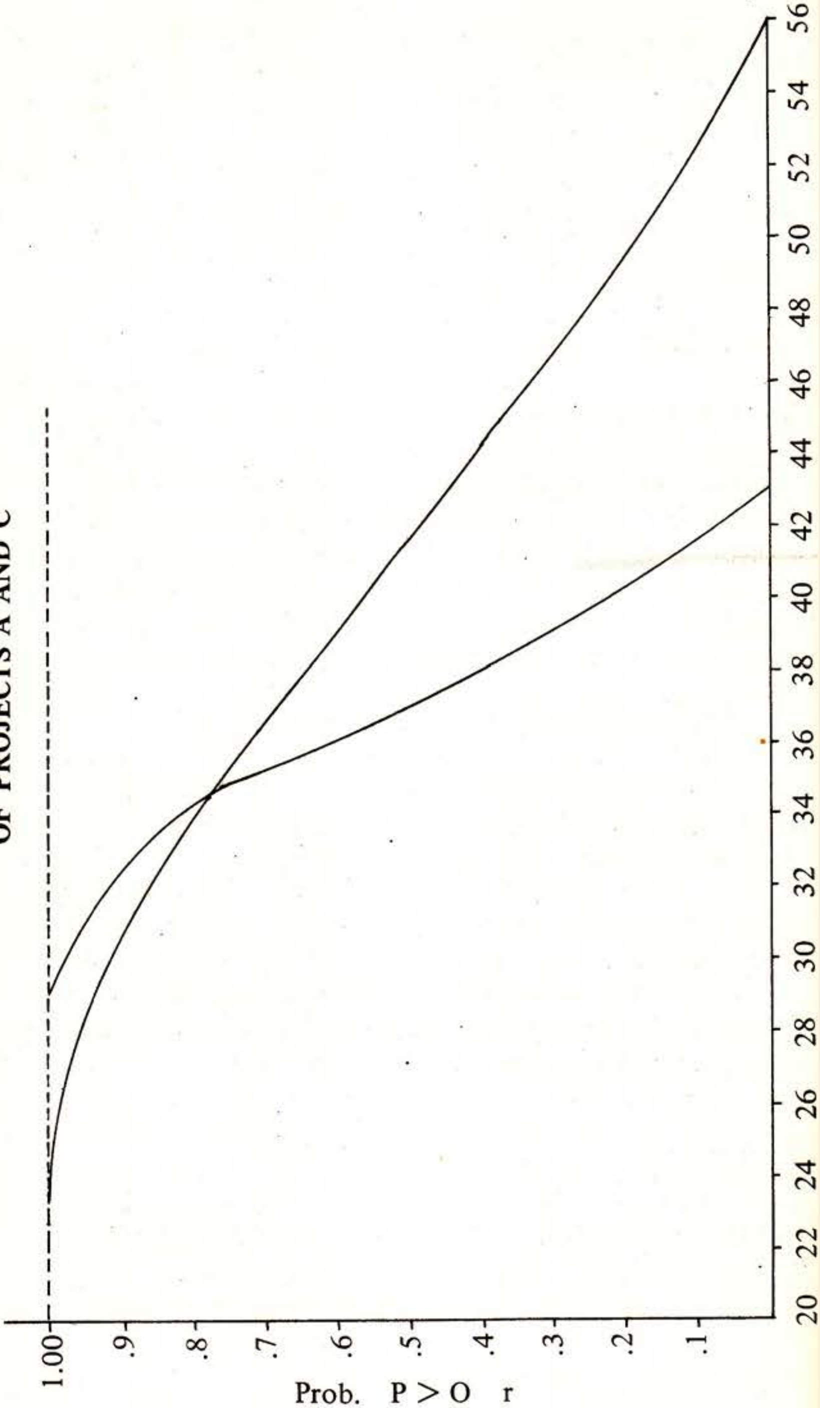
[11]For a proof of this equality, see F. S. Hillier, "The Derivation of Probabilistic Information For The Evaluation of Risky Investments," Management Science, Vol. IX (April 1963) pp. 443-57. For a given value of r, $P < O$ if and only if $r^* < r$.

FIGURE 1

A COMPARISON OF THE RISK-RETURN CHARACTERISTICS OF PROJECTS A AND C



In terms of the procedure for decision-making, the firm has two decision variables: *the cut-off rate* and *the minimum allowable risk exposure.* Going back to Figure 1, a project would be adopted if it fulfills both conditions, e.g., a firm may set as its double standards: a minimum return of 35% with a minimum risk exposure of 15%. In this case, neither Project A nor Project C above could pass the test. At 35%, Project S's risk exposure is 20.4%, while that of Project A is around 45%.

III. CONCLUSIONS

The preceding discussions were intended to illustrate a method of incorporating the risk factor in investment decisions. The indiscriminate application of a procedure designed for situations under certainty to what are essentially risky situations creates unwarranted added responsibility for the decision-maker, i.e., that the actual return would come out to be greater than or equal to the project's internal rate of return. This may also account for the popularity among practitioners of the payback-period over the discounted cash flow method, because the former, for all its crudeness, after all, is designed in part to cope with the uncertainty, or risk, factor.

At another level, the risk factor adds another dimension to the problems of developing economies. Ordinarily, the risk factor exists under normal conditions as individual firms try to plan for the future. Even under normal conditions, it has been shown that a low level of interest rates may be necessary, but not a sufficient condition for inducing a high level of net investments. If, in addition to normal risks, the added uncertainty arising from general economic and social instability has to be considered, then the obstacles to capital formation indeed become compounded.