GROUP DECISIONS INVOLVING RISK[1]

By

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I. Introduction

Most formal analyses of group choice in risk situations have two features in common: they assume von Neumann-Morgenstern utility functions (which give certainty equivalents for risk choices) and they tend to have a primarily normative orientation.[2] By a formal analysis here, we mean one that expresses the group choice as a well defined function of the preferences of the individuals composing the group. The aim of this paper is to present a hypothesis based on lexicographic preferences which appears to have explanatory value for some group decision situations. A group of individuals is assumed to make a choice from a set of feasible alternatives whose outcomes depend on the true state of nature, and the possible states of nature are considered to have known probabilities. The members of the group (e.g., the board of directors of a business firm, a management team, an economic planning commission) share certain values or objectives in common but they may have different preference orderings over the possible outcomes. The main result is that the alternative chosen can be thought of as being reached through a series of steps each of which is by simple majority rule.

We recall briefly first the concept of lexicographic preferences.[3] Let f_i ($i = 1, 2, \ldots$) be an index of preference in the sense that if $f_i(x) > f_i(y)$, x is preferred to y *on the basis of the ith criterion of choice*. A vector $f(x) = [f_1(x), f_2(x), \ldots]$ is thus associated with each alternative x, characterizing x in terms of the various dimensions of choice. Let f_i^* denote a critical or satisfactory level regarding f_i, and write $g_i(x) = \min [f_i(x), f_i^*]$. Suppose that x is preferred to y if and only if the first non-zero difference $g_i(x) - g_i(y)$, $i = 1, 2, \ldots$, is positive. In other words, suppose that the preference ordering of the alternatives is given by the lexicographic ordering of the vectors $g(x) = [g_1(x), g_2(x), \ldots]$. Then we shall say for short that the preference ordering is L*-determined by f_i ($i = 1, 2, \ldots$). If we are referring to the preference system of individual k ($k = 1, 2, \ldots, N$), we shall attach a superscript k to the appropriate symbols. In this conception of choice, the decision maker maximizes some f_h subject to appropriate constraints on f_i for $i < h$. For instance, if there exist alternatives satisfying f_1^*, the choice is constrained by $f_1(x) \geqslant f_1^*$.

The above formulation permits the same choice criterion to appear more than once in f(x). For example, in the theory of the firm, f_i^* could correspond to a critical minimum rate of profit, below which is financial disaster, and f_j^* $(j > i)$ to a satisfactory rate of profit, beyond which the firm may choose to promote sales rather than still greater profit. Also, f_i may be a function of time, and different f_i^* may pertain to different times or time periods. In a development program, f_i^* could correspond to a minimum consumption level in the current period and f_j^* to a similar level at some later time.[4]

II. Certain Alternatives

It will be convenient to consider the simpler case of certainty before proceeding to that of risk. Let u_i $(i = 1, 2, \ldots)$ be a preference index on the basis of the group's ith value, so that given any two alternatives x and y, the members agree on which of the following holds: $u_i(x) > u_i(y)$, $u_i(x) < u_i(y)$, $u_i(x) = u_i(y)$. There is thus a unique ranking of the alternatives on the basis of any particular criterion even if the rankings by the various criteria are different. The essential requirement is that the outcome of x can be represented by a vector $u(x) = [u_1(x), u_2(x), \ldots]$ which is the same for every member.

Our basic assumption is that individual k's preference ordering is L*-determined by u_i^k $(i = 1, 2, \ldots)$, $k = 1, \ldots, N$, and the group preference ordering by u_i $(i = 1, 2, \ldots)$. Since k's ordering depends on the parameters $u_1^{k*}, u_2^{k*}, \ldots$, differences among the members' u_i^{k*} levels account for their different preferences even though their priority rankings of the criteria $u_1, u_2, \ldots$, are identical. For example, although the members of a planning commission may agree on lesser unemployment and higher per capita income as values to be promoted, in that order, they may very well hold different views as to what constitutes a tolerable level of unemployment or a satisfactory increase in income. Within the firm, board directors may disagree on the satisfactory rate of profit and the extent to which sales should be increased. Despite common agreement on the values governing the group's decisions and their relative importance, we may expect that the members will have different preferences over the possible outcomes.

It is natural to try to define the group's u_i^* levels in terms of the members' u_i^{k*} values. Consider first the selection of u_1^* on the basis of individual choices for this parameter. Everyone will want u_1^* to be as close as possible to his own u_1^{k*}, since any higher value implies an unnecessarily high constraint on u_1 while any lower value is less than satisfactory. Assuming an odd number of members (or an extra tie-breaking vote by the chairman in the case of an even number), the conditions of D. Black's theorem on single-peaked preferences [3, p. 16] are clearly satisfied; therefore the median of the u_1^{k*} is the only choice for u_1^* that can win by at least a simple majority against any other possible choice. We accordingly define u_1^* as the median of the u_1^{k*}, or $u_1^* = \text{med}(u_1^{k*})$ for short. By a similar argument, $u_2^* = \text{med}(u_2^{k*})$, etc., and the group preference ordering is completely determined.

The u_i^* may be called the group objectives, for the group decision is one that attains as many of the u_i^* as possible, starting with the most important. It is worth observing that this formulation gives determinateness to Tinbergen's well known analysis of the use of "target variables" in macro-economic policy [19]. If not all the target variables (the objectives) can be reached, the least important are dropped from consideration. A similar remark applies to Simon's concept of "satisficing" [16].

From the assumptions it follows that the group prefers x to y if and only if there is a u_h such that: $u_h(x) > u_h(y)$ and a majority finds y less than satisfactory with respect to u_h; and for very $i < h$, $u_i(x) = u_i(y)$ or else a majority considers x and y satisfactory on the basis of u_i. For, note that since $u_i^* = \text{med}(u_i^{k*})$, $u_i(z) \geqslant u_i^*$ if a majority considers z satisfactory regarding u_i, and $u_i(z) < u_i^*$ otherwise. The "if" part of the statement therefore implies $\min[u_h(x), u_h^*] > \min[u_h(y), u_h^*]$ and $\min[u_i(x), u_i^*] = \min[u_i(y), u_i^*]$ for $i < h$, giving the preference relation between x and y. The "only if" part is equally clear.

Accordingly, the group reaches its decision through a series of steps. Confronted with a choice between alternatives, the members consider them first on the basis of u_1; if a majority finds them satisfactory, the alternatives are then examined in regard to u_2. Again, if a majority (which may be different from the preceeding majority) finds them satisfactory, the third criterion of choice becomes relevant. The procedure continues until one alternative comes out as definitely superior by majority vote. The alternative chosen, i.e. the group decision, is thus the result of a series of separate decisions each of which is made by simple majority rule. This is certainly consistent with what we know of actual group decision processes, for issues are in fact considered one at a time and settled usually by majority vote.

III. The Case of Risk

In the absence of certainty, the outcome of x when s is the true state of nature may be represented by $u(x, s) = [u_1(x, s), u_2(x, s), \ldots]$. We assume an objectively known probability measure on the set of possible states S, or at least that the members of the group use the same measure in forming their judgments. Although this is a severe simplification, it seems a fair approximation where the members of the group are supplied with the same statistical or technical report by a professional staff. This would seem to be the typical situation in the decision making of a board of directors or a planning commission.

A natural question that is asked in a risk situation is, which course of action has the better chance of achieving a desired result? A second relevant question is, what would be the worst result of each act? The following elements thus appear as criteria of choice:

$$p_i^k(x) = \Pr[u_i(x, s) \geqslant u_i^{k*}]$$

$$\bar{u}_i(x) = \min_{s \in S} u_i(x, s) \qquad (i = 1, 2, \ldots),$$

$p_i^k(x)$ being the probability that the outcome of x will give u_i^{k*} or better, $\bar{u}_i(x)$ the worst possible outcome of x in terms of u_i. By analogy,

$$p_i(x) = \Pr[u_i(x, s) \geqslant u_i^*]$$

would be relevant in a group's decision making. Various forms of this criterion have been discussed in the literature, though not explicitly within the context of group decision. For instance, H. Cramér interprets "the probability that income will fall below some critical level as the criterion for ordering" (cited by Arrow [1, p. 423]) the alternatives facing an insurance company. Marschak has observed, in the course of discussing various determinants of choice, that "another important parameter is the probability that a certain variable (cash reserves, annual profit, etc.) fall below a constant, for example, below zero" [16, p. 120, n. 10].[5]

If $p_i^k(x) \geqslant p_i^{k*}$, we shall say that individual k considers x satisfactory in regard to p_i^k. In other words, k takes the probability level p_i^{k*} as satisfactory for attaining u_i^{k*}. The greater is p_i^{k*}, the less is k willing to risk this objective. This concept of a satisfactory probability is not unfamiliar. There is, for example, the standard statistical practice of taking some probability level as good enough for the purpose of detecting batches of goods containing more than a certain fraction of defectives. The classical Neyman-Pearson rule can be understood in a similar way: a given probability of avoiding a type 1 error is considered satisfactory. More recently, Charnes and Cooper [4] [5] have analyzed programming problems in which the usual inequality constraints are required to hold only at stated probability levels.[6]

Given that $p_i^k(x) = p_i^k(y)$, one may still distinguish between x and y as to their worst possible outcomes: x would be preferable on this account if $\bar{u}_i(x) > \bar{u}_i(y)$. By considering $\bar{u}_i$, one puts a floor under the worst results of a decision. It might be thought that the mathematical expectation of u_i would be just as plausible a criterion of choice as its minimum; however, the function u_i may be purely "ordinal," in which case the expected value is quite ambiguous as a choice criterion. Moreover, nothing prevents the criterion u_i itself from being the expected value of an appropriate variable. There is thus no loss of generality in this regard.

We now assume that k's preference ordering is L*-determined by f_j^k ($j = 1, 2, \ldots$), where

$$f_{2i-1}^k(x) = p_i^k(x), \quad f_{2i-1}^{k*} = p_i^{k*}$$
$$f_{2i}^k(x) = \bar{u}_i(x), \quad f_{2i}^{k*} = u_i^{k*} \qquad (i = 1, 2, \ldots)$$

and that the group preference ordering is similarly defined, omitting the index k throughout. Since the u_i^* are already known from Section 2, the only parameters to be determined are the p_i^*.

According to this formulation, k as the sole decision maker would first consider the alternatives that give a satisfactory probability of attaining u_1^{k*}; if there are none, he maximizes $p_1^k(x)$. If a further selection is possible, he narrows his field of choice so that if possible, u_1^{k*} is surely attained. There will be such alternatives only if $p_1^k(x) = 1$ for some x; otherwise, he simply maximizes $\bar{u}_1(x)$, thereby ensuring that the result will not be any lower. The procedure is repeated for the second objective, etc., until only one alternative is left.

However, k is not the sole decision maker, and the questions facing the group concern the group objectives u_i^* rather than his u_i^{k*}. We assume that each individual adapts himself to this fact and makes appropriate adjustments in his parameters of choice. More precisely, instead of

$$f_{2i-1}^k(x) = \Pr[u_i(x, s) \geqslant u_i^{k*}]$$

k takes

$$f_{2i-1}(x) = \Pr[u_i(x, s) \geqslant u_i^*]$$

as the relevant criterion in a group decision context. Now if $u_i^* > u_i^{k*}$, k would consider u_i^* excessive and its attainment can accordingly be exposed to greater risk; he would therefore select a satisfactory probability level for u_i^*, say $p_i^{k*\prime}$, such that $p_i^{k*\prime} < p_i^{k*}$. If $u_i^* < u_i^{k*}$, k would want a higher probability that the group decision should yield at least u_i^*, in which case $p_i^{k*\prime} > p_i^{k*}$. By such adjustments, each member evaluates the prospects of reaching the group objectives in terms of his own preferences.

Returning to the problem of defining the group's p_i^* levels, we see that the theorem on single-peaked preferences is applicable again, for each individual k will want p_i^* to be as close as possible to $p_i^{k*\prime}$. Accordingly, $p_i^* = \text{med}(p_i^{k*\prime})$ for each i, and the group decision is uniquely determined.

As in the case of certain alternatives, we can derive a proposition relating group preference to the use of majority rule: the group prefers x to y if and only if there is an f_h such that $f_h(x) > f_h(y)$ and a majority finds y less than satisfactory with regard to f_h, and for every $j < h$, $f_j(x) = f_j(y)$ or else a majority considers x and y satisfactory regarding f_j. This is formally the same as the earlier statement in Section 2 and is shown in an analogous way. We merely note that if a majority considers z satisfactory with respect to f_j, then for $j = 2i - 1$, $p_i(z) \geqslant p_i^*$ since $p_i^* = \text{med}(p_i^{k*\prime})$, and for $j = 2i$, $\bar{u}_i(z) \geqslant u_i^*$ since $u_i^* = \text{med}(u_i^{k*})$.

From a theoretical standpoint, the above property of the group decision model is important in two respects: (1) it gives a role to majority decisions, (2) the group arrives at a choice between alternatives even without any explicit agreement on the group's p_i^* levels. If a majority considers x and y satisfactory with regard to the probability of reaching the first objective, they are compared on the basis of their worst possible outcomes. Should x and y be equal in this regard, the probability of reaching the second objective becomes relevant. Again, if x and y are found satisfactory by a majority, the next criterion is considered, etc., until a majority vote selects one alternative as superior. At no stage is there any need for the members to discuss the satisfactory probability level for attaining an objective. What is necessary is a common knowledge of the group objectives, which does not seem unrealistic for the class of group decision situations we are considering. For example, it may be clear from previous experience that the board of directors aims at a 10% rate of return on capital, or that the planning commission finds an unemployment figure greater than 5% intolerable.

In practice, of course, a group does not always go through the motions of evaluating the alternatives according to the choice criteria *seriatim* every time a decision is made. The reason for this would be that the alternatives being considered satisfy many of the more important criteria to begin with. It is only when the outcomes being yielded by the state of nature are falling short of important objectives would we find the group paying explicit attention to the means for achieving such objectives. Thus, for example, the question of avoiding bankruptcy is not a usual item on the agenda of board meetings.

As one may require, the formulation in Section 2 of group choice under certainty is the case where a state of nature has probability 1 and all other states have probability 0. In this case, $p_i(x)$ can only be 0 or 1. Writing $g_j(x) = \min[f_j(x), f_j^*]$, consider the ranking of the alternatives according to $g_{2i-1}(x)$ and $g_{2i}(x)$. If $p_i(x) = 1$, $g_{2i-1}(x) = p_i^*$ and $g_{2i}(x) = u_i^*$; if $p_i(x) = 0$, $g_{2i-1}(x) = 0$ and $g_{2i}(x) = \bar{u}_i(x) = u_i(x)$ simply. Since $p_i^* > 0$, this means in brief that the ranking is given by $\min[u_i(x), u_i^*]$. The group preference ordering is thus the same as was defined in Section 2, and a similar remark obviously holds for each individual's preference ordering.

IV. Some Implications for Welfare Economics

In general, i.e. when more than one choice criterion is relevant, the group decision does not depend on the members' preferences among the alternatives themselves; it depends instead on the members' choices for the parameters of the group preference function. Even if "cyclical majorities" (where different majorities would choose x over y, y over z, and z over x) should exist, since the group preference ordering is of course transitive, the group decision is well defined. The fact that the group's choice is thus not a function of majority preferences among the alternatives can sometimes lead to a violation of the Pareto principle [2, p. 96], which requires the group to choose x over y if every member does.

To show this possibility, let there be two alternatives x, y such that $u_i(x) > u_i(y)$ $(i = 1, 2, 3)$, and let

$$G = m_1 \cup m_2 \cup m_3, m_i \cap m_j = 0 \text{ for } i \neq j$$

$$= M_i \cup m_i \quad (i = 1, 2, 3)$$

where M_i is a majority and m_i a minority in the group G. Suppose that the individuals in M_i find x and y satisfactory as regards u_i $(i = 1, 2, 3)$; then the group would choose y over x if $u_4(x) < u_4(y) \leqslant u_4^*$. At the same time, it is also possible that the individuals in m_i consider y less than satisfactory as regards u_i $(i = 1, 2, 3)$. In this case, everyone in m_1 obviously prefers x to y. Since m_2 is contained in $M_1 = m_2 \cup m_3$, individuals in m_2 base their preference on u_2 and therefore prefer x. And since m_3 is contained in M_1 and M_2, so do the individuals in m_3. Therefore everyone in G prefers x to y, but y is the group's choice. The members may not realize when a violation of the Pareto principle occurs, since alternatives are multi-dimensional and the method of group decision considers the various dimensions of choice only one at a time.

It would seem awkward for a decision model to admit violations of the Pareto principle, but the present model is intended to explain actual decision making in certain circumstances; it is not required to satisfy any normative principle as such. If violations do occur, the Pareto principle is simply false as an empirical generalization.

But to turn to normative analysis for a moment, must the Pareto principle have universal scope? It could be argued that occasional violations would be accepted by the members of a group whose decision process is basically democratic and has a value of its own. "For example, the belief in democracy may be so strong that any decision on the distribution of goods arrived at democratically may be preferred to such a decision arrived at in other ways, even though all individuals might have preferred the second distribution of goods to the first if it had been arrived at democratically" [2, p. 90]. Moreover, suppose that in effect a group makes decisions for a larger body (as in the case of a board of directors, a national planning commission, or an executive committee). Every member could well accept the group decision y in spite of his own preference for x, especially if the larger body would choose y over x. The group may not know the larger body's preference, but the point is simply that the validity of the Pareto principle is not as universally compelling as might seem at first sight.

A conceptual framework based on lexicographic preferences, if broadly correct, has far-reaching consequences for normative economics. We should have to take explicit account of the multiplicity of values and objectives by means of a vector-valued welfare function. Obviously this makes analysis more complicated. For instance, the standard analysis of resource allocation in terms of a single set of shadow prices – which is premised on just one objective – would be quite misleading; each objective would require a corresponding set of shadow prices for evaluating resources. On the other hand, we avoid difficulties inherent in the conception of social welfare as a scalar quantity, for as Strotz [17] has shown, unacceptable income distributions result from certain seemingly innocuous value judgments that presuppose a real-valued welfare function. Furthermore, the representation of welfare as a vector permits a natural basis for international comparisons. Given a common priority ranking of objectives, it would make sense to say that a country pursuing an objective with a higher index – the more important objectives having been reached – is better off than one that is still concerned with a lower index.

V. Concluding Remark

There has been no attempt here to consider the general problem where individuals may have opposite interests regarding some objectives. But the class of group decision situations where the members do have common objectives seems sufficiently important to deserve specific study. In the model of this paper, risk-taking attitudes are represented in an extremely simple way and majority rule plays an important part. The widespread practice of majority rule thus finds an explanation that seems lacking otherwise.

FOOTNOTES

1. The author is grateful to the Rockefeller and Ford Foundations for research support.

2. See for instance Hildreth [12], Savage [15, Chap. 10], Harsanyi [11], Strotz [17], and Theil [18, Chap. 7].

3. The important articles by Georgescu-Roegen [10] and Chipman [6] are basic. See Encarnacion [8] [9].

4. Cf. Day [7].

5. See also Roy [14] where a similar argument is made.

6. Cf. also the suggestion by Tisdell that the firm attempts to maximize expected income "subject to the restriction that the probability of its income being equal to or below some present level be less than or equal to a specified probability" [20, p. 109].

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