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ON THE APPLICATION OF INFORMATION THEORY IN THE ANALYSIS OF FINANCIAL STATEMENT: A REPORT ON AN ONGOING RESEARCH PROJECT

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Introduction

Information theory, as formulated and developed in electrical engineering applications with the pioneering work of Shannon (1948), is concerned with the definition and measurement of the amount of information contained in a message. Its application in the analysis of financial statements is based on the inherent characteristics of these statements, that a number of items add up to a total, or, to put it differently, that the total can be decomposed into parts. Theil (1969) has shown that informational concepts may be applied to such decompositions to measure the size of their shares (say, current assets), the changes in size over time, as well as the changes in their sizes in relation to some other group of items (say, current assets to current liabilities).

It is argued that information measures are superior to the conventional ratio measures found in the analysis of financial statements. Unlike these traditional financial ratios which are limited to an accounting period and are therefore static and directionless, information measures are "measures of variability" since they are applied to financial statements of two or more accounting periods. Thus, information measures are dynamic. They are also "distance measures" in that they can indicate how far a financial statement item of one year deviates from that of another year, although one cannot infer the direction of change from them just as in the case of conventional financial ratios. All told, however, these information measures are useful only to the extent that they have predictive

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validity.

Lev (1969) tested the usefulness of these information measures in Accounting and found, among other things, that these measures can discriminate between failing and non-failing firms at least five years prior to the occurrence of failure.[1] Thus, the ability of these measures to predict financial failure with such a lead time makes them ideal components of a model on an early warning system on financial performance.

In the following sections of this paper, we proceed as follows: Section 2 describes the basic information theory concepts. Section 3 deals with the disaggregation of balance sheet information into assets and liabilities decomposition and a time-horizon decomposition. Finally, Section 4 discusses the information balance sheets derived, using the computer programs developed as Phase I of the research project, for seven marketing companies in the oil industry on the basis of their balance sheets for the years 1970 and 1971.

Although information concepts can be applied to income statements, to sales, costs and expenses of a multiple product or multiple territory firm (Theil 1969, 467-72), and to budgets (Lev 1969, 35-54), our research project will be confined to the generation of information balance sheets for a large number of companies, say, comparable to *Business Day's* list of the Top 1,000 Philippine Corporations, in order to be able to make inter-temporal comparisons and have a basis to determine their predictive validity in relation to the conventional financial ratios. Also, inter-company comparison will be provided on the basis of an industry-wide information balance sheet especially for industries of interest like the banking sector.

Information Theory Concepts

The quantification of information is premised on whether or not probabilities can be attached to a number of possible answers to a problem. If this can be done, then knowledge about the changes in probabilities permit measurement of the amount of information con-

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[1] Failure was defined by Lev (1969, 28) as the inability of a firm to pay its financial obligations as they mature. He operationalized this in terms of the occurrence of the following events: bankruptcy, bond default, overdrawn bank account or non-payment of a preferred stock dividend. Lev used the same data that Beaver (1966) used in his study on financial ratios as predictors of failure. These ratios deteriorated greatest between the third and second year prior to failure.

tained in the message that induced these changes. Thus, when information is received about the problem, the probabilities undergo transformation so that information becomes a function of two sets of probabilities: one before the message is received and the other after it.

Let us first consider the case of a definite or certain message. Given a set of *n* mutually exclusive and collectively exhaustive events $S = (E_1 \ldots, E_n)$ with probabilities $p_1, \ldots p_n$, a message on the realization of a specific event E_1, can be said to be *more* informative if the event was *less* likely to occur. In other words, the amount of information conveyed by a message is said to be small if the probability of the occurrence of an event is close to 1 since its occurrence is not surprising or unexpected. Thus, the smaller the probability, the larger the unexpectedness caused by the message and hence, the larger its informational content.

The information content of a message can be specified, following Shannon (1948), as a function:

(1) $h(p) = \log(1/p) = -\log p$

which decreases monotonically from infinity to zero when p increases from 0 to 1 as shown by Figure 1. Thus, when $p = 1$, $h(p) = 0$ since no information at all is contained by the message.

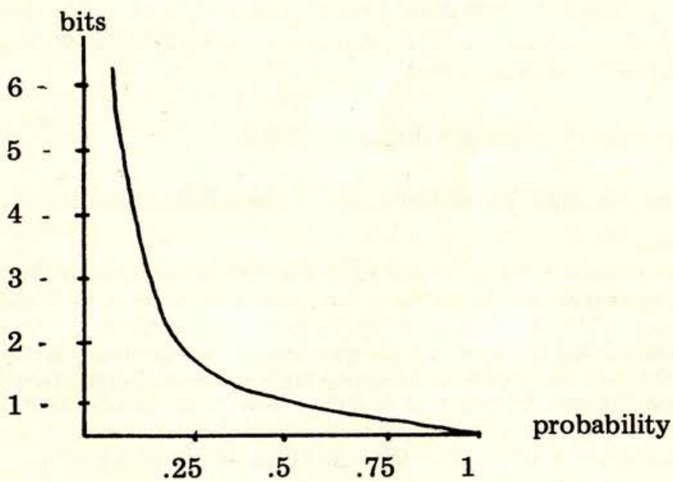

Figure 1 — Information Function $h(p) = -\log p$

The unit of information is determined by the base of the logarithm. In information theory, logarithms to the base 2 are used and information is said to be measured in *bits* (binary digits).

It should be noted that we can apply the information function (1) only if we know that E has occurred. Before the message comes in, however, we can only determine the *expected* information of the message that states which of the events $E_1, \ldots E_n$ occurred. This is referred to as a nondefinite message. The expected information content of the forthcoming or nondefinite message is:.

(2) $H = \Sigma_{i=1}^{n} \quad p_i \log (1/p_i) \ = \ - \Sigma_{i=1}^{n} \quad p_i \log p_i$

which is also known as the *entropy* of the distribution whose probabilities are $p_i, \ldots p_n$. The minimum value of H is zero, which corresponds to the case where one of the p's is equal to 1 and all others are equal to 0. The maximum value of H is log *n*, which corresponds to the case where all the p's are equal to 1/n. Clearly log *n* increases as the number of possible outcomes, *n*, increases corresponding to the greater uncertainty of the specific outcome.[2]

To generalize the information concept in terms of a nondefinite message, we now consider changes in the probability of an event. The message may indicate that the odds in favor of E have changed such that the original or prior probability p is replaced by the posterior probability q. The information received with this definite message is $h(p) = -\log p$ based on the prior probability and $h(q) = -\log q$ based on the revised probability. The change in information content is therefore the difference:[3]

(3) $h(p) - h(q) = -\log p + \log q = \log q/p$

When the message leaves the original probability unchanged, i.e.,

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[2] The relationship between uncertainty and information stems from the former being present before the arrival of the message while the latter is supplied by the message. Thus the larger the uncertainty, the larger is the expected amount of information conveyed by the message. An analogous application can be found in the revision of prior to posterior probabilities in Bayesian Statistics. A prior probability distribution that is diffuse (events are equally likely) and, therefore, informationless is very sensitive to sample information such that the posterior distribution, in effect, is a reflection of the sample information.

[3] The logarithmic functions h(p) and h(q) have additive properties; i.e., information is an additive concept (Lev 1969, 4-5).

p = q, then the measures (3) equal zero, indicating that no information is gained. It is negative when $p > q$ or when the prior probability is decreased by the new information. It is positive when a message brings about an increase in the prior probability ($p < q$).

As in the above discussion, we cannot compute (3) unless we know whether the event will ultimately occur. In the case of a nondefinite message, we can only compute the expected information content of a message about a set of events of which one is bound to occur. This is given by:

$$(4) \qquad I(q{:}p) = \Sigma_{i=1}^{n} \quad q_i \log q_i/p_i$$

which can be shown to be always nonnegative (Lev 1969, 71) It is zero if and only if $p_i = q_i$ for all i, in which case no information is obtained. The measure, I(q:p), will be positive if $p_i \neq q_i$ for some i and increases as the discrepancy between the p_i's and the q_i's increases.

The Information Balance Sheet

A balance sheet is composed of asset items and items in the liabilities side [equity is regarded by Theil (1969, 462) as part of total liabilities]. Thus, one can convert any kind of decomposition into a set of fractions. For example, total assets can be decomposed into current and fixed assets for which fractions can be computed. Since these fractions are nonnegative and add up to one, probabilities are thus measured in terms of the proportion of a particular item to the total.[4]

Consider the following two-day table of probabilities pertaining to a firm's balance sheet (Figure 2)

	Assets	Liabilities	Total
Current	p_{11}/q_{11}	p_{12}/q_{12}	$p_{1.}/q_{1.}$
Medium-term	p_{21}/q_{21}	p_{22}/q_{22}	$p_{2.}/q_{2.}$
Fixed	p_{31}/q_{31}	p_{32}/q_{32}	$p_{3.}/q_{3.}$
Total	$p_{.1}/q_{.1}$ (= 1/2)	$p_{.2}/q_{.2}$ (= 1/2)	1

Figure 2. A Two-way Balance Sheet Table

[4]The probabilistic interpretation of these fractions can be justified by the answer to the question: If one draws at random a peso of the company's asset, what is the chance that it will fall under asset i?

The bivariate probabilities p_{ij} and q_{ij} pertain to time periods t-k and t, respectively, where k is the number of lagged periods. As indicated above, the p_{ij}'s are prior probabilities while q_{ij}'s are posterior probabilities. The index j (two columns in Figure 2) refers to the side of the balance sheet (assets and liabilities) while the index i (three rows) refers to the time horizon of the item (current, medium and fixed).

The balance sheet information measure for period t is:

$$(5) \qquad I(q:p) = \Sigma_{i=1}^{3} \quad \Sigma_{j=1}^{2} \; q_{ij} \log q_{ij}/p_{ij}$$

This measure incorporates assets and liabilities simultaneously. It is an overall measure of the change in the relative positions of the six groups represented in the six cells of Figure 2. The larger the instability of these groups relative to a proportional development ($p_{ij} = q_{ij}$), the larger the measure I(q:p). It is thus useful when compared with past measures and/or corresponding measures for other firms.

Proceeding now to the decomposition of (5), we can first derive the assets information and liabilities information based on the columns of Figure 2. These are:

$$(6) \qquad I_A(q:p) = \Sigma_{i=1}^{3} \quad q_{i1} \quad \log q_{i1}/p_{i1}$$

$$(7) \qquad I_L(q:p) = \Sigma_{i=1}^{3} \quad q_{i2} \quad \log q_{i2}/p_{i2}$$

The assets information measure (6) describes the behavior of the individual asset items (current, medium and fixed) over the time spanned by two balance sheets. The larger the discrepancy between p_{i1} and q_{i1}, the larger $I_A(q:p)$. It, therefore, represents the end result of management's policy with respect to resource allocation.

Similarly, the liabilities information measure (7) indicates the degree to which liabilities deviated from a proportional development.[5] A large $I_L(q:p)$ is indicative of the efforts of a firm to correct excessive leverage through the reorganization and consolidation of

[5] Note that it is possible to have negative items among liabilities such as negative retained earnings. Information measures, being in logarithmic form, cannot handle negative numbers. This can be remedied by aggregating it with positive, larger items such as a common stock. Thus, one will have to be careful in increasing the number of rows of Figure 2.

debts and equities.

It can be shown that the arithmetic mean of the assets information measure and the liabilities information is equal to the balance sheet information measure (Theil 1969, 474):

$$(8) \qquad I(q{:}p) = (I_A(q{:}p) + I_L \ (q{:}p)/2$$

The alternative decomposition of I(q:p) is based on the rows of Figure 2: current, medium term and fixed. Here we are interested in the relationship of the probabilities in the row cells and row marginal probabilities as well as the relationship of the prior and posterior marginal probabilities. These are the current items information (I_1), the medium-term information (I_2), the fixed item information (I_3) and the time horizon information (I_T), Thus

$$(9) \qquad I_1 = \Sigma_{j=1}^{2} \quad q_{1j}/q_{1.} \quad \log \quad \frac{q_{1j}/q_{1.}}{p_{1j}/p_{1.}}$$

$$(10) \qquad I_2 = \Sigma_{j=1}^{2} \quad q_{2j}/q_{2.} \quad \log \quad \frac{q_{2j}/q_{2.}}{p_{2j}/p_{2.}}$$

$$(11) \qquad I_3 = \Sigma_{j=1}^{2} \quad q_{3j}/q_{3.} \quad \log \quad \frac{q_{3j}/q_{3.}}{p_{3j}/p_{3.}}$$

$$(12) \qquad I_T = \Sigma_{i=1}^{3} \quad q_{i.} \quad \log \quad q_{i.}/p_{i.}$$

The time horizon information at time $t(I_T)$ is concerned with changes in the relative positions of all current items (assets and liabilities combined), all medium-term items, and all fixed items. On the other hand, the current item information (I_1) distinguishes between changes in current assets and current liabilities during two periods to which the well known financial ratio "working capital ratio" is a static counterpart. Similarly, the medium-term item information (I_2) and fixed item information (I_3) discriminates between changes in assets and liabilities.

The decomposition of the balance sheet information measure (5) is now:

$$(13) \qquad I(q{:}p) = q_{1.}\ I_1 + q_{2.}\ I_2 + q_{3.}\ I_3 + I_T$$

which says that the balance sheet information is equal to the weighted average of 1_1, 1_2 and 1_3, using the posterior marginal probabilities as weights, plus the time horizon information.

The two alternative decompositions (8) and (13) can be summarized in an information balance sheet. This will be presented in the next section for each of the oil marketing companies.

For intercompany comparison, we want to derive an industry-wide information balance sheet. As in the discussion above, we are interested in alternative decompositions of the balance sheet information given as follows:

$$(14) \qquad I(q:p) = \Sigma_{k=1}^{N} \quad w_k \, \Sigma_{i=1}^{3} \quad \Sigma_{j=1}^{2} \quad q_{ijk} \quad \log \quad q_{ijk}/\gamma_{ij}$$

where $\gamma_{ij} = \Sigma_{k=1}^{N} w_k \, q_{ijk}$ with q_{ijk} defined as the fraction of the kth company's (k=1,. . .,N) asset or liability to its total assets plus liabilities, and w_k is the total assets (liabilities) of the kth company divided by the total assets (liabilities) of the industry. The decompositions into assets and liabilities and time-horizon are a straight forward extension of what has been developed earlier. (See Theil 1969, 475-79). In the next section, we present the industry-wide balance sheets of the oil companies for the years 1970 and 1971.

Information Balance Sheet of Oil Companies

We now present the information balance sheet for each of the seven marketing oil companies spanning the periods 1970 and 1971[6] (See Table 1). These two accounting periods were selected to determine the effects of the devaluation of February 1970 and the increase in the landed costs of crude oil in February 1971 on their balance sheets. The companies are letter coded from A to G.

We make the following observations on Table 1:

1. Company C has the highest balance sheet information measure, followed closely by company D which indicates that a number of substantial simultaneous changes affecting the balance sheet items were made by their management between 1970 and 1971. On the basis of the assets and liabilities information measures, it can be inferred that liabilities were less stable than assets in both

[6]One of the seven companies, Caltex (Philippines), Inc., has integrated refining and marketing operations.

Table 1. Information Balance Sheet, 1970-1971
Oil Companies
(In Bits)

	COMPANIES							Industry-Wide	
Assets and Liabilities Decomposition	A	B	C	D	E	F	G	1970	1971
Assets Information	.0019	.0041	.0455	.0447	*	.0080	.0048	.0829	.0804
Liabilities Information	.0273	.0075	.0812	.0612	.0102	.0007	.0020	.2784	.1982
Balance Sheet Information	.0145	.0058	.0633	.0529	.0051	.0043	.0034	.1807	.1393
Time-Horizon Decomposition									
Current Items Information	.0070	*	.0097	*	.0001	.0001	*	.0104	.0047
Medium-Term Items Information	.0470	.0134	.0032	1.0099	.0712	.0757	0	.2255	.1579
Fixed Items Information	.0002	*	.0042	*	.0015	.0006	.0007	.0166	.0150
Weighted Mean	.0053	.0005	.0062	.0290	.0041	.0017	.0003	.0329	.0208
Time-Horizon Information	.0092	.0053	.0571	.0239	.0010	.0026	.0031	.1478	.1185
Balance Sheet Information	.0145	.0058	.0633	.0529	.0051	.0043	.0034	.1807	.1393

Note: *denotes information measure less than .0001 bit.

companies C and D, as well as companies A and B. The reverse is true for companies E, F and G whose balance sheet information measures are relatively small. Perhaps this indicates the extent of the adjustments that had to be made as a result of the repayment of foreign loans and the higher peso cost of crude oil that were affected by the devaluation of the peso.

2. On the intercompany comparison, we find that the 1971 industry-wide balance sheet measure is smaller than the 1970 measure. This is true also for the assets information measures and liabilities information measures. We thus infer that the assets and liabilities structure of these companies became more similar over the two-year period.

3. The current items information measures and fixed items information measures indicate relative stability between current assets and liabilities as well as fixed assets and fixed liabilities over the period 1970 to 1971. However, the medium-term items information measures are higher than either of these two measures except for company G which does not have any medium-term liabilities. This means that medium-term assets such as Investment, Advances, to Affiliates, Long-term Receivables, Special Deposits Funds and Deferred Charges and medium-term liabilities such as Payables to Affiliates, Long-term Debts, Deferred Credits, and Reserves for Retirement Benefits changed substantially relative to each other between 1970 and 1971. This is particularly true for company D. It should be noted that these liabilities items are the ones that are readily affected by foreign exchange adjustments and changes in crude oil prices. It also reflects the fact that the oil industry taps foreign sources for funding its debts to a large extent.

4. In terms of the time-horizon information measures, we find that changes in the relative positions of all current medium-term and fixed items between 1970 and 1971 must have been larger for companies C and D than for the others.

5. The decomposition of the industry-wide balance sheet information into time-horizon components shows that the 1971 measures are all lower than the 1970 measure. This shows that the companies in the oil industry have become more similar in the time structure of their assets and liabilities.

In summary, we find that information measures are helpful in pinpointing the relative changes in balance sheet items over time and describing the relative effects on different companies. For the oil companies, we find that the main effects of foreign exchange and crude oil price adjustments were confined to the medium-term liabilities items. Specifically, companies C and D were more affected than the rest of the industry. As a result of these changes, the relative positions of these companies in relation to each other have become more similar between 1970 and 1971.

As a final note, we plan to extend the study to the years 1973 and 1974 so as to quantify the effects of the oil crisis on the balance sheets of oil companies.

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