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A THEORY OF PLANT LOCATION

By

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Introduction

The absence of a satisfactory theory of optimal plant location owes much to the misunderstanding of the meaning of the principle of profit maximization as it applies to plant location decisions. For in location theory, profit maximization is confused with the behavior of firms to seek the site which offers the greatest *positive* spread between revenues and costs among all possible locations.[1] This, of course, does not make sense both mathematically and economically for it implies that firms are considering "absolutes" and not "relatives", and are therefore unduly concerned with revenue and cost *levels* in their search for location.

Indeed the theory of plant location has developed along two contending rather than complementary lines in regard to this, one emphasizing the search for the least-cost site by abstracting from demand, the other emphasizing demand, by abstracting from cost.[2] Consequently, and what is perhaps more central to the problem is that, if profit maximization is interpreted strictly in the above sense in the theory of optimal plant location, the generally observed phe-

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[1] See for example Harry W. Richardson, *Regional Economics.* (New York: Praeger Publishers, 1969), Ch. 4, esp. pp. 90-100; also D.M. Smith, "A Theoretical Framework for Geographical Studies of Industrial Location," *Economic Geography*, 42 (April 1966), pp. 95-113; Leon N. Moses, "Location and the Theory of Production," *Quarterly Journal of Economics*, 72 (1959), pp. 259-272; Arthur Smithies, "Optimum Location in Spatial Competition," *Journal of Political Economy*, 49 (June 1941), pp. 423-439; Melvin L. Greenhut, "Integrating the Leading Theories of Plant Location," *Southern Economic Journal*, 18 (April 1952), pp. 526-538. In this article, Greenhut concludes that the theories of the least-cost location and the interdependent location are, despite their differences, quite similar; both emphasize the search for the site which offers the greatest spread between total costs and total revenues."

[2] Melvin L. Greenhut, *op. cit.*, pp. 526-527.

nomenon of firms locating at "less than maximum profit locatio
or alternatively, the absence of observed clustering of *all* firms (in
same industry) at the one location that offers the greatest posi
spread between revenue and cost relative to all other possi
locations, cannot be explained.[3]

As a result of this failure to emerge with a satisfactory theory
optimal plant site, profit maximization as a rational behavior of
firm has been questioned and increasing attention has been given
so-called "non-economic" considerations of firms in their choice
location.[4] This of course, suggests a trade-off between profits
"non-economic" considerations — whatever these may contain.

Properly understood, profit maximization simply refers to
behavior of firms to *equalize* the slopes of the revenue and c
functions by finding the combination of factors that minimize c
and the scale of output that equalizes the marginal cost and the p
of the product.[5]

The objective of this study is to sketch a theory of plant locat
which will show that if profit maximization is taken to mean sim
the behavior of firms to equalize marginal revenue (or price i
perfectly competitive market) and marginal cost, then it is possi
to generate a simple but general explanation of observed pl
locations. In addition, the result contributes to the analysis
locational interdependence factors influencing spatial concentrat
(dispersion) of firms, and perhaps more significantly, a satisfacto
theory of general equilibrium of location and Pareto optimality
production (and possibly consumption) over space can also
worked out. Lastly, one result of the approach which may be wo
noting is the integration of location theory with the orthodox dim
sionless micro-theory of the firm by showing that "extra-econom
factors in the choice of site are not inconsistent with notions

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[3] See references in footnote 1.

[4] Harry W. Richardson, *op. cit.*, pp. 90-100; Charles Tiebout, "Locat
Theory, Empirical Evidence, and Economic Evolution," *Papers and Proceedi
of the Regional Science Association*. 3 (1957), p. 81; Melvin L. Greenhut, *Pl
Location in Theory and Practice* (Chapel Hill: University of North Caro
Press, 1956), pp. 175-176; Melvin L. Greenhut, "Observation of Motives to Pl
Location," *Southern Economic Journal*, 18 (October 1951) pp. 225-228.

[5] Oskar Lange, "On the Economic Theory of Socialism," *Review of Econo
Studies*, IV, Nos. 1 and 2 (Oct. 1936 and Feb. 1937).

rational behavior in the context of the orthodox theory of the firm, hence doubts concerning the profit motive of firms in the choice of location are dispelled. This is an important result since it allows retention of conventional resource allocation efficiency criteria.

The Problem

In this study we envision a firm whose choice-of-location problem involves essentially two stages: first, consideration of the profitability of all possible sites; second, actual choice of site is made.

To begin with, let there be *at a point in time* a finite number of possible locations which a firm is initially faced with. (Mathematically, there is an infinite number of points over economic space, but here we consider only *economically feasible* points.) If profit maximization simply means that whoever makes the decision (we refer to him here as the entrepreneur or the firm) is "equalizing the slopes of the revenue and cost functions", then at each and every point in economic space, there is for a firm a $\hat{\Pi}(\hat{X}) \gtreqless 0$. Or alternatively, every point in economic space can be represented as $\hat{\Pi}(\hat{X})$ where $\hat{X}$ is a vector of inputs that maximize profits. Needless to say, this means that at every point in economic space there is for a firm a revenue function and a cost function. This $\hat{\Pi}(\hat{X})$ is the "maximized" profit, that is, the result of "solving" for the maximum of the profit equation, Π = Total Revenue − Total Cost. A firm confronted with the problem of choosing a site among alternative locations "solves" one profit equation for each of the n locations that it is considering. Thus all in all a firm solves n profit equations and comes up with $\hat{\Pi}$s. For the ith location, for example, a prospective firm faced with the problem of choice of plant site first "solves" the profit equation and obtains $\hat{\Pi}_i(\hat{X}_i)$:

$$\Pi_i = P_iQ_i - C_i - T_i \text{ , } (i = 1, 2, 3, \ldots, n)$$

$$= (P_i^F + t_id_i)Q_i(X_i) - r_iX_i - t_id_iQ_i(X_i) \qquad (1)$$

where:

PQ = total revenue

rX = total (input) cost C

$P^F + td$ = delivered price P

P^F = factory price

$Q = Q(X)$ = production function

X = vector of inputs

r = vector of input prices

T = total transport cost = $tdQ(X)$

t = transport rate

d = distance

Some remarks on (1) are appropriate at this juncture. The m initial concern of a firm confronted with a location problem is to what the $\hat{\Pi}$ is at each of the n possible locations. This means t although the firm pays attention to the various components of revenue and cost functions to the extent that they affect profit, th are not, however, its main concern. Perhaps the firm becomes m concerned over the behavior of revenue and cost functions as su only *after* decision on the site has been made and the plant set u since at this stage of the life of the firm survival becomes the ov riding objective. At the outset, however, the profit equation simp *summarizes* the effect of the components and determinants revenue and cost, and the profit that the firm obtains after "maximizes" the equation is what it considers in its choice of site.

The function of delivered price P_i in (1) is, together with t delivered prices of other firms, to determine the market area. The is no apparent reason for transport cost to receive special attention location theory any more than input costs. The significant role transport cost in location theory is not as the principal object minimization but as *a* determinant of the market area. It is in th perspective that the role of transport cost is viewed here. Wi respect to the input terms it matters less for a firm where and ho they can be made available, and as regards transport costs of inpu these are assumed to be reflected by input prices. All this mea allowing spatial cost variations.

Having solved for each of the n $\hat{\Pi}$s, the firm is now faced with th following set of all nonnegative $\hat{\Pi}$s arranged in order of magnitud (negative $\hat{\Pi}$s are out of consideration) where $\hat{\Pi}_i$ = maximum prof obtaining at the ith location ($i = 1, 2, \ldots, n$);

$$0 \leq \hat{\Pi}_1 \leq \hat{\Pi}_2 \leq \hat{\Pi}_3 \leq \ldots \leq \hat{\Pi}_n \qquad (2)$$

With m firms, we have m sets of (2). Because of locational interdependence factors and allowing for differences in the way firms would combine the inputs, the firms do not necessarily obtain identical profits at the *same* location, that is, at the ith location for example,

$$\hat{\Pi}_i^1 \gtreqless \hat{\Pi}_i^2 \gtreqless \hat{\Pi}_i^3 \gtreqless \ldots \gtreqless \hat{\Pi}_i^m$$

Since the study is concerned with how *a* firm — not all firms are taken together — solves its location decision problem, we focus attention on just one of the m sets of maximized profits, the set of $\hat{\Pi}$s shown in (2). It is conceivable that some of the equalities in (2) may hold. Realistically, however, the chances are very slim that even at any two different sites of similar physical features exactly the same maximized profits occur because of the imperfect character of spatial markets and differences in locational interdependence factors among the possible locations. Consequently, to have a mapping of the profits on the real line, and for uniqueness of solution to the location decision problem, we assume that

$$0 \leq \hat{\Pi}_1 < \hat{\Pi}_2 < \hat{\Pi}_3 < \ldots < \hat{\Pi}_n \qquad (3)$$

The consequent question then is: which of these $\hat{\Pi}$s and hence location will the firm choose? Two approaches to this question will be explored. One approach explicitly takes into account so-called non-economic factors in the choice of location by positing a trade-off between these factors and the maximized profits in (3) and on this basis considers an objective function of the firm; the other approach envisions the firm as making a decision under conditions of uncertainty into which is lumped all non-economic and locational interdependence factors, and posits a preference ordering over the probability distributions in economic space according to their respective certainty equivalents.

Plant Location Under Certainty

Consider the first approach. As pointed out elsewhere, existing location theories are unable to settle the observed location of firms at "less than *maximum* profit" sites where "maximum profit" refers to the greatest positive difference between revenues and costs amo the various possible locations. For this reason, recent studies advan such motives as "personal preferences and constraints not clos related to any calculus of money cost, revenue, or profit[6]", "limited objectives, or psychic income[7]" as considerations vis-a-v profits in location decisions. In a study by Tiebout particularly, th extra-economic factors in the firm's location decision were giv empirical justification.[8] It is evident that all these point to the exi ence of a trade-off between profits and extra-economic facto (whatever these are) in location decision. With this, and with the a of the axioms in consumer theory which build up the util function[9], we have a justification for assuming an analogo objective function for the firm confronted with the problem choice of location, which we will call the firm's welfare function.

Why has not the orthodox dimensionless theory of the fi assigned a welfare function to the firm? The answer is precis because the firm, in a spaceless setting, is not faced with a cho among profit alternatives and extra-economic considerations so t no trade-off exists. In the theory of the firm in which the spat dimension is absent the firm has but a singular objective, namely, maximize profit by which is meant both the behavior of equalizi slopes and the search for the greatest positive gap between reven and cost, hence no conflict between these notions arises. Here l the synthesis of the orthodox micro-theory of the firm and locati theory. The firm's welfare function is relevant only insofar as it deciding where to locate. Once a decision is made and the pla *actually* set up at the chosen site, the welfare function may drop o

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[6] Edgar M. Hoover, *An Introduction to Regional Economics*, First ed., (N York: Alfred A. Knopf, 1971) pp. 60-62.

[7] Harry W. Richardson, *op. cit.*, pp. 90-100.

[8] Charles Tiebout, *op. cit.*, pp. 74-86.

[9] For which, see Peter Newman, *The Theory of Exchange* (New Jers Prentice-Hall, Inc., 1965). Ch. 2.

[10] If this does not suffice, we can call forth Arrow's possibility theorem si the board of directors in the case of a corporation; the partners in the case o partnership; the individual in the case of a single-proprietorship, who make t decisions, play the role of "dictators" and the organizational structures of th types of firms can satisfy or be made to satisfy Arrow's axioms [for which s James M. Henderson and Richard E. Quandt, *Micro-economic Theory*. Seco ed. (New York: McGraw-Hill Book Co., 1971), pp. 284-286].

of importance and the firm can be viewed simply as having that singular objective of maximizing profit in the sense of both equalizing slopes and attaining the greatest spread between revenue and cost at the chosen site where it *must* operate for the duration of its life.

Now, let μ represent all locational considerations of a firm other than profit. Then on the basis of the existence of a trade-off between $\hat{\Pi}$ and μ, we write the firm's welfare function as

$$W_F = f(\hat{\Pi}, \mu) \tag{4}$$

where:

$$\hat{\Pi} \geqq 0, \mu \geqq 0,$$

$$\frac{\partial W_F}{\partial \hat{\Pi}} > 0, \text{ and } \frac{\partial W_F}{\partial \mu} > 0$$

The constraint is taken to be of the linear form:

$$Z = \alpha\hat{\Pi} + \beta\mu \tag{5}$$

We interprete Z as the maximum monetary value of total income consisting of money income from profits $\alpha\Pi$ and the monetary value of "psychic" income $\beta\mu$. Defined in this manner, the monetary value of total income Z is equal to the greatest profit that a firm can obtain among the various sites, that is, the $\hat{\Pi}$-intercept of (5) is $\hat{\Pi}_n$.

α may be interpreted as the average of the weights which firms give to profit. In other words α is taken as the average of the indices of "attitudes" — which vary from 0 to 1 — of firms toward profit. At most $\alpha = 1$ when firms take profit simply for what it is. The existence of extra-economic factors, however, suggests that $0 \leqq \alpha \leqq 1$, disregarding a negative attitude towards profit. Since extra-economic factors are already explicitly treated separately, we get rid of α in the constraint by simply taking profit as the consideration, thus making $\alpha = 1$.

The existence of μ brings up the idea of foregone profit (locational opportunity cost). Thus β may be interpreted as some average profit differential among locations in the industry. Hence what we have referred to as the "monetary value of psychic income" $\beta\mu$ is the profit foregone by a firm if $\mu > 0$. By interpreting β in this manner we have in effect assumed that β is *common* to all firms. In additi having linked β with profits obtained among the possible sites, to extent that locational interdependence factors giving rise to agglo ration (deglomeration) and the economies (diseconomies) associa with it would affect the firms' production functions and hence co β will be affected through the $\hat{\Pi}$s. Thus β may change due to lo tional interdependence factors. This will be dealt with later.

From (4) and (5) the choice-of-location problem of a firm simply the maximization problem:

$$\text{maximize } W_F = f(\hat{\Pi}, \mu)$$

$$\text{subject to } Z = \alpha\hat{\Pi} + \beta\mu$$

Since the solution to this problem gives a $\hat{\hat{\Pi}} = \hat{\Pi}_i$, the fi location is also known; that is, in solving (6) for $\hat{\Pi}$ we also solve the firm's location. Graphically the solution to (6) is shown by tangency of the constraint AC to the highest attainable W_F — c which is W_F°. This tangency occurs at point B in Fig. 1 where we h

$$\frac{\frac{\partial f}{\partial \mu}}{\frac{\partial f}{\partial \hat{\Pi}}} = \beta$$

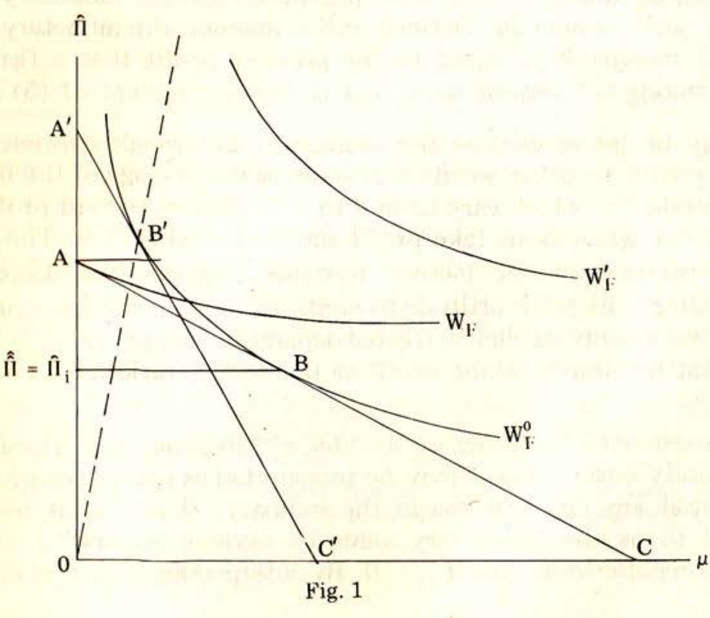

Fig. 1

As shown by the graphical solution, it does not necessarily mean that the firm locating at the ith location is not maximizing profit since $\hat{\Pi} = \hat{\Pi}_i$, the maximum profit obtaining at the ith location. Since $\hat{\Pi}_i < \hat{\Pi}_n$ = OA, we have explained theoretically the phenomenon of firms locating at "less than maximum profit locations" where "maximum profit" refers to the widest gap between revenues and costs among the sites. Moreover, as Fig. 1 shows, the behavior of firms to search for the location offering the highest positive profit among the various alternative sites is not entirely ruled out but may be taken as just a *special case* in that these firms possess iso-welfare curves of the shape of W_F in which case a so-called corner solution occurs at point A where $\hat{\Pi}_n$ obtains. In general the non-economic considerations of firms in their location decision problem are *not inconsistent* with the notion of economic rationality. Lastly, if $\hat{\mu} = \bar{\mu}$ is a constant (after the site has been chosen and the plant set up), then

$$(\hat{\Pi}_i^k, \bar{\mu}) > \ldots > (\hat{\Pi}_i^2, \bar{\mu}) > (\hat{\Pi}_i^1, \bar{\mu}) > (\hat{\Pi}_i, \bar{\mu}) \qquad (7)$$

if and only if $\hat{\Pi}_i < \hat{\Pi}_i^1 < \hat{\Pi}_i^2 < \ldots < \hat{\Pi}_i^k$, in which case the firm can then be viewed as having that singular objective of maximizing profit now taken not only as equalizing slopes but as the greatest *positive* spread between revenue and cost.

As mentioned earlier, the profit equation offers a convenient summary of *all* the combined effects of revenue and cost. The location decision of firms will be affected only to the extent that revenue and cost factors influence profit. Therefore, locational interdependence factors that influence location pattern — that is, whether firms will tend to agglomerate or disperse over space — come into focus only as they affect profit through revenue and/or cost. Schematically, the relationship between locational interdependence factors and location pattern is shown below:

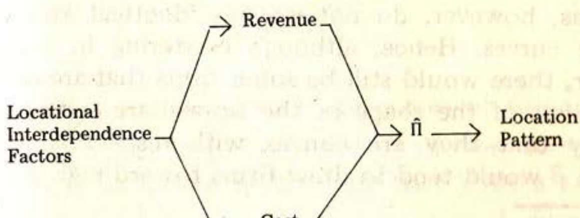

Fig. 2

We will not go into a detailed discussion of the various locati interdependence factors (demand elasticities, relationship betw freight rate and selling price, shapes of the cost curves, spatial variations, etc.[11]). Rather, on the basis of the schematic relation shown in Fig. 2, we will investigate the influence of locational in dependence factors on location pattern through the $\hat{\Pi}$s.

In Fig. 1, we see that the $\hat{\Pi}$, and hence the location that is cho by the firm together with μ, is determined largely by two thi namely, the shape of the firm's iso-welfare curves and the rela weights of $\hat{\Pi}$ and μ, $\frac{\beta}{\alpha}$. From its definition and since the $\hat{\Pi}$s non-negative, $\beta \geqq 0$. β varies positively as the profit differen among the locations. Since $\alpha = 1$, the slope of the constraint depend mainly on β. Having interpreted β in such a way as to lin with the $\hat{\Pi}$s, agglomeration (dispersion) tendencies can be inv gated by simply looking into the behavior of β together, of cou with the firm's iso-welfare curves.

Rational behavior would imply that firms in general put relati more *weight* to profits than to non-economic factors. This, of cou does not necessarily mean that $\mu = 0$. It simply means that altho philanthropic firms do exist, they are not however a general stat affair. We state this technically in the assumption that the wel levels of firms are "profit-intensive." With this, we proceed to l into the effects of changes in β.

If firms generally have identical and homogeneous iso-wel curves of the shape of (say) W_F°, agglomeration for any $\beta > O$ wo tend to take place at such location as the ith where $\hat{\Pi}_i$ obtains (1). Allowing β to vary within a certain limit, if firms generally po identical iso-welfare curves of the shape of W_F, that is, the slop the iso-welfare curve is *everywhere* less than β, agglomeration t place at the location offering the greatest profit $\hat{\Pi}_n$.

All firms, however, do not possess identical and homogen iso-welfare curves. Hence, although clustering in certain locat does occur, there would still be some firms that are fairly dispe But regardless of the shape of the iso-welfare curves of firms, vided only that they are convex with respect to the origin increase in β would tend to draw firms toward high profit locati

[11] Melvin L. Greenhut, *Plant Location in Theory and Practice*. (Chapel University of North Carolina Press, 1956), esp. Chs. II and VI.

In general, as the *price* β of μ increases, firms would tend to agglomerate at relatively high profit locations. On the other hand, as β decreases firms would tend to disperse. The limiting cases are an infinitely large β so that the constraint approaches the $\hat{\Pi}$-axis, in which case only corner solutions would occur, that is, firms agglomerate at the nth location; $\beta = 0$ so that the constraint becomes a horizontal line, in which case $0 \leqq \hat{\Pi}_1 = \hat{\Pi}_2 = \hat{\Pi}_3 = \ldots = \hat{\Pi}_n$ location is indeterminate. This is interpreted as a tendency toward dispersion over space, all locations being *equally* profitable. These results are shown graphically in Fig. 3.

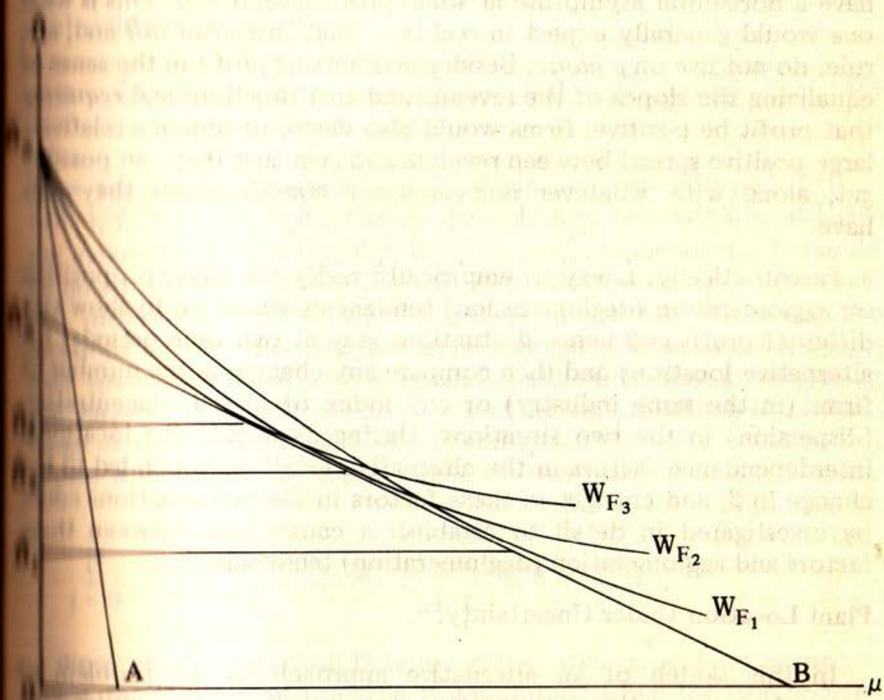

Fig. 3

W_{F_1}, W_{F_2}, W_{F_3} are the iso-welfare curves of firms 1, 2, and 3 respectively. With a very large β depicted by the very steep constraint $\hat{\Pi}_n A$, all three firms locate at the nth site. With a very small β depicted by line $\hat{\Pi}'_n B$, firm 1 chooses location 2, firm 3 chooses location 3.

That an increase in β would lead to a tendency toward agglome tion at relatively high profit locations while a decrease in β leads dispersion (indicating asymmetric response to changes in β) hinges the assumption that the shape of the iso-welfare curves of firms ha the common feature of being biased in favor of profit and agai extra-economic factors, that is, firms' welfare levels are "prof intensive". In other words, while it can happen that corner soluti could occur on the $\hat{\Pi}$-axis, that is the slope of the iso-welfare curv of firms can be everywhere smaller than β, the opposite can n occur. Corner solutions can not occur on the μ-axis, that is, the slo of the iso-welfare curves of firms can not be everywhere greater th β. One may state this technically by saying that the iso-welfare curv have a horizontal asymptote at some profit level $\hat{\Pi} > 0$. This is wh one would generally expect in reality — that firms *can not* and, a rule, do not live on μ *alone*. Besides maximizing profit in the sense equalizing the slopes of the revenue and cost functions and *requiri* that profit be positive, firms would also desire to obtain a relativ large positive spread between revenue and cost that they can possib get, along with whatever non-economic considerations they m have.

Parenthetically, a way to empirically verify the above propositi on agglomeration (deglomeration) tendencies would be to show t different profit and hence β-situations (say at two time periods) f alternative locations and then compare any changes in the number firms (in the same industry) or any index of spatial concentrati (dispersion) in the two situations. Having done this, the locatio interdependence factors in the alternative locations which led to t change in β, and changes in these factors in the two situations cou be investigated in detail to establish a causal link between th factors and agglomeration (deglomeration) tendencies.

Plant Location Under Uncertainty[12]

In this sketch of an alternative approach to the problem optimal plant location, the location decision of the firm under co dition of uncertainty consists in selecting a probability distributi from a given set of such distributions. Rational behavior then me selecting the *best* of the available distributions. This means t location decision under uncertainty must be based on a prefere

[12] This section makes much use of the method elaborated by Karl He Borch in his *The Economics of Uncertainty*. (Princeton, New Jersey: Prince University Press, 1968), esp. Ch. III.

ordering over the probability distributions in a set of such distributions. We attempt to construct such preference ordering for a firm confronted with the problem of choice of location among the various possible sites, based on the Bernoulli principle.

For simplicity, we consider only discrete distributions. As in the first approach, the prospective firm is confronted with n economically feasible locations, each of which offers as gain the stochastic variable $\hat{\Pi}_j \geq 0$ (negative $\hat{\Pi}$s are out of consideration), the *maximum* profit, i.e. in the sense of equating the slopes of the revenue and cost functions, that could occur at the ith location, with probability distributions $f_i(\hat{\Pi}_j)$, $i = 1, 2, \ldots, n; j = 0, 1, 2, \ldots, m$. In symbols, the firm is confronted with a set D the elements of which are the n probability distributions, i.e.,

$$D = \{f_1(\hat{\Pi}_j), f_2(\hat{\Pi}_j), \ldots, f_n(\hat{\Pi}_j)\}, \quad j = 0, 1, 2, \ldots, m \tag{8}$$

Thus for the ith location, for example, we can interpret $f_i(\hat{\Pi}_o)$, $f_i(\hat{\Pi}_1)$, $f_i(\hat{\Pi}_2)$, ... $f_i(\hat{\Pi}_m)$ as the probabilities that the site will give the *maximum* profits $\hat{\Pi}_0, \hat{\Pi}_1, \hat{\Pi}_2, \ldots, \hat{\Pi}_m$, respectively. It should be noted that as a result of the imperfect character of spatial markets and differences in locational interdependence factors among the locations all of which would imply demand and/or cost variations over space, the range of the $\hat{\Pi}$s that could occur may differ among the various sites. This is taken care of by simply taking the largest range of $\hat{\Pi}_j$ and assigning zero probabilities to those $\hat{\Pi}$s which do not appear where the range is small. It is obvious that

$$\sum_{j=0}^{M} f_i(\hat{\Pi}_j) = 1 \text{ for all } i = 1, 2, 3, \ldots, n$$

Since each element of D is associated with a point in economic space, the location of the firm is determined when the firm chooses the *best* distribution in C. Hence to solve the firm's location problem, we seek a preference ordering over the n elements of the set D. Assuming that this ordering can be represented by a utility function, our objective then is to associate a real number U(f) with each of the n elements (the probability distributions $f_i(\hat{\Pi}_j)$, hereafter called "prospects") of the set D such that

$$U\{f_i(\hat{\Pi}_j)\} > U\{f_k(\hat{\Pi}_j)\}$$

if and only if $f_i(\hat{\Pi}_j) > f_k(\hat{\Pi})$. Mathematically, the problem is t find a mapping from the space of all discrete probability distribu tions or the prospects in D to the real line. To do this, we employ th axioms laid down by Borch.[13]

Axiom 1. To any probability distribution $f_i(\hat{\Pi}_j)$ in the set D, there corresponds a certainty equivalent $\overline{X}_i$.

In symbols, Axiom 1 says $(1, \overline{X}_i) \sim f_i(\hat{\Pi}_j)$ ("$\sim$" denotes equ valence relation.)

Set D includes all binary type distributions in which the only two possible outcomes are

$\hat{\Pi}_M$ with probability p

0 with probability (1 - p)

If $(p, \hat{\Pi}_M)$ stands for such binary distribution, we have from Axiom 1 that for any p, there is an X_p so that

$$(1, X_p) \quad (p, \hat{\Pi}_M)$$

Axiom 2. As p increases from 0 to 1, X_p will increas from 0 to $\hat{\Pi}_M$.

Axiom 3. $f_i(\hat{\Pi}_j)$ and $f_i^*(\hat{\Pi}_j)$, the equivalent binary distribution o $f_i(\hat{\Pi}_j)$, have the same certainty equivalent.

With Axiom 1 we determine the certainty equivalent of ea of the $\hat{\Pi}$s with their respective probabilities $f_i(\hat{\Pi}_o)$, $f_i(\hat{\Pi}_1)$, . . . $f_i(\hat{\Pi}_m)$. With Axiom 2 we form the equivalent binary type distrib tion of the original prospects. Axiom 3 together with Axiom 2 allow replacement of $\hat{\Pi}_j$, j = 0, 1, 2, . . . , m = M, except $\hat{\Pi}_o = 0$ an $\hat{\Pi}_m = \hat{\Pi}_M$, in the ith location with the equivalent binary form (p $\hat{\Pi}_M$) to give a modified prospect

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[13] Ibid., pp. 25-26

$$f_i^*(\hat{\Pi}_0 = 0) = f_i(0) + f_i(\hat{\Pi}_1)(1 - p_1) + f_i(\hat{\Pi}_2)(1 - p_2) + \ldots + f_i(\hat{\Pi}_M)(1 - p_M)$$

$$f_i^*(\hat{\Pi}_1) = 0 \tag{9}$$

$$f_i^*(\hat{\Pi}_2) = 0$$

$$\ldots\ldots\ldots$$

$$f_i^*(\hat{\Pi}_M) = p_1 f_i(\hat{\Pi}_1) + p_2 f_i(\hat{\Pi}_2) + \ldots + p_M f_i(\hat{\Pi}_M)$$

In this manner we obtain for the ith location a prospect of the type $(P_i, \hat{\Pi}_M)$ which has the same certainty equivalent as the original prospect $f_i(\hat{\Pi}_j)$. For the ith location, P_i is determined by the last equation in (10), i.e.:

$$P_i = \sum_{j=0}^{M} P_j f_i(\hat{\Pi}_j) \tag{10}$$

Applying this method to each of the prospects $f_i(\hat{\Pi}_j)$, $i = 1, 2, \ldots, n$, in the set D, we obtain a complete preference ordering over the elements of D and hence of the corresponding locations. Thus, for two arbitrary distributions in D, $f_i(\hat{\Pi}_j)$ and $f_k(\hat{\Pi}_j)$, we can determine the corresponding binary prospects $(P_i, \hat{\Pi}_M)$ and $(P_k, \hat{\Pi}_M)$ and their certainty equivalents. The ordering is then that $f_i(\hat{\Pi}_j)$ is preferred to $f_k(\hat{\Pi}_j)$ if and only if $P_i > P_k$ (or equivalently if $f_i(\hat{\Pi}_j)$ has the greater certainty equivalent). And since P_i is associated with the ith point in economic space, the firm chooses this location.

To represent this preference ordering by a utility function, we define

$$U\{f_i(\hat{\Pi}_j)\} = P_i = \sum_{j=0}^{M} P_j f_i(\hat{\Pi}_j), (i = 1, 2, 3, \ldots, n) \tag{11}$$

As a final point in this discussion, we note that since the certainty equivalent of a prospect varies positively as its probability and independently with $\hat{\Pi}_j$ the distribution and therefore the location that is chosen does not necessarily have to offer the greatest positive spread between revenues and costs among all the possible locations.

In this approach to plant location determination, there two sources of support for the firm's rational behavior in choice of site: the first arises from the fact that the *best* terms of the certainty equivalent) of the available distributi is chosen; the second is that the distribution and hence location that is chosen has a $\hat{\Pi}$ which is the result equalizing the slopes of the revenue and cost functions, i.e. the profit maximizing behavior of the firm.

Towards a General Equilibrium of Location and Pareto Optimality Production Over Space

The objective of this section is simply to sketch an appro to general equilibrium of location and Pareto optimality production over space based on the first of the two alternat approaches to plant location problem that have been present Consider two firms A and B at contiguous locations i and respectively. The equality of the firms' delivered pri determines the market areas, i.e.

$$P_i = P_j$$

$$P_i^A + t_i d_i = P_j^B + t_j d_j$$

$$d_j - \frac{t_i}{t_j} d_i = \frac{P_i^A - P_i^B}{t_j}$$

defines the boundary between areas tributary to two geographic competing markets for homogeneous goods, where P_i^A and P_j^B the firms' factory prices; t_i and t_j are transport rates which are giv and d_i and d_j are distances. The determination of market areas and M_B is illustrated graphically in Fig. 4.[14]

[14] For a detailed discussion of the law of market areas, see Frank Fetter, "The Economic Law of Market Areas," *Quarterly Journal Economics*, Vol. 29 (May 1924), pp. 520-529; C. D. Hyson and W. Hyson, "The Economic Law of Market Areas," *Quarterly Journal Economics*, Vol. 64 (1950), pp. 319-324; Melvin L. Greenhut, "The and Shape of the Market Area of a Firm," *Southern Economic Jour* (July 1952), pp. 37-50.

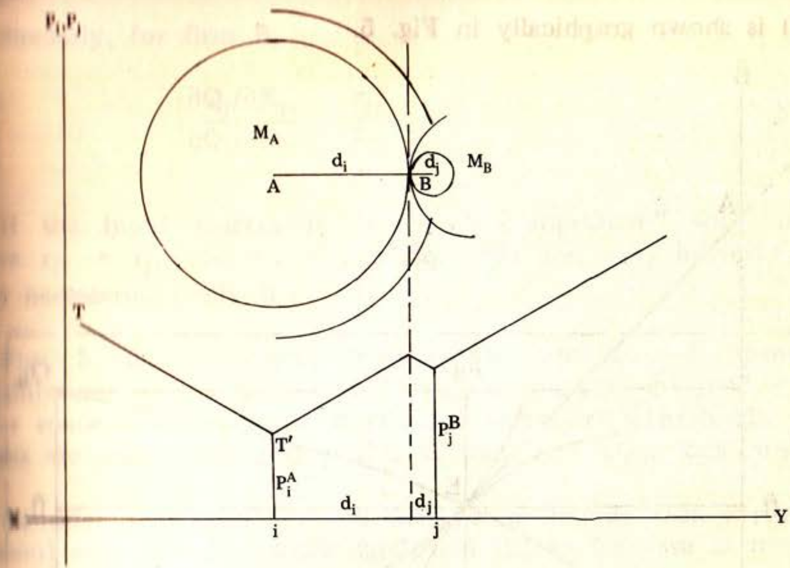

Fig. 4

The slope of TT′ is equal to t_i.

With these delivered prices, profits are maximized at the respective locations. The firms' having chosen locations i and j means that

$$\text{For firm A: } \frac{\partial f_{A/\partial\mu_A}}{\partial f_{A/\partial\hat{\Pi}_i}} = -\frac{d\hat{\Pi}}{d\mu_A} = -\frac{\beta}{\alpha} \tag{13}$$

$$\text{For firm B: } \frac{\partial f_{B/\partial\mu_B}}{\partial f_{B/\partial\hat{\Pi}_j}} = -\frac{d\hat{\Pi}}{d\mu_B} = -\frac{\beta}{\alpha} \tag{14}$$

And since β is *common to all firms* (in the same industry), and $\alpha = 1$, and taking the firms as sharing the total profits for both locations, that is, $\hat{\Pi}_T = \hat{\Pi}_i + \hat{\Pi}_j$, we have

$$\frac{\partial f_{A/\partial\mu_A}}{\partial f_{A/\partial\hat{\Pi}_i}} = \frac{\partial f_{B/\partial\mu_B}}{f_{B/\partial\hat{\Pi}_j}} = \frac{\beta}{\alpha} \tag{15}$$

(15) is shown graphically in Fig. 5.

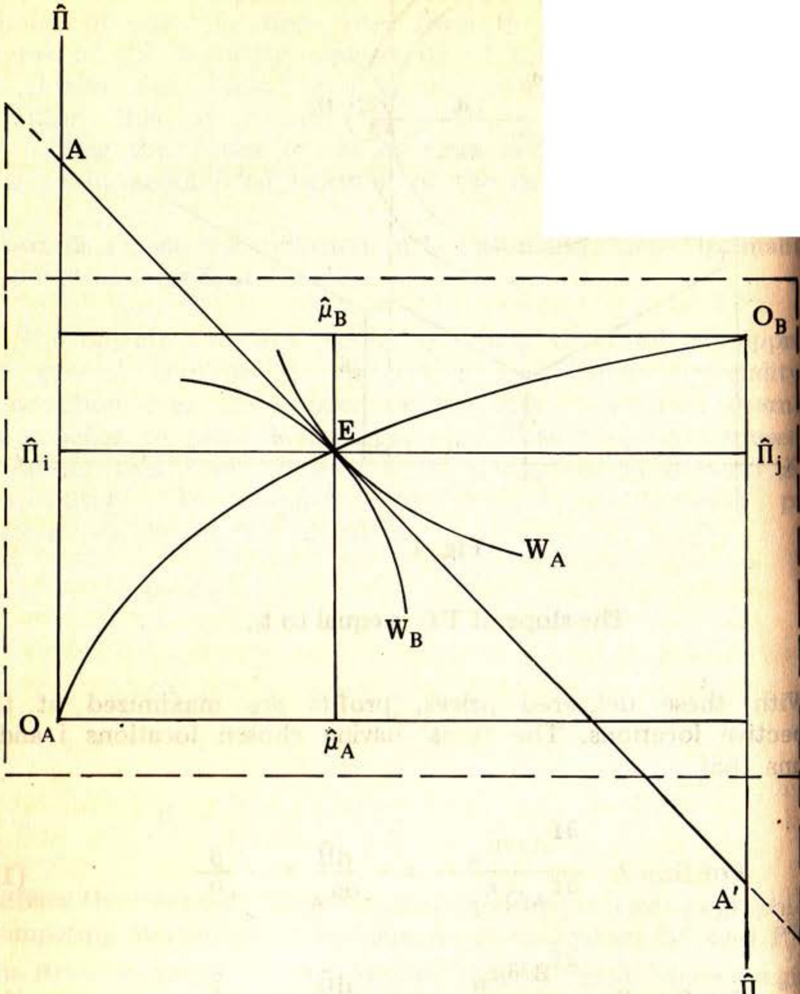

Fig. 5

Since firm A is maximizing profit at the ith location, we have

$$\frac{\partial Q_i / \partial X_{i1}}{\partial Q_i / \partial X_{i2}} = \frac{r_{i1}}{r_{i2}} \quad (1$$

where r_{i1} and r_{i2} are the prices of inputs X_{i1} and X respectively.

Similarly, for firm B,

$$\frac{\partial Q_j/\partial X_{j1}}{\partial Q_j/\partial X_{j2}} = \frac{r_{j1}}{r_{j2}} \tag{17}$$

If the input market is "perfectly competitive" we should have $r_{i1} = r_{j1}$ and $r_{i2} = r_{j2}$. Eq. (16) and (17) however do not necessarily imply this.

Figs. 3 and 4 together illustrate the conditions for general equilibrium of location and Pareto optimality in production over space since at point E (Fig. 5) where eq. (15) holds, the firms are also maximizing profits at their respective locations.

Consider two cases: (1) an increase in profits (for whatever reason) such that β is unchanged, and (2) an increase in profits that affects β. In case (1) we simply have a higher intercept on the $\hat{\Pi}$-axis and an enlargement of all sides of the box diagram (shown by broken lines, Fig. 4) so there is no incentive for the firms to change locations. Furthermore, provided the shape of the iso-welfare curves particularly of potential firms is not affected by this manner of increase in profits, location pattern will not also be affected. In case (2), the constraint AA′ (Fig. 5) becomes steeper than before. Here two things could occur: either the firms (existing and potential) move to high profit locations but total profits which the firms share increase, i.e. the width of the box diagram increases and we still have equality of the slopes, or, the firms remain in their respective locations which means that there has to be a change in the shape of their iso-welfare curves (analogous to the "change in taste" in consumer theory). The tendency toward agglomeration (deglomeration in the case where β decreases) is thus present in case (2) where there is a change in the slope of the constraint AA′.

Conclusion

The theory that has been presented takes into account extra-economic factors in the firm's choice-of-location problem without impairing the notion of rational behavior in the context of existing dimensionless theory of the firm. Moreover, the theory is free from highly unrealistic assumptions such as spatially homogeneous and uniformly distributed resources, homogeneous plain, uniform population densities, uniform transport costs, instantaneous costless relocation, etc.,[15] which have burdened existing location theories. Besides purporting to explain plant site determination and location pattern, the theory may also serve as an explanation of capital movement over space. Although locational interdependence factors have not been examined in detail, the influence of these in location pattern has not been ignored but is captured in the behavior of the constraint to which firms react. Thus, location pattern is systematically linked with foregone profit, that is, with "locational opportunity cost."

As a final point, it should be noted that theories of plant location would perhaps be applicable only to firms that cater essentially to localized markets, that is to say, the markets that are not as large as the national market — not to mention international market. Where the firm is large as in the case of steel or transport equipment firms in highly industrialized economies, and the market of which is the whole country and/or the world, demand can be taken as more or less given so that least-cost locations would be the optimal plant sites.[16] In this case, so-called extra-economic factors would have virtually no significance although conceivably, they may still be present in the firm's location decision problem (for example, providing employment to certain "depressed areas"). Clearly, in this instance, it makes no difference whether profit maximization is taken to mean the behavior of equalizing the slopes of the revenue and cost functions or the attainment of the widest positive gap between revenue and cost among locations.

[15] See for example August Losch, *The Economics of Location*, tr. by William H. Woglom with Wolfgang F. Stolper. (New Haven: Yale University Press, 1954); Harold Hotelling, "Stability in Competition," *Economic Journal*, 39 (1929), pp. 41-57; Walter Isard, *Location and Space-Economy*, (Cambridge, Mass.: The MIT Press, 1956).

[16] This is perhaps one reason for international capital movement.