The Philippine Review of Business and Economics Vol. XIII No. 2 December 1976

SECTORAL PER CAPITA GROWTH AND EQUITABLE INCOME DISTRIBUTION

by

Alberto R. Ilano*

Relatively high levels of income inequality have been common phenomena for developing countries. As a consequence, one would think that income redistribution would have been afforded top priority as a national economic goal. In reality, however, most of the efforts currently being made at economic development cannot be characterized as being directly aimed at changing such existing high levels of income inequality. It may even be said and understandably so that income redistribution has never been a popular national economic goal for the governments of developing nations saddled by a feudal past.

In a way, economic theory has contributed to this state of affairs because of the acceptance of the direct proportionality between the savings rate and the income level. This has resulted in relegating the problem of income inequality to the background. The alleged justification has been that eventually, across the board increases in income levels will "trickle down" to the low income groups. But generally, this has not occurred. Because of the propensity of governments to try to achieve the appearances of industrialization, the goal of redistributing income has not only been largely ignored, but what has occurred is a "trickling up" effect — a long run shift towards more inequitable income distributions.[1]

To be sure, the process of development is a highly constrained one. The nature as well as the severity of these constraints vary from one

*Associate Professor of Finance, University of the Philippines and Concordia University, Montreal.

[1] For a similar view, see J. Viner, "The Economics of Development," in A. N. Agarwala and S. P. Singh (eds.), *The Economics of Underdevelopment* (New York: Oxford University Press, 1963), p. 15.

developing country to another. But one problem the majority of developing countries share is population growth. Because a population control program necessarily involves very long-term effects, it usually does not enter materially in the realm of shorter run problems faced by governments. And while governments may come and go, the population problem remains.

This paper does not belong to the genre of population trap models. Instead, the objective here is to analyze the effects of growth rates by sectors of both population and income levels. Attention will be focused on the conditions that prevent the majority of people within a developing economy from not only sharing proportionately in the total growth but also from improving their material levels of subsistence. Hopefully, such an approach can lead to describe the conditions of the problem and to prescribe as well the directions towards mitigating their undesired effects.

Part I contains the background presentation of the measurement of economic performance. Here it is underscored that average increases in real per capita incomes can give a myopic view of the real situation. Part II presents as an alternative a sectoral approach to analyzing economic performance. At this stage, only sectoral income effects are considered and population growth is assumed to be uniform for all income levels. In Part III, the implications of this sectoral approach for economic planning are specified by way of rendering planned reductions of existing levels of inequality into operational terms. Varying degrees of reduction can be incorporated into an economic plan. The effects of sectoral population growth on the general analytical framework are discussed in Part IV. Finally, the policy considerations of this sectoral approach are analyzed in Part V, where it is stressed that the choice between economic growth and income equality need not be made on a mutually exclusive basis. The sectoral approach to per capita growth as propounded in this paper can hopefully serve to provide a workable balance between these two in arriving at more equitable distribution of income.

I. Background

Let us suppose that in a developing economy, population and income growth for a given period can be described by the following exponential paths:

(1.0) $$P_t = P_o e^{gt}$$

where P_t — population at time t,
P_o — current population,
g — geometric average growth rate; and

(2.0) $$Y_t = Y_o{}^{vt}$$

where Y_t — real income at time t,
Y_o — current real income,
v — geometric average growth rate.

By dividing Eq. (2.0) by Eq. (1.0), we can derive the equation for the growth rate of real per capita income over the same period —

(3.0) $$y_t = y_o e^{(v-g)t}$$

where $y_t = Y_t/P_t$, the real per capita income at t;
$y_o = Y_o/P_o$, the current real per capita income.

If $(v\text{-}g)<0$, then $y_t<y_o$. For y_t to be greater than y_o, the term (v-g) must be positive, i.e., real income must grow faster than population if the per capita income level of the economy is to improve. From the point of view of reducing income inequality, however, the fact that $(v\text{-}g)>0$ is not adequate for measuring economic performance. It becomes necessary to look deeper into the manner in which the growths in income and in population have been distributed among the sectors comprising the economy.

II. The Notion of Sectoral Per Capita Incomes

This section will deal with the formulation of a sectoral approach for measuring the effects of per capita growth. Fig. 1 shows a Lorenz curve depicting the relationship between percentiles of real income and of population.[2] If a line is projected from the origin to any point on the curve, then the slope of this line corresponds to the per capita income (expressed in percentiles ratio)[3] of the population percentile given by the abscissa of that point. It also follows therefore that the

[2] Throughout this paper, the analysis will abstract from effects of price level variations. Hence the term income shall refer exclusively to real income.

[3] This inconvenience in dealing with per capita income expressed in terms of percentiles ratio will be removed later on with the aid of Fig. 3. At the moment the objective is to base the sectoral approach formulation on the established concept of the Lorenz curve.

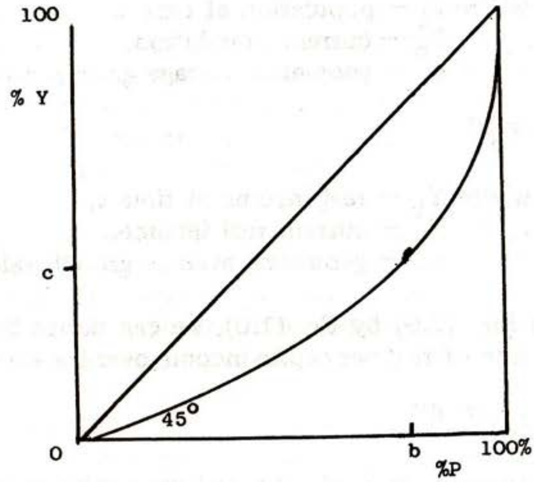

Figure 1
Lorenz curve broken down into two sectors.

45° slope corresponding to the overall per capita income if the percentile scales are reconverted to absolute values.

Going back to the Lorenz curve, the distribution of sectoral incomes per capita can be determined from its curvature. Whereas the slope of a line linking any point on the curve to the origin corresponds to the average per capita income, the slope at any point along the curve corresponds to the marginal per capita income arising from an incremental population percentile.[4] It follows then that at the point on the curve where the slope becomes parallel to the 45° line, the marginal per capita income equals the overall average. Let the horizontal component of this point be denoted by b and its vertical component by c. Therefore b% of the population have per capita incomes lower than the overall average; and c gives the percentage of total income received by this b% of the population. The component per capita income of this group would then amount to 100(c/b)% of the overall average per capita income. A two-sector analysis can hence be made by treating the b% as the low-income group and the remaining (100-b)% as the high-income group.

---

[4] This distinction should be familiar since it is analogous to that between the average propensity to consume and the marginal propensity to consume in reference to the consumption function. Since the curve is continuous, a unit population percentile can likewise be treated as a continuous interval.

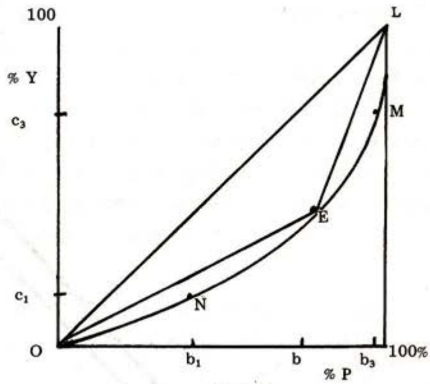

Figure 2
Lorenz curve showing a four-sectoral breakdown.

Actually, this two-sector breakdown can be further disaggregated by employing the same procedure to each of these two sectors. In Fig. 2, the average per capita incomes within the two defined sectors are indicated by slopes of the lines OE and EL. The point N on the lower portion of the Lorenz curve where the slope equals that of OE determines the incremental population percentile whose marginal per capita income equals the average within the low-income group. This is indicated in Fig. 2 as $b_1\%$ of the population with $c_1\%$ of total income The same procedure can be used to determine the point (b_3,c_3) within the high-income sector. Hence we have now established four sectors and if we want, we can have more. For convenience however, the two-sector breakdown will be used hereafter for purposes of analysis since the same procedures as well as the results obtaining in the two-sector analysis in the succeeding sections can also be extended to any more-detailed sectoral analyses should this be desired and warranted.[5]

[5] For one thing, the inequality between the high and low income groups is greater than the inequalities of the two subsectors within each of these two broad sectors. To put it differently, in Fig. 2, the ratio of the slope of EL to that of OE is greater than the ratio, say, of the slope of NE to that of ON. Therefore the relative inequality is more critical between the two broad sectors than the subsectoral inequalities within them. This observation follows from the shape of the Lorenz curve.

The sectoral breakdown discussed above need not be confined to even numbers. For instance, the slope of the line joining the points N and M in Fig. 2 can be interpreted to define the per capita income of the "middle class." If this is considered as one sector for purposes of analysis, then we have now set apart three sectors — the low-income group from O to N, the middle class, and the high-income group from M to L.

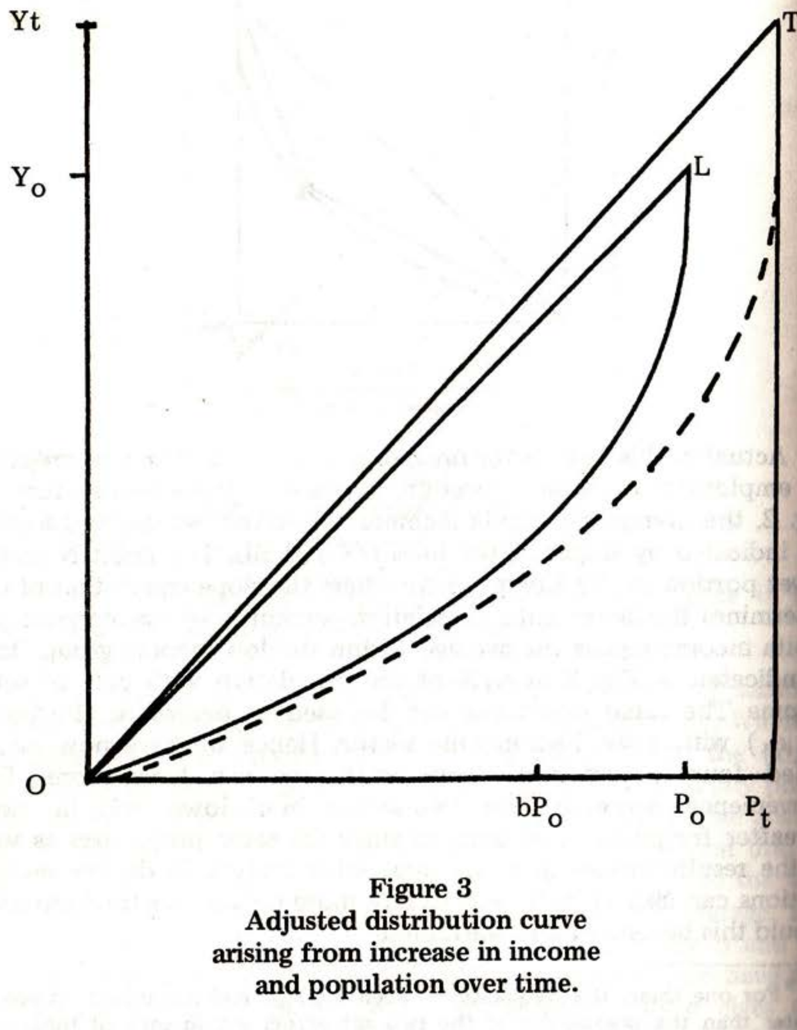

Figure 3
Adjusted distribution curve arising from increase in income and population over time.

In Fig. 3, the percentages are changed to the actual values of income Y_o and population P_o.[6] Supposing that in a future period t, Y_o increases to Y_t and P_o increases to P_t. An improvement in the income per capita level corresponds to an increase in the slope of Y_t/P_t over Y_o/P_o. Since the total population is now P_t, the original percentage b will now correspond to a larger number of people bP_t,

[6] The scaling of the two axes in Fig. 3 as shown is meant to coincide with the percentile scales of Fig. 1. This serves to facilitate comparisons between the two figures.

With this increase in the per capita income level, the resulting curve relating income and population will now be altered such that it will wind up at the point (Y_t, P_t). Let us refer to this new curve as OT and to the original one as OL. The shape of OT will be determined by the sectoral changes in income and in population that have transpired in the interim. To simplify the analysis, it will be assumed throughout this and the next section that the population growth rate is uniform for all income levels.

In Fig. 3 OT is curved in dashes to indicate that this is only one out of the many possible shapes it could take. The actual shape will be determined *ex post*. This can then be deflated to be directly comparable to OL.[7] If the deflated curve is entirely above OL, then there has been a clear case of improvement in the distribution of income. A complete downward shift will likewise indicate a clear case of deterioration. The more usual cases will be where the two curves intersect at one or two interior points, indicating improvements in some sectors and worsening in others. Actually, a downward shift at the very high income levels accompanied by an upward shift at the low levels might be indicative of the formative processes of the middle class.

III. The Integration of the Sectoral Approach with Economic Planning

The sectoral approach discussed in the preceding section can also have some implications for economic planning. Given the existing structure of income distribution, the following relationships can be obtained from Fig. 1:

$y_w = cY_o/bP_o$, the existing average per capita income within the low-income group; and

$y_h = (1-c)Y_o/(1-b)P_o$, the existing average per capita income within the high-income group.

Given a planning period t, the projected ending value Y_t as defined in Eq. (2.0) above can also be expressed as the sum of the

[7] However, in comparing directly two Lorenz curves referring to two different periods, one must remember that the percentile scales refer to different totals of income and population. This direct comparison will also obscure the relation between Y_t/P_t and Y_o/P_o that is evident in Fig. 3.

ending values of the two component sectors:

$$Y_t = Y_{wt} + Y_{ht} \tag{4.0}$$

where Y_{wt} — total income of low-income group at t;

Y_{ht} — total income of high-income group at t.

Following the same formulation in Eq. (2.0), we get

$$Y_o e^{vt} = cY_o e^{ut} + (1-c)\, Y_o e^{zt} \qquad 0<c<0.5 \tag{4.1}$$

where u — component growth rate of income for the low income group; and

z — component growth rate of income for the high income group.

Factoring out the term Y_o,

$$e^{vt} = c\, e^{ut} + (1-c)\, e^{zt}. \tag{4.2}$$

This expression contains the relationship over time of sectoral to total income growth rates given c, the existing proportion of income held by the low income group.

Fig. 4 contains a vector representation of the different per capita incomes defined thus far. In this figure, the existing state of income inequality can be loosely interpreted as the angle between y_h and y_w. An improvement over this current state must therefore be substantiated by a counterclockwise rotation of y_t (relative to y_o) simultaneously accompanied by a relative narrowing of the angle between y_{ht} and y_{wt}.[8] If the value of v in Eq. (4.2) is exogenously determined, then we need another equation to solve for the two unknowns therein — u and z. Once this has been done, then the resulting component vectors y_{ht} and y_{wt} in Fig. 4 can be specified.

[8] Some of the vectors in Fig. 4 have been linearly transformed to the origin to facilitate visual comparisons. The notations y_{ht} and y_{wt} refer to the resultant vectors of y_h and y_w, respectively, at time t. These two resultant vectors are depicted as dashed lines to indicate that their positions are not fixed as long as the values of z and u are still undetermined. Although Fig. 4 is not drawn to scale, the growth rate of v, u, and z will respectively determine both the

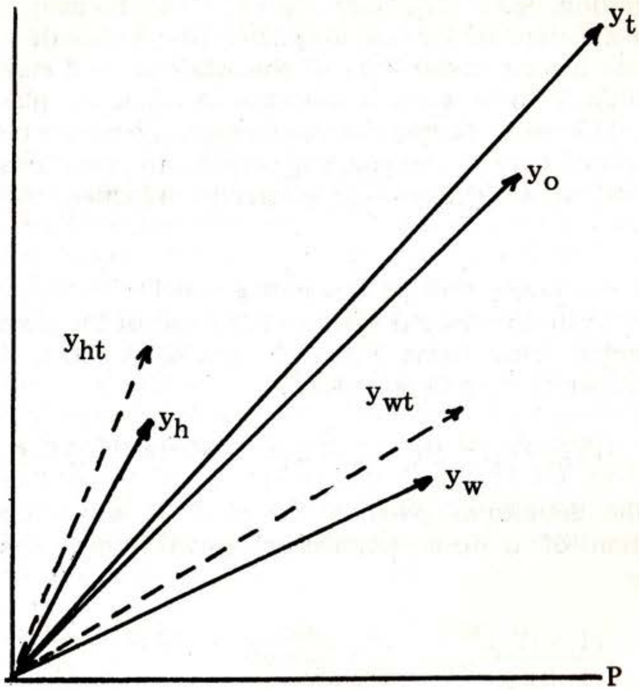

Figure 4
Vector diagram of per capita incomes.

We can now be explicit about the significance of this sectoral approach for economic planning. Let us express the existing level of income inequality as the ratio of the per capita income of the high income group to the per capita income of the low-income group. Let us denote this ratio by Q:

(5.0) $$Q = y_h / y_w = b(1-c) \,/\, c(1-b).$$

---

magnitudes and directions of the corresponding resultant vectors (expressed as per capita incomes) y_t, y_{wt}, and y_{ht}.

Strictly speaking, an improvement over the existing state of inequality can also involve only a narrowing of the angle. However, overall growth should also be considered from the point of view of the entire population.

By definition, $Q>1$. The term "Q-level" will be used hereafter to refer to the state of income inequality (the higher the Q-level, the higher the income inequality). If the attainment of more equitable distribution is to be given importance in economic planning, then this should have an operational counterpart. To render the reduction in the Q-level over a given planning period into operational terms, let us first set up a coefficient of inequality reduction, to be denoted by q.

This coefficient will be a planning variable in the sense that it serves to relate the desired Q-level at the end of the planning period to its initial value. Using Eqs. (4.0) and (5.0) above, the planned Q-level at period t can be stated as[9]

(5.1) $(Y_{ht}/P_{ht}) = qQ\,(Y_{wt}/P_{wt})$ where $(1/Q) < q < 1$.

Using the definitions given at the start of this section and the assumption of uniform population growth, the above equation becomes

(5.2) $$\frac{(1-c)Y_o e^{zt}}{(1-b)P_o\, e^{gt}} = \frac{cY_o\, e^{ut}}{bP_o\, e^{gt}}\, qQ.$$

The terms Q, Y_o/P_o, and e^{gt} will all cancel out, leaving

(5.3) $e^{zt} = qe^{ut}$.

Since q is exogenously set, we now have the other equation needed to solve for the values of u and z in Eq. (4.2) above. Combining these two equations we get

(6.0) $e^{vt} = c\, e^{ut} + q(1-c)\, e^{ut}$

or $e^{vt} = (c + q(1-c))\, e^{ut}$.

---

[9] The existing Q-level is derived from Eq. (5.0) given the values of b and c. For example, if $b = 0.7$ and $c = 0.3$, $Q = 5.44$. A given value of q implies a $100(1\text{-}q)\%$ reduction in the existing Q-level. For instance, if q is set at 0.95, the planned reduction in Eq. (5.1) refers to a 5% lowering of Q at the end of the planning period. Although the lower limit of q approaches $(1/Q)$ theoretically, such low values will probably have no relevance for planning purposes. Approaching the lower limit implies nearing the ideal state of perfect equality.

Taking the logarithms of both sides, the desired income growth rate for the low-income group necessary to achieve a 100(l-q)% reduction in the Q-level is given by the following equation:

$$(6.1) \qquad u = v - (1/t)\,\underline{Ln}\,(c + q(1-c))\,.$$

Note that the expression $(c + q(l-c)) < 1$ and this follows from the definitional range of the planning variable q. The logarithm of this expression will have a negative value; therefore, these desired values of u in the above equation must always be greater than the given values of v. Eq. (6.1) states the second condition for a planned reduction in the Q-level, namely, that the low-income sectoral growth rate must always exceed the overall average. This result is to be expected. Perhaps what is more meaningful is that in the absence or failure of planning, a trend towards income inequality is more likely to occur than an unplanned attainment of Eq. (6.1). As a corollary to the last equation, the converse condition requires that the sectoral income growth rate of the high income group be smaller than the combined average v:

$$(6.2) \qquad z = v - (1/t)\,\underline{Ln}\,(1-c+\frac{c}{q})$$

The term $(l - c + \frac{c}{q}) > 1$ when $q < 1$. Hence $z < v$ and this negative difference decreases over time. Let us now proceed to the next section where the assumption of uniform population growth rates will be dropped.

IV. The Effects of Sectoral Population Growth on Income Distribution

This section will incorporate population growth into the general analytical framework of the preceding section. Henceforth the assumption of uniform population growth will no longer be adhered to. As might be suspected, this assumption tends to cover up certain relationships occurring in the relevant sectors.

Still retaining the two-sectoral breakdown, the overall population growth in Eq. (1.0) can also be expressed as

$$(7.0) \qquad P_o e^{gt} = bP_o e^{dt} - (1-b)P_o e^{kt} \qquad 0.5 < b < 1.$$

where d — sectoral population growth rate of the low-income group; and k — sectoral population growth rate of the high income group.

Factoring out P_0, we get

$$(7.1) \qquad e^{gt} = be^{dt} + (1-b)\, e^{kt}.$$

These sectoral population growth rates can now be incorporated into Eq. (5.2) as follows:

$$(5.2.1) \qquad \frac{(1-c)e^{zt}}{(1-b)e^{kt}} = qQ\frac{ce^{ut}}{be^{dt}}$$

Using Eq. (5.0), Q will cancel out, leaving

$$(5.3.1) \qquad e^{(z-k)t} = qe^{(u-d)t}$$

Taking the logarithms of both sides, we get

$$(5.4) \qquad z - k = u - d + (1/t)\, \underline{\text{Ln}}\, q.$$

Note that (z-k) and (u-d) denote respectively the growth rates of per capital income for the high-income group and for the low-income group. Recalling that Eq. (5.4) was formulated on an *ex ante* basis, the definitional constraint $q < 1$ renders the term (l/t) Ln q negative. It follows therefore that (u-d) must exceed (z-k) by that amount if a 100(l-q)% reduction in the Q-level is to be attained.[10] The following table gives a representative idea of the magnitudes of q, Ln q, and the term (l/t) Ln q for t = 4 years.

TABLE I

Sample Values of q, Ln q, and (1/4) Ln q

q	Ln q	$(\frac{1}{4})$ Ln q
0.98	−0.0202	−0.005
0.95	−0.0513	−0.013
0.90	−0.1053	−0.026
0.85	−0.1625	−0.041
0.80	−0.2232	−0.056
0.75	−0.2871	−0.072

[10] On an *ex-post* basis, q can usually turn out to be greater than one. However, this is no longer within the context of planning, unless of course a higher level of income inequality is actually being planned for the next t years.

For cases where $k \neq d$, Eqs. (4.2) and (5.3.1) can be combined to solve for u and z. The corresponding versions of Eqs. (6.1) and (6.2) adjusted for sectoral population growth become respectively:

$$(6.1.1) \quad u = v - (1/t)\ \underline{\underline{Ln}}\ (c + q(1-c)\ e^{(k-d)\ t}) \quad \text{and}$$

$$(6.2.1) \quad z = v - (1/t)\ \underline{\underline{Ln}}\ (1 - c + \frac{c}{q}\ e^{(d-k)\ t}).$$

Eqs. (6.1.1) and (6.2.1) are the generalized forms of which Eqs. (6.1) and (6.2) are the specific cases wherein $k = d$. To illustrate the significance of Eq. (6.1.1), suppose that for a given planning period of $t = 4$ years, the target overall growth rate of income is $v = 0.06$, and a $100(l\text{-}q)\% = 2\%$ reduction in the existing Q-level is desired. Assume further that $b = 0.70$, $c = 0.30$, i.e., $Q = 5.44$. From Eq. (6.1.1), the only undetermined term left is (k-d), the difference between the sectoral population growth rates. For convenience, let

$$F(k-d)_{c,q} = (c + q(1-c)\ e^{(k-d)t})$$

be the shorthand reference to the bracketed term in Eq. (6.1.1). The subscripts on the left-hand side denote constant values of c and q; however they will be understood as such and dropped hereafter. The following table contains the relevant values leading to the calculation of u in Eq. (6.1.1) as a function of (k-d). These values of u are to be found in the sixth column. The values of z in the seventh column can be found by using either Eq. (6.2.1) or (5.4).

TABLE II

Calculation of Sectoral Income Growth Rates
Given $Q = 5.44$ and $q = .98$.

(1) $(k-d)$	(2) $e^{4(k-d)}$	(3) $F(k-d)$	(4) $Ln\ F(k-d)$	(5) $\frac{1}{4} Ln\ F(k-d)$	(6) u	(7) z
0.015	1.0618	1.0284	0.0280	0.0070	0.053	0.063
0.010	1.0140	1.0140	0.0140	0.0035	0.0565	0.0615
0.005	1.0202	1.0000	0.0000	0.0000	0.060	0.060
0.000	1.0000	0.9860	−0.0140	−0.0035	0.0635	0.0585
−0.005	0.9802	0.9724	−0.0280	−0.0070	0.067	0.057
−0.010	0.9608	0.9591	−0.0420	−0.0105	0.0705	0.0555
−0.015	0.9418	0.9460	−0.0560	−0.0140	0.074	0.054

The following points can be elicited from Table II:

1. When $k = d$, $u = v + 0.0035$. This value of 0.0035 is the sectoral income effect isolated from population effects. In turn, the sectoral income effect arises because $c < (1\text{-}c)$. Put another way, cY_o must grow at a rate $u = 0.0635$; $(1\text{-}c)Y_o$ must grow at a lower rate $z = u - 1/4\underline{Ln}(.98) = 0.0585$ for the Q-level to be reduced by 2%.[11]

2. The sectoral population effect, i.e., the variation in (k-d), is linearly related to Ln F(k-d). Notice that the entries in columns (1) and (4) fit into a straight line:

$$\underline{Ln}\ F(k-d) = 2.8(k-d) - 0.014$$

Similarly, the entries in columns (1) and (5) fall in another straight line obtained by dividing both sides of the last equation by $t = 4$ years:

$$0.25\ \underline{Ln}\ F(k-d) = 0.7\ (k-d) - 0.0035.$$

Plugging this term back into Eq. (6.1.1), we now get a simplified version of the latter as follows:[12]

$$u = v + 0.0035 - 0.7(k-d).$$

This equation serves to isolate the sectoral population effect from the sectoral income effect. Depending on the sign of (k-d), the population effect either reinforces the income effect (which is always positive) or serves to offset it.

3. Some positive level of (k-d) would be needed to compensate for the income effect. In this particular example, $(k\text{-}d) = 0.005$ is

---

[11] To verify this statement, EQ. (5.2) can be used to determine the Q-level at period t. Let this be Q_t. Using Eq. (5.2), $Q_t = b(1\text{-}c)^{.242}/c(1\text{-}b)e^{.254}$

After obtaining the values of the exponential terms, we get $Q_t = Q(1.27/1.29) = 0.98Q$. Therefore $q = 0.98$ as claimed and intended.

[12] It is interesting to note that since we are only concerned with a narrow range of values of (k-d) around zero, Eq. (6.1.1) may be approximated by the following:

$$\text{(6.1.1a)} \qquad u = v + (1/t)\left(\underline{Ln}\ F(0.) - \frac{(k-d)\,(F(s) - F(0.))}{F(0.)}\right)$$

required to exactly offset the income effect. This means that if k = d + 0.005, a uniform growth rate in incomes (u = z = v) will still be accompanied by a 2% reduction in the Q-level.

4. For developing countries, the cases where (k-d) > 0 would be more the exception than the rule. Thus in developing countries the sectoral population effect generally reinforces the sectoral income effect in placing a greater strain against efforts at reducing income inequality.

A more detailed sectoral analysis as outlined in Part II can be done following essentially the same procedural lines. Suppose this is to be done for the two subsectors within the low-income group as shown in Fig. 2. We only have to substitute the subsectoral counterparts of the term contained in Eqs. (6.1.1) and (6.2.1) to solve for the subsectoral counterparts of u and z. Specifically, the term c will be replaced by c_1, the term v by u, and the term (k-d) by the corresponding subsectoral population growth rate differential. The existing level of income inequality within the low income group can be denoted by the term Q_W-level, where $Q_W = b_1 (c-c_1)/c_1 (b-b_1)$ following the notation in Fig. 2. The coefficient of inequality reduction will be denoted by another planning variable q_W. Therefore the subsectoral analysis is dependent on the sectoral analysis in the sense that some results in the latter are used as premises for the former.

---

where s is a very small number close to zero. In the above example, s = 0.005. Comparing Eq. (6.1.1a) with the specific equation above, the sectoral income effect as defined equals (1/t) Ln F(0.); the constant coefficient (0.7) of the sectoral population effect is based on the approximation

$$\frac{F(0.005) - F(0.0)}{F(0.0)} \simeq \underline{Ln}\, F(0.005) - \underline{Ln}\, F(0.0) \simeq \underline{Ln}\, F(.005(n+1)) - \underline{Ln}\, F(.005n)$$

where n denotes integer values.

For instance, therefore, the above coefficient value 0.7 can be obtained as the quotient of the following expression obtained by substitution of the approximation with the relevant values from Table II: + (0. − (−.0140))/4(0.005)(.986).

The above approximation applies as well for other values of c and q.

V. Policy Consideration Arising from the Sectoral Approach

In the introductory part of this paper, a comment was made about the tendency in developing countries for income inequality to increase over time. In the absence of or otherwise failure of economic planning, this trend is not surprising. If we remove the constraints on q in Eq. (5.4) above, this trend may be better understood by rearranging the terms therein:[13]

(5.4a) $(z-u) + (d-k) = (1/t)\ \underline{Ln}\ q.$

Income inequality will increase if the combined sum in the left-hand side of the equation above is positive. These cases occur when (1) $z >$ u jointly with $d > k$; or when (2) $(z + d) > (u + k)$. Note that these are the more likely cases for developing economies.

This brings us now to another question — given a value of q, how long will this planned reduction in income inequality take? To answer this question, let us set

(5.4b) $D = (z-u) + (d-k)$

Lumping the four growth rates into one enables us to temporarily evade the problem of determining the combination of economic growth and population control. To reiterate, D must be negative for there to be a reduction in the current Q-level as time goes by. The answer to the question pose above can be found in the Table III below which solves the following equation

(5.4c) $t = (\underline{Ln}\ q)/D$

for the varying values of D and the accompanying schedule of possible target values of q.

Let us return to the problem of determining the composition of D in Eq. (5.4b). It is interesting to note that the problem of income inequality can be lessened tremendously if only the high-income group has a very high population growth rate. However, due perhaps to the fact that high-income families have had more exposure to and

---

[13] **The difference lies in the treatment of q. In Eq. (5.4), q was a planning variable; in Eq. (5.4a), q will be allowed to vary depending on the combined values of the left-hand side of Eq. (5.4a).**

TABLE III

Relationships of Time Requirements and Inequality Reduction

q	D = −0.005 t	D = −0.01 t	D = −0.02 t
0.98	4.04	2.02	1.01
0.95	10.26	5.13	2.57
0.90	21.08	10.54	5.27
0.85	32.50	16.25	8.13
0.80	44.62	22.31	11.16
0.70	71.34	35.67	17.84
0.60	102.16	51.08	25.54
0.50	128.64	69.32	34.66

can better afford family planning, this situation has had dysfunctional effects on reducing income inequality. In any case, the efforts at controlling population growth must be concentrated largely on the low-income sectors. Failure to at least neutralize the population factor in Eq. (5.4b) will shift the burden totally to incurring sacrifices in economic growth. To reiterate, the choices are easier to make if population and growth can share in achieving a reduction in the level of income inequality.

A more detailed sectoral analysis along the lines discussed in Part II can further provide the basis for aligning a progressive tax rate structure with the existing as well as with the desired distribution of income.

To recapitulate, it has been previously stressed that economic planning can be reoriented such that instead of totally concentrating on per capita growth, a workable balance can be struck between per capita growth and income equality. In effect, this implies that a certain growth rate will have to be foregone in order to achieve a certain amount of improvement in the income distribution. The remaining final portion of this paper will be geared to the task of reducing this trade-off into quantifiable terms.

Going back to the example in Table II of Part IV, supposing that the overall population growth rate accompanying v = 0.06 has a value g = 0.03, i.e., the per capita growth for the whole economy will have a value (v-g) = 0.03. This single value of overall per capita growth can be attained by different combinations of u, z, k, and d. To illustrate, there are various possible combinations of k and d that can result in g = 0.03 for the given value of b = 0.70.[14] These combinations can be found by substituting the values of (k-d) in Table IV below into Eq. (7.1) above. The results are shown in the second and third columns; the fourth and fifth columns use the values of u and z from Table II in order to arrive at the necessary combinations of sectoral per capita growth that will all result in the planned value of q = 0.98.[15]

The values of u and z from Table II and those of k and d from Table IV are plotted against (k-d) in Fig. 5. This reveals the relationships among the sectoral growth rates if a 2% reduction in the Q-level is to be attained. In order to determine the price of achieving this goal in terms of the effects on the z, the high-income growth

---

[14] Following the same approximation procedure outlined in Footnote 12, substituting the value of (k-d) = s = 0.005 into Eq. (7.1) yields the following expression for the sectoral population growth rate of the low-income group:

$$(7.1a) \qquad d = g - (1/t)\,\underline{\text{Ln}}\,(b + (1-b)e^{(k-d)t}).$$

For convenience, let us use the simplifying notation

$$J(k-d)_b = J(k-d) = b + (1-b)e^{(k-d)t}$$

Then we can use the following approximation for values of d:

$$d = g - (1/t)\left(\frac{(k-d)}{s}\,\frac{(J(s) - J(0.))}{J(0.)}\right)$$

For the given values of b and t, the values of d in Table IV are found from the approximation d = 0.03 − 0.3(k-d).

[15] Note that sectoral per capita growth terms are not additive since the mathematical expression relating the overall per capita growth (v-g) to the sectoral terms can be derived from Eq. (3.0) above, namely,

$$(3.1) \qquad e^{(v-g)t} = \frac{ce^{ut} + (1-c)e^{zt}}{be^{dt} + (1-b)e^{kt}}$$

TABLE IV

Variations in Sectoral Per Capita Growth with Sectoral Population Growth Differentials*

(1) (k−d)	(2) k	(3) d	(4) (z−k)	(5) (u−d)
0.015	0.0405	0.0255	0.0225	0.0275
0.010	0.037	0.027	0.0245	0.0295
0.005	0.0335	0.0285	0.0265	0.0315
0.000	0.03	0.03	0.0285	0.0335
−0.005	0.0265	0.0315	0.0305	0.0355
−0.010	0.023	0.033	0.0325	0.0375
−0.015	0.195	0.0345	0.0345	0.0395

*These computations refer to given values of v, g, Q, and q.

rate, we can compare these values of z at q = 0.98 with the corresponding values of z at q = 1. The latter is shown as the dashed line in Fig. 5. These two lines denoting z − values for varying (k-d) values are parallel to each other, and the vertical distance between them to be denoted by z, can be expressed as:[16]

(9.0) $$z = (1/t)\ \underline{\text{Ln}}\ ((1 - c + ce^{(d-k)t})/(1 - c + \frac{c}{q}\ e^{\,(d-k)t}))$$

---

[16] The dashed line can be obtained by substituting q = 1 into Eq. (6.2.1) to get

(6.2.1a) $$z = v - (1/t\ \underline{\text{Ln}}(\ (1 - c + ce^{(d-k)t}\ )$$

The partial derivative of z with respect to q in Eq. (9.0) is negative, indicating that the two lines will be wider apart for larger values of q. In this particular example, Z = 0.015.

Following the same formulation, the values of u at q = 1 can be indicated in Fig. 5 as the dashed line parallel to and below the line of u-values at q = 0.98. Let us denote the vertical distance between these two lines by the term U, where

$$U = (1/t)\ \underline{\text{Ln}}\ ((c + (1-c)qe^{(k-d)t})/(c + (1-c)e^{(k-d)t})) \tag{10.0}$$

We can now relate Eqs. (9.0) and (10.0) to the cost of achieving equitable income distributions. Since the propensity to save increases with higher income levels, the incremental savings of the low-income group arising from U will be more than offset by the savings foregone due to the lowered income growth rate in the high-income group. Therefore, this difference in terms of foregone savings can be regarded as the price or cost of achieving a 100(1 − 1)% reduction in the Q-level. Let us denote this cost by the term C.

$$C = f_h(1-c)Y_o e^{Zt} - f_w c Y_o e^{Ut} \tag{11.0}$$

where f_h — propensity to save of the high-income group,

f_w — propensity to save of the low-income group.

The magnitude of C depends on the difference between f_h and f_w and on the initial level of income inequality.[17] Its constraining effect can also be greatly mitigated by the efficiency with which the government is able to channel its share of the foregone private savings into the investments stream.

Note also that C = C(f_h, f_w, c, t, Y_o, q, k − d) so that C is independent of v. Therefore, the planned growth rate of income that was heretofore assumed exogenously determined can be tied up to the value of C. For instance, if we denote the total domestic saving that can be generated at q = 1 by the term D and if this amount can sustain an income growth rate v*, then (D − C) can likewise sustain a smaller growth rate $v < v^*$.

The determination of the planned value of v will also have to depend on the availability of external financing and foreign aid that can augment the net domestic saving (D − C). After all the available foreign contributions have been considered, the planned value of v

---

[17] **A greater cost will have to be incurred if c = .20 than if the level of income inequality were less, say, at c = .30.**

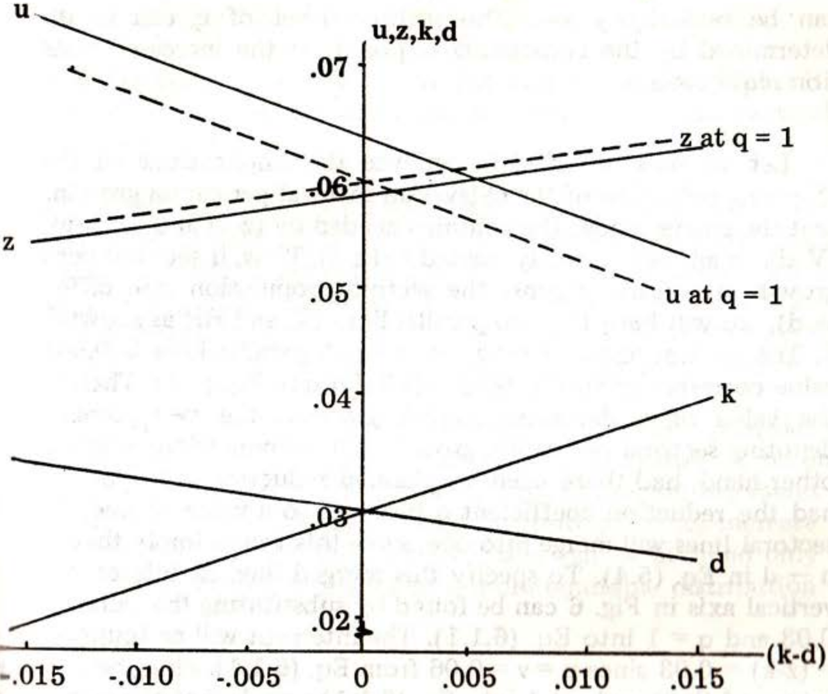

Figure 5
Graphs of Sectoral Growth Rates
(Solid lines for q = .98; dashed line for q = 1)

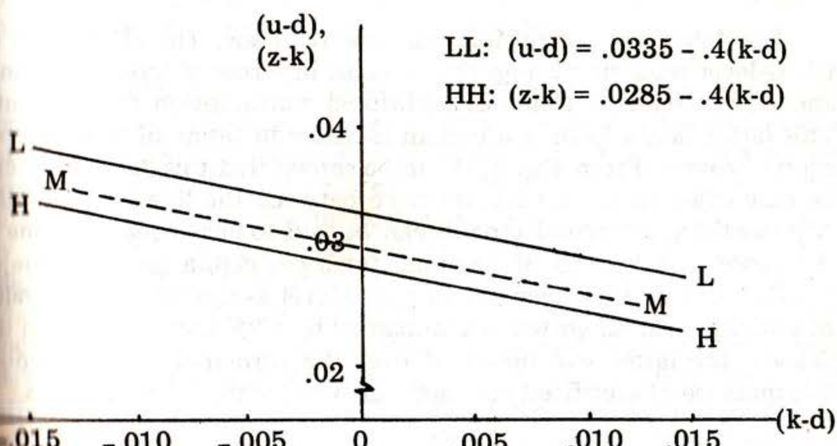

Figure 6
Graphs of Sectoral Per Capita Growth
(LL and HH refer to q = .98; MM to q = 1)

can be realistically set. The realistic level of q can in turn be determined by the constraints imposed by the increased consumption requirements.

Let us now proceed to analyze the implications of the 100 $(1 - c)\%$ reduction of the Q-level on sectoral per capita growth. Note that the entries under the columns headed by (z-k) and (u-d) in Table IV above are both linearly related to (k-d). Thus, if sectoral per capita growth is graphed against the sectoral population rate differential (k-d), we will have the two parallel lines LL and HH as shown in Fig. 6. The vertical distance between the two parallel lines is 0.005; this value corresponds to the term $-(1/t)\underline{Ln}\ q$ in Eq. (5.4). Therefore, as the value of q decreases, *ceteris paribus*, the two parallel lines denoting sectoral per capita growth will become wider apart. On the other hand, had there been no planned reduction in the Q-level, i.e., had the reduction coefficient q been set to a value of one, the two sectoral lines will merge into one, since this would imply that $z - k = u - d$ in Eq. (5.4). To specify this merged line, its intercept on the vertical axis in Fig. 6 can be found by substituting the values $k = d = 0.03$ and $q = 1$ into Eq. (6.1.1). The intercept will be found at $(u\text{-}d) = (z\text{-}k) = 0.03$ since $u = v = 0.06$ from Eq. (6.1.1). Plugging the other values of (k-d) and $q = 1$ into Eq. (6.1.1) reveals that the merged line MM shown as dashes in Fig. 6 will be parallel to and between the two sectoral lines. The difference, of course, is that line MM corresponds to $q = 1$ while the two sectoral lines correspond to $q = 0.98$.

The following conclusion can now be made. The attainment of the Q-level reduction brings about a cost in terms of foregone savings and a constraint in terms of additional consumption requirements. This latter factor implies a certain sacrifice in terms of foregone per capita growth. From Fig. 6, it can be shown that this "sacrifice" can be quantified as the vertical distance between the lines LL and MM. A potential incremental growth can be said to have been foregone in the sense that instead of planning for a per capita growth value of 0.0335 with no improvement in the Q-level as against a lower value of 0.030 per capita growth accompanied by a 2% improvement in the Q-level, the latter was preferred over the former.[18] Let us denote this measure of sacrificed per capita growth by the term S, where

[18] It is implied here that there is a constraint in the maximum per capita growth accruing to the low-income group that can be supported by the economy.

(12.0) $S = -(1/t)\ \underline{Ln}(c + q(1 - c))$

and this has been previously referred to in the analysis of Table II in Part IV above as the "sectoral income effect." In our particular example, $S = 0.0035$.

The partial derivatives of S with respect to q, to c, and to t are all negative. *Ceteris paribus*, the lower the value set for q, the greater will be the sacrifice in per capita growth. Similarly, a greater sacrifice in per capita growth will have to be made starting with a more inequitable income distribution, say, $c = 0.20$ than if one were to start at a relatively less inequitable distribution, say at $c = 0.35$.

On the opposite side of the fence, Eq. (12.0) also provides an explanation for the tendency in developing countries of income distribution to become more inequitable over time. Setting values of $q > 1$ in Eq. (12.0) can be regarded as opting for an increase in potential per capita growth. However, this can be attained only at the price of foregoing a chance at a more equitable distribution of income.

Summary

The measurement of economic growth in terms of increases in average per capita incomes can be grossly misleading. This practice stems from an emphasis that is heavily oriented toward growth, with little or no regard for the accompanying effects that this orientation may have on the structure of income distribution.

The sectoral approach propounded in this paper brings into perspective the dubious nature of economic growth that is achieved at the expense of the majority of people comprising the low-income groups. It casts grave doubts on the validity of claims to the effect that reduction in income inequality will automatically albeit belatedly follow as a consequence of economic growth. It further stresses the need for a shift in orientation of economic planning, namely, to one that incorporates a targeted reduction in the existing levels of income inequality as part of the set of planned objectives for a developing economy. This incorporation will in turn have wide effects encompassing progressive fiscal policy, segmented population control, and perhaps even radical means of affecting the distribution of wealth. Only then will the measures of economic performance have relevance to and bring some measure of hope to the masses in developing countries.