

A NOTE ON DECOMPOSITIONS OF THE GINI RATIO BY FAMILY AND BY TYPE OF INCOME

By

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Clarifying the Decompositions

It has recently been demonstrated that the Gini measure of income inequality is decomposable when data are available (a) for income recipients or *families* classified into mutually exclusive sets or sectors (Mangahas, 1974), or (b) for *income* classified into mutually exclusive sources (Fei and Ranis, 1974). The purpose of this note is to clarify the relationship between the two decomposition formulae and to present the results of the application of the latter formula to Philippine income distribution data for 1971. Some results on the application of the first formula have been published earlier (Mangahas, 1975).

The Gini ratio may be written

$$(1) \quad L = 1 - (1/m) f' (2C - 1) Xf \\ = 1 - (1/m) f' Pf$$

where m is overall mean income, f is a vector of proportions of families found in the various income classes, X is a diagonal matrix with typical element equal to mean income in a corresponding income class, and C is a matrix with ones on and below the diagonal and zeros elsewhere. If there are K income classes, then f is $K \times 1$ and C , X and P are all $K \times K$.

The details on the decomposition by families are found in the appendix to this paper. Briefly, if the data are available for families classified into sectors, then for the i^{th} sector ($i = 1, \dots, R$) one can define f_i and m_i and hence $L_i = 1 - (1/m_i) f_i' P f_i$, the Gini measure of income inequality *within* the i^{th} sector. The *Gini-difference* between

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two sectors i and j is defined as

$$D_{ij} = (f_i - f_j) 'P (f_i - f_j),$$

a positive-definite quadratic form which vanishes only when income distributions (not merely mean incomes) in sets i and j are identical. National and sectoral income inequality are then related by

$$(2) \quad L = \sum_j \theta_j L_j + \sum_{i>j} \phi_i \phi_j (D_{ij}/m)$$

where θ_j is the proportional share of sector j in total income and ϕ_j is the proportional share of sector j in total families. The first term, which is the within-section inequality component, is an income-weighted average of the sectoral Gini ratios. The second term, which is the between-sector inequality component, is a weighted sum of all possible pair-wise Gini differences. Since P is positive definite, there are no negative terms in decomposition (2).

Fei and Ranis have considered the case where data are decomposed as to type of income, as opposed to type of income recipient. For example, the former decomposition might pertain to *wages* and *rents*, whereas the latter would pertain to *workers* and *rentiers*. (As will be seen later, the difficulties of relating the two arise when workers earn some rents in addition to their wages, and/or rentiers earn some wages in addition to their rents.)

An alternative derivation of the Fei-Ranis decomposition is as follows. The Gini ratio can also be written¹

$$(3) \quad L = 1 - \sum_k f_k (y_k^* + y_{k-1}^*)$$

where f_k is the proportion of families in the k^{th} income class (i.e., the k^{th} element of the vector f), and y_k^* is the *cumulative* proportion of families up to and including income class k . The income classes $k = 1, \dots, K$ proceed from poorest to richest; this is an important specification, since the Gini ratio is computed only after the data are ordered as to size of income.

Now let α be a type of income, such as wages, and $y_{k\alpha}$ be the proportion of all income which is of type α and in income class k .

¹ Cf. equation (1) of the Appendix.

Then y_k , the proportion of all income in income class k , is

$$y_k = \sum_{\alpha} y_{k\alpha}$$

On the other hand, the cumulative income proportion y_k^* is

$$\begin{aligned} y_k^* &= \sum_{h=1}^k y_h = \sum_{h=1}^k \sum_{\alpha} y_{h\alpha} \quad , \\ &= \sum_{h=1}^k (\sum_{\alpha} y_{h\alpha}) = \sum y_{k\alpha}^* \quad . \end{aligned}$$

Substituting this into (3) gives

$$L = 1 - \sum_k f_k (\sum_{\alpha} y_{k\alpha}^* + \sum_{\alpha} y_{k-1,\alpha}^*) \quad ,$$

$$(4) \quad L = 1 - \sum_k (\sum_{\alpha} f_k (y_{k\alpha}^* + y_{k-1,\alpha}^*))$$

It is important to recognize that, for given α , the set $y_{k\alpha}$ is not necessarily ordered according to income. For instance, the average wage income per family in $k = 2$ may be smaller than the average wage income per family in $k = 1$, though, of course, average income of all types must be greater in $k = 2$. The Gini formula can nevertheless be applied on such unordered data, and Fei-Ranis term the result a *pseudo-Gini ratio*. In particular, the pseudo-Gini ratio for income type α would be

$$G_{\alpha} = 1 - \sum_k f_k \frac{y_{k\alpha}^*}{\theta_{\alpha}} + \frac{y_{k-1,\alpha}^*}{\theta_{\alpha}}$$

where $\theta_{\alpha} = \sum_k y_{k\alpha}$, the proportional share of type α in all income, standardizes the income shares to the total income of type α . Then we have

$$G_{\alpha} = 1 - \frac{1}{\theta_{\alpha}} \sum_k f_k (y_{k\alpha}^* + y_{k-1,\alpha}^*) \quad ,$$

$$\theta_{\alpha} (1 - G_{\alpha}) = \sum_k f_k (y_{k\alpha}^* + y_{k-1,\alpha}^*) \quad .$$

Combining this with (4) gives

$$(5) \quad L = 1 - \sum_{\alpha} \theta_{\alpha} (1 - G_{\alpha}) = \sum_{\alpha} \theta_{\alpha} G_{\alpha}$$

since $\sum_{\alpha} \theta_{\alpha} = 1$. Thus the national Gini ratio can also be expressed as an income-weighted average of the pseudo-Gini ratios corresponding to the income types.

Now let L_{α} be the Gini ratio for set α ; this ratio is computed only after the data are properly ordered. Fei-Ranis then define a *Gini-error*

$$E_{\alpha} = L_{\alpha} - G_{\alpha} ,$$

which is necessarily non-negative.² Then (5) becomes

$$(6) \quad L = \sum_{\alpha} \theta_{\alpha} L_{\alpha} - \sum_{\alpha} \theta_{\alpha} E_{\alpha}$$

Fei-Ranis call the first term of this decomposition the "inequality effect" and the second term the "correlation effect."³ If we now compare decompositions (6) and (2), at first glance there would seem to be an inconsistency. The first terms appear identical; but the second term of (2) is supposed to be non-negative whereas the second term of (6) is supposed to be non-positive. Actually, there is no inconsistency, since decomposing according to j is substantially different from decomposing according to α .

Example. Consider a society of 10 families, identified a, b, c, . . . , j. There are two types of income, wages and rents. Five families earn no rental income, and are classified as workers. The other five have no wage income, and are classified as rentiers.

²The proof of this is diagrammatical. The Lorenz curve, shaped like a bow, is drawn from cumulative relative frequencies of families and of income, given that the relative frequencies are ordered from poorest group to richest group. This ordering, out of all possible orderings, results in the fattest possible bow. The pseudo-Gini ratio corresponds to an arbitrary ordering, which cannot produce a bow fatter than the Lorenz curve. Not only will the bow be typically thinner, it may even extend, wholly or partially, above the 45° line of perfect equality, thus producing a *negative* pseudo-Gini ratio.

³If, for instance, wage income is very closely correlated to total income, then the size orderings of wage income and total income may be identical, in which case $L_{\alpha} = G_{\alpha}$, and E_{α} vanish.

Family	Wage Income	Rental Income	Total Income	
a	1	0	1	
b	2	0	2	
c	3	0	3	Workers
d	4	0	4	
e	5	0	5	
f	0	2	2	
g	0	4	4	
h	0	6	6	Rentiers
i	0	8	8	
j	0	10	10	
Total Income	15	30	45	

Mean wage income	1.5	Share of wages	1/3
Mean worker income	3.0	Share of workers	1/3
Mean rental income	3.0	Share of rents	2/3
Mean rentier income	6.0	Share of rentiers	2/3
Overall mean income	4.5		

Gini ratios:	Across wages	.63333
	Across workers	.26664
	Across rents	.63333
	Across rentiers	.26664

The reason wage income and rental income are more unequal than workers income and rentiers income is that the former set includes zero income whereas the latter does not. In this simple case, there is a straightforward relationship between the two types of Gini ratios. Define the no-income income class as $k = 0$, and the proportion of families in that class as f_0 . Then the Gini ratio for all families, with or without income, is

$$(7) \quad L_A = 1 - \sum_{k=0}^k f_k (y_k^* + y_{k-1}^*) = 1 - \sum_{k=1}^k f_k (y_k^* + y_{k-1}^*)$$

where $y_0^* = y_{-1}^* = 0$ by convention. For families with positive incomes only the Gini ratio is

$$(8) L_B = 1 - \sum_{k=1}^k (f_k / (1 - f_0)) (y_k^* + y_{k-1}^*) .$$

Note that $\sum_{k=1}^k f_k / (1 - f_0) = 1$. Then

$$\begin{aligned} L_A - L_B &= \sum_{k=1}^k \frac{f_k}{1 - f_0} - f_k (y_k^* + y_{k-1}^*) \\ &= \frac{f_0}{1 - f_0} \sum_{k=1}^k f_k (y_k^* + y_{k-1}^*) \\ &= \frac{f_0}{1 - f_0} (1 - L_A) \quad ; \end{aligned}$$

$$(L_A - L_B) (1 - f_0) = f_0 (1 - L_A) \quad ;$$

$$(9) L_A = f_0 + (1 - f_0) L_B$$

In our numerical example, this is verified:

$$L_A = 0.5 + 0.5 (.2666) = .6333$$

In the computation of the overall Gini ratio, families are ranked according to income, starting with the poorest:

Families	Total Income	Wages	Rentals	Cumulative Frequencies	
				of Wages	of Rentals
a	1	1	0	.0667	0
b	2	2	0	.2000	0
f	2	0	2	.2000	.0667
c	3	3	0	.4000	.0667
d	4	4	0	.6667	.0667
g	4	0	4	.6667	.2000
e	5	5	0	1.0	.2000
h	6	0	6	1.0	.4000
i	8	0	8	1.0	.6667
j	10	0	10	1.0	1.0

$$L = .3312$$

$$G_W = -0.14002 \quad G_R = .56662$$

In this example, the overall Gini ratio is $L = .3312$. The relative and cumulative frequencies needed to compute it are not shown.⁴ Note that when total income from all types is ordered, corresponding wage income and rental income are not. The cumulative frequencies from these unordered income components are on the right side of the table. The pseudo-Ginis are computed from them. Note that the pseudo-Gini for wages is negative. When weighted by factor shares, the pseudo-Ginis give the overall Gini:

$$(1/3) (-0.14002) + (2/3) (.56662) = .3311$$

However, the Gini ratios for wages and for rents are both .6333, implying a Gini-error of $.3021 = .6333 - .3312$. On the other hand, the Gini ratios for workers and rentiers are both .2666, implying a between-group inequality of $.0646 = .3312 - .2666$. Since this is equal to

$$\left(\begin{array}{c} \text{proportion} \\ \text{workers} \end{array} \right) \left(\begin{array}{c} \text{proportion} \\ \text{rentiers} \end{array} \right) \left(\begin{array}{c} \text{Gini-difference} \\ \text{relative to} \\ \text{the mean} \end{array} \right)$$

then the Gini difference relative to the mean is $0.2584 = 4(.0646)$.

The Philippine Case: Decomposition of the Gini Ratio by Type of Income

The purpose of decomposing numerical measures of income inequality is to highlight explanatory components which are excluded in computing aggregate measures. The theoretical framework adopted in the decomposition analysis defines the set of components and provides the context within which the otherwise sterile mathematical manipulations of the decomposition are interpreted.

In the following analysis, the national Gini ratio is decomposed in an attempt to explain the structure of the distribution of family income in terms of the unequal effects of the various factor incomes that make up total family income. These effects are consequences of the distribution of factor ownership among families and of the prevailing factor prices. The factor incomes considered in this paper are wages, entrepreneurial income, rents, and other types of income which will be explained later.

⁴The formula used here is $L = .9 - .2 \sum_1^9 y_k^*$, which is applicable when data are in deciles.

Given an initial distribution of family income across income classes, the first step in the analysis is to divide the total family income in each income class into mutually exclusive and exhaustive component factor incomes. Next, the means of all incomes, total and factor components, are computed for each income class. The ranking of factor component means may be positively or negatively correlated with the ranking of total income (all sources) means. If the rank correlation between the factor income means and the family income means is positive, the factor adds to overall inequality, whereas if negative, it subtracts from it. The degree of correlation and the relative share of the factor income in the total family income account for the direction and magnitude of the factor's final effect.

The various sources of family income defined in the Bureau of Census and Statistics (BCS) Survey of Households Bulletin, Family Income and Expenditure (FIES), 1971, series no. 34, can be classified under four major factor income groups:

1. *Wages* — income derived from work; includes agricultural and non-agricultural wages and salaries.
2. *Entrepreneurial Income* — income derived from work such as operating family enterprises or self-employment. This includes all income from trading, manufacturing, transport, other enterprises, practice of profession or trade, farming (including livestock and poultry raising), fishing, forestry, and hunting.
3. *Rents* — a non-work source of income which includes income in the form of rent received for lands, for buildings or rooms and for other properties, rental value of owner-occupied houses, share of crops, livestock and poultry raised by others. Also included are interest earned and dividends received from investments.
4. *Others* — a catchall group for sources not mentioned previously. This includes production of articles from own use, profit from sale of stocks and bonds, pension, retirement benefits, back pay and proceeds from insurance, gifts, support, assistance and relief, winnings from gambling, sweepstakes and lotteries, inheritance, and other sources.

Although one can conceptually combine groups (2) and (3) as *Property Income*, the distinction between work and non-work sources is maintained in this analysis. Group (4), which includes transfer payments and non-recurrent incomes, accounts for the smallest share of total family income. Using this grouping scheme, four sets of decompositions are presented, one for each of the following sectors: (1) *National*, (2) *Manila and Suburbs*, (3) *Other Urban*, and (4) *Rural*. Tables 1 to 4 show the distribution of families and incomes in absolute terms by sector whereas Tables 5 to 8 show the same distributions in per cent units. The factor income means by income class and by sector are shown in Tables 9 to 12.

Note that computations of the Gini ratio which are based on grouped income data tend to be under estimates because they ignore the possibility of income inequality within an income class. In other words, the families belonging to the same income class are assumed to be earning identical amounts of *Wages*, *Entrepreneurial Income*, etc., which are taken to be equal to the means of the corresponding factor incomes for that class.

The results of the sectoral decompositions are presented in Tables 13 to 16. The factor-Gini, which measures the inequality in the distribution of a particular factor income, is computed after rearranging the *factor* income in a monotonic nondecreasing order. The pseudo-Gini, however, is computed using the original factor income distribution derived from the given monotonic nondecreasing order across income classes of *total* family income. In the latter case, therefore, if the factor income and total family income have a rank correlation of + 1.0, the pseudo-Gini will be equal to the factor-Gini. The Gini Error, on the other hand, is the difference between the two Gini ratios in case the rank correlation is other than + 1.0.

Using factor income shares in the total income as weights, the weighted sum of factor-Gini ratios less the weighted sum of the Gini Errors gives the overall Gini ratio, which, as shown in the first part of this paper, is also equal to the weighted sum of the pseudo-Gini ratios. For a particular factor income, the weighted Gini Error is the reduction in the inequality contribution of that factor and can be construed, therefore, as its contribution to overall equality. The Correlation Effect is this contribution expressed as a percentage of the overall Gini ratio.

The problem is to determine in what manner factor incomes ultimately affect overall inequality. Numerically, this is equivalent to computing for that part of the overall Gini ratio attributable to each factor income. Given our formulation of the overall Gini ratio as the weighted sum of the pseudo-Gini ratios, only two things need to be considered: (a) the factor income share in total income, and (b) the factor income pseudo-Gini. The relative contribution of each factor is called the Factor Inequality Weight, which is the weighted pseudo-Gini expressed as a percentage of the overall Gini ratio.

The factor shares (item (1) in Tables 13 to 16) indicate that *Wages* account for the largest share of total family income in all sectors except *Rural*. The range is from 55.8 per cent in *Manila and Suburbs* to 35.1 per cent in the *Rural* sector (where it is surpassed only by *Entrepreneurial Income* at 48.9 per cent). *Entrepreneurial Income* ranks second in magnitude in all sectors, except *Rural* where it ranks first; and *Rents* and *Other Income* rank third and fourth in all sectors.

The factor pseudo-Gini ratios, however, have different rank positions. In all sectors except *Manila and Suburbs*, *Other Income* posted the highest ratio followed in decreasing order by *Wages*, *Rents* and *Entrepreneurial income*. Reflecting a higher concentration of property owners in the upper income groups, *Manila and Suburbs* has the highest pseudo-Gini ratios for *Rents* and *Entrepreneurial income* and the lowest ratio for *Wages*.

In terms of relative contribution to overall inequality, the following order of decreasing magnitude holds for all sectors: *Wages*, *Entrepreneurial Income*, *Rents*, and *Other Income*. Apparently, factor share in total income is the major determinant since relative differences in factor shares are much greater than those in factor pseudo-Gini ratios.

The decomposition analysis presented provides a method by which component inequalities can be identified and measured. It can be used in evaluating alternative factor-specific redistribution policies aimed at reducing the measured overall inequality. Another important application is in intertemporal comparisons where the analysis can be used in monitoring the effects of economic development insofar as it influences income distribution through differential growth rates in factor ownership and prices.

TABLE 1

1971

Income Class	Number of families 000	Total Income P000	Wages P000	Entrepreneurial		Other Income P000
				Income P000	Rents P000	
PHILIPPINES						
1 Under 500	329	110939	11870	63117	23075	7877
2 500 to 999	768	578431	90235	370196	87922	30078
3 1,000 to 1,499	774	965995	222179	579597	115919	48300
4 1,500 to 1,999	748	1304067	392524	706805	143447	61291
5 2,000 to 2,499	611	1372351	514632	646377	135863	75479
6 2,500 to 2,999	517	1418773	627098	591628	129108	70939
7 3,000 to 3,999	794	2736975	1319222	1067420	240854	109479
8 4,000 to 4,999	475	2112879	1014182	771201	207062	120434
9 5,000 to 5,999	316	1723439	851379	598033	165450	108577
10 6,000 to 7,999	403	2769251	1536934	814160	252002	166155
11 8,000 to 9,999	226	2017560	1026938	579040	215879	195703
12 10,000 to 14,999	234	2811168	1487108	773071	295173	255816
13 15,000 to 19,999	71	1220236	625980	313601	106161	174494
14 20,000 and above	81	2572221	737100	853977	576178	354966
TOTAL	6347	23714285	10507381	8733223	2694093	1779588

Source: Bureau of Census and Statistics
Survey of Households Bulletin
Family Income and Expenditure, 1971
 Series No. 34.

TABLE 2

Income Class	Number of families 000	Total Income P000	Wages P000	Entrepreneurial Income P000	Rents P000	Other Income P000
MANILA AND SUBURBS						
1 Under 500	1	218	76	41	000	101
2 500 to 999	5	3714	2555	327	401	431
3 1,000 to 1,499	9	11583	6162	1587	1135	2699
4 1,500 to 1,999	21	37472	22408	8581	2361	4122
5 2,000 to 2,499	36	83006	60263	13779	2656	6308
6 2,500 to 2,999	45	123973	95460	15620	7562	5331
7 3,000 to 3,999	80	277147	214513	30763	17737	14134
8 4,000 to 4,999	57	254660	173933	35398	23938	21391
9 5,000 to 5,999	40	218334	146939	22707	23580	25108
10 6,000 to 7,999	70	479767	343513	58532	42219	35503
11 8,000 to 9,999	48	431722	287527	62168	37560	44467
12 10,000 to 14,999	64	780268	461919	161515	78807	78027
13 15,000 to 19,999	23	391679	227566	89111	30551	44651
14 20,000 and above	26	992087	240085	266871	352191	132940
TOTAL	525	4085630	2282919	766800	620698	415213

Source: Bureau of Census and Statistics
Survey of Households Bulletin
Family Income and Expenditure, 1971
 Series No. 34.

TABLE 3

Income Class	Number of families 000	Total Income P000	Wages P000	Entrepreneurial Income P000	Rents P000	Other Income P000
<u>OTHER URBAN</u>						
1 Under 500	35	10996	2111	5278	2628	979
2 500 to 999	62	45688	12062	22478	8087	3061
3 1,000 to 1,499	98	123218	49411	44974	16265	12568
4 1,500 to 1,999	112	198518	84172	73253	25807	15286
5 2,000 to 2,499	128	289890	153063	75661	35366	25800
6 2,500 to 2,999	122	333827	192284	77448	29711	34384
7 3,000 to 3,999	210	722679	415541	195123	83108	28907
8 4,000 to 4,999	130	577901	321891	154877	66459	34674
9 5,000 to 5,999	108	585966	328142	147663	63284	46877
10 6,000 to 7,999	142	971845	555895	247820	115650	52480
11 8,000 to 9,999	87	773941	393935	214382	109126	56498
12 10,000 to 14,999	96	1160720	672058	271608	110268	106786
13 15,000 to 19,999	29	505307	291562	100051	44467	69227
14 20,000 and above	29	834743	367287	229554	134394	103508
TOTAL	1388	7135239	3839414	1860170	844620	591035

Source: Bureau of Census and Statistics
Survey of Households Bulletin
Family Income and Expenditure, 1971
 Series No. 34.

1971

Income Class	Number of families 000	Total Income P000	Wages P000	Entrepreneurial Income P000	Rents P000	Other Income P000
<u>RURAL</u>						
1 Under 500	292	99725	9673	62827	20344	6881
2 500 to 999	700	529029	75122	347043	79354	27510
3 1,000 to 1,499	666	831193	166239	533625	98081	33248
4 1,500 to 1,999	615	1068076	286244	623757	116420	41655
5 2,000 to 2,499	447	999455	301835	557697	97946	41977
6 2,500 to 2,999	351	960974	339224	498745	92254	30751
7 3,000 to 3,999	505	1737149	689648	840780	140709	66012
8 4,000 to 4,999	288	1280319	517249	581265	116509	65296
9 5,000 to 5,999	168	919139	375928	427399	78127	37685
10 6,000 to 7,999	192	1317640	639056	507291	97505	43788
11 8,000 to 9,999	91	811897	345056	301214	69011	96616
12 10,000 to 14,999	74	870180	352424	340240	103551	73965
13 15,000 to 19,999	19	323250	106996	124128	30708	61418
14 20,000 and above	26	745390	180384	356297	88701	120008
TOTAL	4434	12493416	4385078	6102308	1229220	776810

Source: Bureau of Census and Statistics
Survey of Households Bulletin
Family Income and Expenditure, 1971
 Series No. 34

TABLE 5

1971

Income Class	Number of families %	Total Income %	Wages %	Entrepreneurial Income %	Rents %	Other Income %
PHILIPPINES						
1 Under 500	5.2	.5	.1	.8	.9	.4
2 500 to 999	12.1	2.4	.9	4.2	3.3	1.7
3 1,000 to 1,499	12.2	4.1	2.1	6.6	4.3	2.7
4 1,500 to 1,999	11.8	5.5	3.7	8.1	5.3	3.4
5 2,000 to 2,499	9.6	5.8	4.9	7.4	5.0	4.2
6 2,500 to 2,999	8.1	6.0	6.0	6.8	4.8	4.0
7 3,000 to 3,999	12.4	11.5	12.6	12.2	8.9	6.2
8 4,000 to 4,999	7.5	8.9	9.6	8.8	7.7	6.8
9 5,000 to 5,999	5.0	7.3	8.1	6.9	6.1	6.1
10 6,000 to 7,999	6.4	11.7	14.5	9.3	9.4	9.3
11 8,000 to 9,999	3.6	8.5	9.8	6.6	8.0	11.0
12 10,000 to 14,999	3.7	11.9	14.2	8.9	11.0	14.4
13 15,000 to 19,999	1.1	5.1	6.0	3.6	3.9	9.8
14 20,000 and above	1.3	10.8	7.5	9.8	21.4	20.0
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0

TABLE 6

1971

Income Class	Number of families %	Total Income %	Wages %	Entrepreneurial Income %	Rents %	Other Income %
<u>MANILA AND SUBURBS</u>						
1 Under 500	.2	.0	.0	.0	.0	.0
2 500 to 999	.9	.1	.1	.0	.1	.1
3 1,000 to 1,499	1.7	.3	.3	.2	.2	.1
4 1,500 to 1,999	4.0	.9	1.0	1.1	.4	1.0
5 2,000 to 2,499	6.9	2.0	2.6	1.8	.4	1.5
6 2,500 to 2,999	8.6	3.0	4.2	2.0	1.2	1.3
7 3,000 to 3,999	15.4	6.8	9.4	4.0	2.9	3.4
8 4,000 to 4,999	10.9	6.2	7.6	4.6	3.9	5.2
9 5,000 to 5,999	7.6	5.3	6.4	3.0	3.8	6.0
10 6,000 to 7,999	13.3	11.7	15.0	7.6	6.8	8.6
11 8,000 to 9,999	9.1	10.6	12.6	8.1	6.0	10.7
12 10,000 to 14,999	12.2	19.1	20.3	21.1	12.7	18.8
13 15,000 to 19,999	4.3	9.6	10.0	11.6	4.9	10.8
14 20,000 and above	4.9	24.4	10.5	34.9	56.7	32.0
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0

TABLE 7

1971

Income Class	Number of families %	Total Income %	Wages %	Entrepreneurial Income %	Rents %	Other Income %
<u>OTHER URBAN</u>						
1 Under 500	2.6	.2	.0	.3	.3	.2
2 500 to 999	4.4	.6	.3	1.2	1.0	.5
3 1,000 to 1,499	7.1	1.7	1.3	2.4	1.9	2.1
4 1,500 to 1,999	8.1	2.8	2.2	3.9	3.1	2.6
5 2,000 to 2,499	9.2	4.1	4.0	4.1	4.2	4.4
6 2,500 to 2,999	8.8	4.7	5.0	4.2	3.5	5.8
7 3,000 to 3,999	15.1	10.1	10.8	10.5	9.8	4.9
8 4,000 to 4,999	9.3	8.1	8.4	8.3	7.9	5.9
9 5,000 to 5,999	7.8	8.2	8.6	7.9	7.5	7.9
10 6,000 to 7,999	10.2	13.6	14.5	13.3	13.7	8.9
11 8,000 to 9,999	6.3	10.8	10.3	11.5	12.9	9.6
12 10,000 to 14,999	6.9	16.3	17.4	14.7	13.1	18.0
13 15,000 to 19,999	2.1	7.1	7.6	5.4	5.3	11.7
14 20,000 and above	2.1	11.7	9.6	12.3	15.8	17.5
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0

TABLE 8

1971

	Income Class	Number of families %	Total Income %	Wages %	Entrepreneurial Income %	Rents %	Other Income %
<u>RURAL</u>							
1	Under 500	6.6	.8	.2	1.0	1.7	.9
2	500 to 999	15.8	4.2	1.7	5.7	6.5	3.5
3	1,000 to 1,499	15.0	6.6	3.8	8.7	8.0	4.3
4	1,500 to 1,999	13.9	8.5	6.5	10.2	9.5	5.4
5	2,000 to 2,499	10.1	8.0	6.9	9.1	8.0	5.4
6	2,500 to 2,999	7.9	7.7	7.7	8.2	7.5	4.0
7	3,000 to 3,999	11.4	14.0	15.7	14.0	11.3	8.5
8	4,000 to 4,999	6.5	10.2	11.8	9.5	9.5	8.4
9	5,000 to 5,999	3.8	7.4	8.6	7.0	6.4	4.9
10	6,000 to 7,999	4.3	10.5	14.7	8.3	7.9	9.5
11	8,000 to 9,999	2.0	6.5	7.9	4.9	5.6	12.4
12	10,000 to 14,999	1.7	7.0	8.0	5.6	8.4	9.5
13	15,000 to 19,999	.4	2.6	2.4	2.0	2.5	7.9
14	20,000 to above	.6	6.0	4.1	5.8	7.2	15.4
TOTAL		100.0	100.0	100.0	100.0	100.0	100.0

TABLE 9

1971 Mean Incomes

	Income Class	Total Income		Wages		Entrepreneurial Income		Rents		Other Income	
		P	P	P	P	P	P	P	P	P	
1	Under 500	336	36	207	70	23					
2	500 to 999	752	117	482	114	39					
3	1,000 to 1,499	1,247	287	748	150	62					
4	1,500 to 1,999	1,744	525	945	192	82					
5	2,000 to 2,499	2,246	842	1,058	222	124					
6	2,500 to 2,999	2,744	1,213	1,144	250	137					
7	3,000 to 3,999	3,446	1,661	1,344	303	138					
8	4,000 to 4,999	4,449	2,135	1,624	436	254					
9	5,000 to 5,999	5,454	2,694	1,892	524	344					
10	6,000 to 7,999	6,871	3,814	2,020	625	412					
11	8,000 to 9,999	8,927	4,544	2,562	955	866					
12	10,000 to 14,999	12,013	6,355	3,304	1,261	1,093					
13	15,000 to 19,999	17,186	8,816	4,417	1,495	2,458					
14	20,000 and above	31,755	3,717	10,543	7,113	4,382					

TABLE 10

Income Class	Total Income		Wages		Entrepreneurial Income		Rents		Other Income	
	P	P	P	P	P	P	P	P	P	P
<u>MANILA AND SUBURBS</u>										
1	Under 500	218	76	41	0	101				
2	500 to 999	742	511	65	80	86				
3	1,000 to 1,499	1,287	685	176	126	300				
4	1,500 to 1,999	1,783	1,067	408	112	196				
5	2,000 to 2,499	2,305	1,674	383	73	175				
6	2,500 to 2,999	2,754	2,121	347	168	118				
7	3,000 to 3,999	3,465	2,681	385	222	177				
8	4,000 to 4,999	4,467	3,051	621	420	375				
9	5,000 to 5,999	5,459	3,673	568	590	628				
10	6,000 to 7,999	6,853	4,907	836	603	507				
11	8,000 to 9,999	8,993	5,990	1,295	782	926				
12	10,000 to 14,999	12,190	7,217	2,523	1,231	1,219				
13	15,000 to 19,999	17,029	9,894	3,866	1,328	1,941				
14	20,000 and above	38,156	9,234	10,264	13,545	5,113				

TABLE 11

	Income Class	Total	Wages	Entrepreneurial	Rents	Other
		Income P	P	Income P	P	Income P
1	Under 500	314	60	151	75	28
2	500 to 999	736	195	362	130	49
3	1,000 to 1,499	1,257	504	459	166	128
4	1,500 to 1,999	1,772	752	654	230	136
5	2,000 to 2,499	2,265	1,196	591	276	202
6	2,500 to 2,999	2,737	1,576	635	244	282
7	3,000 to 3,999	3,442	1,979	929	396	138
8	4,000 to 4,999	4,445	2,476	1,191	511	267
9	5,000 to 5,999	5,425	3,038	1,367	586	434
10	6,000 to 7,999	6,844	3,915	1,745	814	370
11	8,000 to 9,999	8,895	4,528	2,464	1,254	649
12	10,000 to 14,999	12,090	7,000	2,829	1,149	1,112
13	15,000 to 19,999	17,424	10,054	3,450	1,533	2,387
14	20,000 and above	28,784	12,665	7,916	4,634	3,569

OTHER URBAN

TABLE 12

Income Class	Total Income		Wages		Entrepreneurial Income		Rents		Other Income	
	P	P	P	P	P	P	P	P	P	P
<u>RURAL</u>										
1	Under 500	342	33	215	70	24				
2	500 to 999	755	107	496	113	39				
3	1,000 to 1,499	1,248	250	801	147	50				
4	1,500 to 1,999	1,736	465	1,014	189	68				
5	2,000 to 2,499	2,236	675	1,248	219	94				
6	2,500 to 2,999	2,738	966	1,421	263	88				
7	3,000 to 3,999	3,441	1,366	1,665	279	131				
8	4,000 to 4,999	4,446	1,796	2,018	405	227				
9	5,000 to 5,999	5,471	2,238	2,544	465	224				
10	6,000 to 7,999	6,862	3,328	2,642	508	384				
11	8,000 to 9,999	8,922	3,792	3,310	758	1,062				
12	10,000 to 14,999	11,758	4,762	4,598	1,399	999				
13	15,000 to 19,999	17,012	5,631	6,533	1,616	3,232				
14	20,000 and above	28,669	6,938	13,704	3,412	4,615				

1971

		Wages	Entrepreneurial Income	Rents	Other Income	Total	
<u>PHILIPPINES</u>							
(1)	Factor Share	θ_{α}	.443	.368	.114	.075	1.00
(2)	Factor Gini	L_{α}	.561520	.364153	.519021	.635945	
(3)	Weighted Gini	$\theta_{\alpha} L_{\alpha}$.248753	.134008	.059168	.047696	.489625
(4)	Factor Inequality Effect	$\theta_{\alpha} L_{\alpha}/L$.508048	.273695	.120844	.097413	
(5)	Pseudo-Gini	G_{α}	.561520	.364153	.519021	.635945	
(6)	Weighted Pseudo-Gini	$\theta_{\alpha} G_{\alpha}$.248753	.134008	.059168	.047696	.489625
(7)	Factor Inequality Weight	$\theta_{\alpha} G_{\alpha}/L$.508048	.273695	.120844	.097413	1.00
(8)	Gini Error	E_{α}	.0	.0	.0	.0	
(9)	Weighted Error	$\theta_{\alpha} E_{\alpha}$.0	.0	.0	.0	.0
(10)	Correlation Effect	$\theta_{\alpha} E_{\alpha}/L$.0	.0	.0	.0	.0

TABLE 14

		Wages	Entrepreneurial Income	Rents	Other Income	Total	
<u>MANILA AND SUBURBS</u>							
(1)	Factor Share	θ_{α}	.558	.188	.152	.102	1.00
(2)	Factor Gini	L_{α}	.321163	.577978	.683740	.553525	
(3)	Weighted Gini	$\theta_{\alpha}L_{\alpha}$.179209	.108660	.103928	.056460	.448257
(4)	Factor Inequality Effect	$\theta_{\alpha}L_{\alpha}/L$.401037	.243161	.232572	.126347	
(5)	Pseudo-Gini	G_{α}	.320393	.576632	.683278	.547251	
(6)	Weighted Pseudo-Gini	$\theta_{\alpha}G_{\alpha}$.178779	.108407	.103858	.055820	.446864
(7)	Factor Inequality Weight	$\theta_{\alpha}G_{\alpha}/L$.400075	.242595	.232415	.124915	1.00
(8)	Gini Error	E_{α}	.000770	.001346	.000462	.006274	
(9)	Weighted Error	$\theta_{\alpha}E_{\alpha}$.000430	.000253	.000070	.000640	.001393
(10)	Correlation Effect	$\theta_{\alpha}E_{\alpha}/L$.000962	.000566	.000157	.001432	.003117

		Wages	Entrepreneurial Income	Rents	Other Income	Total	
<u>OTHER URBAN</u>							
(1)	Factor Share	θ_{α}	.538	.261	.118	.083	1.00
(2)	Factor Gini	L_{α}	.449902	.404014	.448066	.529083	
(3)	Weighted Gini	$\theta_{\alpha} L_{\alpha}$.242047	.105448	.052872	.043914	.444281
(4)	Factor Inequality Effect	$\theta_{\alpha} L_{\alpha}/L$.546938	.238274	.119472	.099230	
(5)	Pseudo-Gini	G_{α}	.449902	.403420	.445818	.513283	
(6)	Weighted Pseudo-Gini	$\theta_{\alpha} G_{\alpha}$.242047	.105293	.052607	.042602	.442549
(7)	Factor Inequality Weight	$\theta_{\alpha} G_{\alpha}/L$.546938	.237924	.118873	.096265	1.00
(8)	Gini Error	E_{α}	.0	.000594	.002248	.015800	
(9)	Weighted Error	$\theta_{\alpha} E_{\alpha}$.0	.000155	.000265	.001311	.001731
(10)	Correlation Effect	$\theta_{\alpha} E_{\alpha}/L$.0	.000350	.000599	.002962	.003911

TABLE 10

		Wages	Entrepreneurial Income	Rents	Other Income	Total
<u>RURAL</u>						
(1)	Factor Share	θ_{α}	.489	.098	.062	1.00
(2)	Factor Gini	L_{α}	.377929	.396408	.620336	
(3)	Weighted Gini	$\theta_{\alpha}L_{\alpha}$.184807	.038848	.038461	.459779
(4)	Factor Inequality Effect	$\theta_{\alpha}L_{\alpha}/L$.401996	.084503	.083661	
(5)	Pseudo-Gini	G_{α}	.377929	.396408	.619454	
(6)	Weighted Pseudo-Gini	$\theta_{\alpha}G_{\alpha}$.184807	.038848	.038406	.459724
(7)	Factor Inequality Weight	$\theta_{\alpha}G_{\alpha}/L$.401996	.084503	.083541	1.00
(8)	Gini Error	E_{α}	.0	.0	.000882	
(9)	Weighted Error	$\theta_{\alpha}E_{\alpha}$.0	.0	.000055	.000055
(10)	Correlation Effect	$\theta_{\alpha}E_{\alpha}/L$.0	.0	.000120	.000120

APPENDIX

Decomposition Of The Gini Ratio According To Type Of Recipient (Family)

Let f_k^* be the cumulative proportion of families up to the k^{th} income class, and y_k^* the cumulative proportion of income received by those families, for $k = 1, \dots, G$. The Gini ratio is defined as

$$L = 1 - 2 \sum_{k=1}^G [1/2 (f_k^* - f_{k-1}^*) (y_k^* - y_{k-1}^*) + (f_k^* - f_{k-1}^*) y_{k-1}^*]$$

where $f_0^* = y_0^* = 0$. The summation expression on the right-hand-side is the area underneath the Lorenz "curve", where plotted points are joined by straight lines. This reduces to

$$L = 1 - 2 \sum_{k=1}^G [1/2 (f_k^* - f_{k-1}^*) y_k^* + 1/2 (f_k^* - f_{k-1}^*) y_{k-1}^*]$$

$$L = 1 - \sum_{k=1}^G (f_k^* - f_{k-1}^*) (y_k^* + y_{k-1}^*)$$

$$(1) \quad L = 1 - \sum_{k=1}^G f_k (y_k^* + y_{k-1}^*)$$

where $f_k = f_k^* - f_{k-1}^*$ is simply the proportion of families within the k^{th} income class. We also define $y_k = y_k^* - y_{k-1}^*$ as the proportion of total incomes enjoyed by families within the k^{th} income class.

Now define

$$f = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_G \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_G \end{bmatrix}$$

Then

$$y^* = \begin{bmatrix} y_1^* \\ y_2^* \\ \vdots \\ y_G^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_G \end{bmatrix} = Cy$$

where C is the matrix with ones on and below the diagonal, and zeros elsewhere. Furthermore,

$$y_{-1}^* = \begin{bmatrix} y_0^* \\ y_1^* \\ \cdot \\ \cdot \\ y_{G-1}^* \end{bmatrix} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1 & 1 & \dots & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_G \end{bmatrix} = (C - I)y$$

where I is the $G \times G$ identity matrix. In matrix notation, the Gini ratio is then

$$L = 1 - f' (y^* + t_{-1}^*) \\ = 1 - f' (Cy + (C - I)y)$$

$$(2) \quad L = 1 - f'H y$$

where

$$H = (2C - I) = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 2 & 1 & 0 & \dots & 0 \\ 2 & 2 & 1 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 2 & 2 & 2 & \dots & 1 \end{bmatrix},$$

a matrix with twos below the diagonal, ones on the diagonal, and zeros above the diagonal. In particular, let the vectors f and y refer to national-level data and let f_j and y_j be $G \times 1$ vectors similarly defined for the j^{th} region, with $j = 1, \dots, R$. Then the regional-level Gini ratios are

$$(3) \quad L_j = 1 - f_j' H y_j, \quad j = 1, \dots, R.$$

If n is the total number of families in the nation, then nf is the $G \times 1$ vector whose k^{th} element is the total number of families in the k^{th} income class. Let X be a $G \times G$ diagonal matrix whose k^{th} diagonal element is mean family income in the k^{th} income class. Then nXf is the $G \times 1$ vector whose k^{th} element is the total family income earned by families belonging to the k^{th} income class. Total family income in the nation is then

$$(4) \quad v = \iota' Xf \cdot n$$

where ι is a $G \times 1$ vector of ones. Then y is given by

$$(5) \quad y = (n/v)Xf = (\iota' Xf)^{-1} Xf = (1/m)Xf$$

where m is the mean family income in the nation. Since f determines y , f is the basic data vector, and may be considered synonymous with "the size distribution of income".

The mean income levels per class, or the diagonals of X , depend on the distribution of families within each class's upper and lower bounds. As a simplification, X may be considered identical for each region and for the nation as a whole; in principle at least one can always arrive at approximately equal X s by simply constructing a large enough number of income classes, with very narrow intervals.

From (2) and (5) we obtain

$$(6) \quad 1 - L(1/m) f' HXf = (1/m) f' Pf$$

where $P = HX$ may be viewed as a matrix of constants, on account of the argument in the preceding paragraph. With H triangular, X diagonal, and all elements in H and X positive, it follows that the matrix P is positive definite. (Thus, strictly speaking, L may get very close to one, but never quite reaches it.) For the regions, we similarly obtain

$$1 - L_j = (1/m_j) f_j' P f_j, \quad j = 1, \dots, R$$

where $m_j = \iota' X f_j$ is the mean family income in the j^{th} region.

A *pure* redistribution of income may be defined as one which alters the distribution of families (and hence of income) by income class without altering mean family income. The effect of such a

redistribution on the Gini ratio may be seen by differentiating L with respect to the vector f , on the assumption that m is a constant. We obtain

$$\begin{aligned}\frac{\partial L}{\partial f} &= - (2/m) P f = - (2/m) (2C - I) X f \\ &= - 2 (2C - I) y = 2y - 4Cy\end{aligned}$$

$$(7) \quad \frac{\partial L}{\partial f} = 2y - 4y^*$$

The redistribution would be described by a vector of changes in the proportions of families by income class: $df = (df_1 \ df_2 \ \dots \ df_G)'$, with elements summing to zero since the elements of f always sum to one. Then the effect of df on L is $dL = (2y - 4y^*)' df$.

The next problem is to determine how L and the L_j are related. Define

$$\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \cdot \\ \cdot \\ \cdot \\ \phi_R \end{bmatrix}$$

where ϕ_j is the proportion of all families in the nation who live in region j ; thus $\sum \phi_j = 1$. Consolidating the regional size distributions of income into a $G \times R$ matrix F , where

$$f = (f_1 \ f_2 \ \dots \ f_R) \quad ,$$

then we have

$$f = F\phi$$

Therefore (6) becomes

$$(8) \quad 1 - L = (1/m)\phi'F'PF\phi$$

We now recognize that $1 - L$ is the sum of all the terms of an $R \times R$ matrix whose diagonal elements are

$$(9) \quad (1/m) \phi_j^2 f_j' P f_j = (m_j/m) \phi_j^2 (1 - L_j) \quad , \quad j = 1, \dots, R$$

and whose off-diagonals are

$$(10) \quad (1/m) \phi_i \phi_j f_i' P f_j \quad , \quad i \neq j; i, j = 1, \dots, R$$

Note that

$$(11) \quad (f_i = f_j)' P (f_i - f_j) = f_i' P f_i + f_j' P f_j - f_i' P f_j - f_j' P f_i \quad ,$$

where the last two terms on the right-hand-side are elements of "Gini cross-ratios" such as those in (10). Then the sum of the elements in (10) is

$$\sum_{i > j} ((\phi_i \phi_j / m) f_i' P f_i + (\phi_i \phi_j / m) f_j' P f_j - (\phi_i \phi_j / m) (f_i - f_j)' P (f_i - f_j)) \quad .$$

We now focus on the expression $(f_i - f_j)' P (f_i - f_j)$. Consider two regions whose size distributions of income are identical except that the first region has relatively more families in income class k_1 , by an amount α , and, correspondingly, fewer families in a different class k_2 , i.e., suppose that

$$(f_1 - f_2)' = (\dots \alpha \dots - \alpha \dots) \quad ,$$

containing zeros except in elements k_1 and k_2 as indicated; arbitrarily we have $k_2 > k_1$. In this case,

$$\begin{aligned} (f_1 - f_2)' P (f_1 - f_2) &= \alpha^2 (\dots 1 \dots -1 \dots) H X \begin{bmatrix} \dots \\ \dots \\ i \\ \dots \\ \dots \\ -i \\ \dots \\ \dots \end{bmatrix} \\ &= \alpha^2 (\dots 1 \dots -1 \dots) H \begin{bmatrix} \dots \\ \dots \\ x_{k_1} \\ \dots \\ -x_{k_2} \\ \dots \end{bmatrix} \end{aligned}$$

dicating no between-region inequality at all, whereas the various D_{ij} would be positive.

The sum of the elements in (10) may now be written

$$\sum_{i>j} ((\phi_i \phi_j m_i / m) (1 - L_i) + (\phi_i \phi_j m_j / m) (1 - L_j) - (\phi_i \phi_j / m) D_{ij})$$

Combining this sum with the sum of the terms in (9) gives

$$\begin{aligned} 1 - L &= \sum_i \sum_j \frac{\phi_i \phi_j m_i}{m} (1 - L_j) - \sum_{i>j} \frac{\phi_i \phi_j D_{ij}}{m} \\ &= \sum_j \frac{\phi_j m_j}{m} (1 - L_j) - \sum_{i>j} \frac{\phi_i \phi_j D_{ij}}{m} \end{aligned}$$

Since $m = \sum \phi_j m_j$, therefore

$$L = \sum_j \frac{\phi_j m_j}{m} L_j + \sum_{i>j} \frac{\phi_i \phi_j D_{ij}}{m}$$

$$(13) \quad L = \sum_j \theta_j L_j + \sum_{i>j} \frac{\phi_i \phi_j D_{ij}}{m}$$

where $\theta_j = \phi_j m_j / m$ is the proportion of national family income enjoyed by families in the j^{th} region. This is a decomposition of the national Gini ratio as the sum of an average, weighted by income shares, of the regional Gini ratios and a weighted sum* of all possible Gini-differences. Thus the first expression measures the contribution of "within-region inequality" whereas the second measures the contribution of "between-region inequality". Obviously, the decomposition becomes more meaningful when the between-set component is relatively large. In the (ideal) case where all $L_j = 0$, then the Gini ratio simplifies into

$$(14) \quad L = \sum_{i>j} \phi_i \phi_j |m_i - m_j| / m,$$

which is a simple weighted sum of the absolute differences between pairs of sectoral means.

*The sum of the weights is $(1 - \sum \phi_i^2) / 2m$. For R regions of equal size in terms of population, the sum of the weights is $(R - 1) / 2Rm$.

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