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Time-varying conditional Johnson S_U density in Value-at-Risk methodology

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Value-at-Risk (VaR) is a standard method of forecasting future losses in a portfolio of financial assets. An alternative method of estimating VaR using time-varying conditional Johnson S_U distribution is introduced in this paper, and the method is compared with other existing VaR models. Two estimation procedures using the Johnson distribution are developed in the paper: (1) the joint estimation of the volatility; and (2) the two-step procedure where estimation of the volatility is separated from the estimation of higher parameters, i.e., skewness and kurtosis. Empirical analyses of the two procedures are illustrated using data on foreign exchange rates and the Philippine Stock Exchange index. The methods are assessed using the standard forecast evaluation measures used in VaR models. Modeling procedures where estimation of higher parameters can be integrated in VaR methodology are introduced in the paper.

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1. Introduction

Financial institutions engage in investment activities to expand their assets so that they can provide their clients with quality financial products and services. In these activities, financial institutions incur risks of loss from their investment; losses may cause bankruptcy. In the intricate web of the financial sector, the downfall of large institutions or many firms may lead to a financial crisis.

Central banks as financial regulators require financial institutions to comply with levels of allowable incurred risk in financial activities. Risks in financial activities are categorized into three kinds: (1) credit risk, which is incurred by lending to other institutions; (2) market risk, which is incurred by keeping a portfolio of assets where prices are determined by market forces, e.g., stocks, commodities, and currencies; and (3) operational risks, which are incurred from internal operations of the institutions, such as electricity and office equipment failures [BSP Memo Circular No. 538]. Financial regulators observe international guidelines on risk capital adequacy over financial institutions, as stipulated by the Basel Committee on Banking Supervision [2004].

This paper focuses on market risks where time series analysis and econometric modeling are used in building models. In managing market risks, one of the common tools used to measure risk is Value-at-Risk (VaR). VaR is the maximum potential loss in a financial position or portfolio during a given time period such that there is a low and pre-specified probability that the actual loss may be larger. A common approach to VaR estimation is to treat the conditional mean and variance of the return series as changing over time. The conditional mean is usually estimated using the AutoRegressive-Moving-Average models and its conditional variance using the Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) models.

Empirical evidence on asset returns shows unequal leverage effects due to negative shifts and fat tails in the distribution of the data. These data characteristics correspond to the left-side skewness (extremely large losses) and leptokurtosis (fat tails), which are changes in the shapes of distributions of returns [Tsay 2002]. In the existence of means and variances that are changing over time, the concept of time-varying densities in financial asset returns is gaining ground, and questions arise as to whether or not there is strong evidence for these behaviors in profits and losses [Jondeau et al. 2007].

The objective of the paper is to derive a VaR methodology that incorporates time-varying shape characteristics in the estimation framework. The Johnson S_U distribution with time-varying parameters is assumed in the VaR model as the underlying distribution of the returns [Yan 2005]. Two procedures are devised that incorporate changes in the density: (1) a joint estimation in which mean and variance models are incorporated in the likelihood function; and (2) a two-step approach where the mean and variance models generate residuals to be fitted with the Johnson S_U density. These procedures in estimating VaR are then compared with other existing methods using Philippine financial time series data and are evaluated for accuracy using the different statistical tests.

1.1. Returns from asset prices

Returns are relative capital gains from possessing financial assets and equities (Jorion [2007]; Tsay [2002]). For an asset of price P_t at time t, an arithmetic return describes the relative change of price based on most recent previous price. The arithmetic return at time t is defined as [Jorion 2007]:

$$r_t = (P_t - P_{t-1}) / P_{t-1} \quad (1)$$

The geometric return, also known as log-return, of an asset at time t is defined as [Tsay 2002]:

$$r_t = \log(P_t / P_{t-1}) \quad (2)$$

The logarithm is of base e, the Euler number. It is favorable to use the log-returns due to its additive property [Jorion 2007]. The logarithmic transformation of the data is favored since it restricts prices as positive values compared to the arithmetic returns and reduces the magnitude of volatility in price changes [Chatterjee et al. 2000]. With the statistically favorable advantages, returns computed in the paper are log-returns.

1.2. The definition of Value-at-Risk

VaR is the maximum potential loss of a financial position or portfolio during a given time period such that there is a low and pre-specified probability that the actual loss may be larger. Tsay [2002] gives a more formal definition that deals with probability. Suppose that at current time t, a VaR value is to be estimated for k periods ahead. Let r_t be the financial asset return series of interest to be evaluated with a distribution function $F_{r_t}(x)$, where a negative return means loss in the long position. Define $F_{r_t}^{-1}(q) = \inf\{F_{r_t}(x) \geq q\}$ to be the quantile function for a left-tail probability q. Let the risk probability for extreme loss be p, commonly used values are 0.01, 0.05, or 0.10. Then the $100(1-p)\%$ value-at-risk of possessing 1 unit of an asset k periods ahead is equal to [Tsay 2002]:

$$VaR : p = P(r_{t+k} \leq VaR) = F_{r_{t+k}}(VaR) \text{ or } VaR = F_{r_{t+k}}^{-1}(p) \quad (3)$$

In estimating VaR for an asset, the following elements are needed: (1) the probability p; (2) the forecast horizon k; (3) the data frequency, e.g., daily or weekly; (4) the distribution of asset returns; and (5) the amount of position for the asset [Tsay 2002].

1.3. The family of Value-at-Risk methods

The historical method. In evaluating the quantile of a distribution for VaR, a simple approach is to solve for sample quantiles based on historical data on asset returns. If $\{r_1, r_2, ..., r_t\}$ is a subset of data on consecutive periods of the return series of an asset with window length t, and $r_{(i)}$ is the i^{th} smallest return in the window, then the one-period ahead $100(1-p)\%$ VaR is equal to:

$$VaR_{Hist} = r_{([tp])} + (tp - [tp])\,(r_{([tp]+1)} - r_{([tp])}) \tag{4}$$

where $[q]$ means the integer part of the real number q (Tsay [2002]; Fallon and Sarmiento-Sabogal [2003]). For example, in a window of an asset return series with 1,500 data points, the long-position 99 percent VaR would be the first percentile of the data, which is the 15th smallest return value.

In this method of estimation, the assumed distribution of the data is the empirical distribution of returns. It avoids the possible misspecification by not assuming mathematical probability distributions. It is an easy method of estimating VaR without dealing with statistical complexity. A caveat of the method is that it assumes the distribution is similar between past observed values and future unobserved values of the returns [Tsay 2002]. The static approach ignores the time-varying nature of asset returns, especially its volatility. In addition, the use of sample quantiles in estimating the true quantiles at the tails of the distribution is very unreliable with very high variation [Danielsson and de Vries 1997].

Econometric methods. Using these methods in VaR estimation involves the specification of the following: (1) conditional mean structure μ_t as a function of time t, e.g. using AutoRegressive-Moving-Average models [Box et al. 1994] or regression models with exogenous explanatory variables; (2) conditional variance equation h_t for volatility as a function of time, e.g., using the AutoRegressive Conditional Heteroscedasticity (ARCH) class of models (Engle [1982]; Bollerslev [1986]; Nelson [1991]); and (3) specification of the standardized error distribution $\varepsilon_t \sim F_\varepsilon$, e.g., using the standard normal distribution (Engle [1982]; Longerstaey and Spencer [1996]), the standardized t distribution [Tsay 2002], or the generalized error distribution [Nelson 1991].

Given that these three elements are fully specified and all model parameters are estimated, the one-period ahead $100(1 - p)\%$ VaR is equal to:

$$VaR_{Econ} = \hat{\mu}_{t+1} + \sqrt{\hat{h}_{t+1}}F_\varepsilon^{-1}(p) \tag{5}$$

The hats over the mean and variance specification imply one-step ahead forecasts for the mean and variance of the return series, respectively. The function F_ε^{-1} is the quantile function of the standardized error distribution.

An example of popular models in the econometric method is the RiskMetrics model of J. P. Morgan [Longerstaey and Spencer 1996]. The model assumes that the conditional mean of the returns is zero and the conditional variance follows an Integrated GARCH model. Moreover, the assumed error distribution of the data is the standard normal distribution. In equation form for the log-return series r_t:

$$r_t = 0 + \sqrt{h_t}\varepsilon_t\,;\ \varepsilon_t \sim N(0,1);\ h_t = \lambda h_{t-1} + (1 - \lambda)\varepsilon_{t-1}^2 \tag{6}$$

The parameter λ describes the variance process as an exponentially weighted moving average and is determined to be any number between 0.9 and 1 (Longerstaey and Spencer [1996]; Tsay [2002]).

One problem of the RiskMetrics methodology is the assumption of normality of the distribution. The distribution of financial returns tend to deviate from the normal distribution and are more likely to be heavy-tailed, with higher probability of extreme values (tail values) occurring in the changes of asset prices compared to the normal distribution [Tsay 2002]. Since the normal distribution is inadequate in modeling financial returns, another compromise is to use the t-distribution, which has a bell-shaped density curve but with fatter tails compared to the normal distribution. When an appropriate mean and variance model has been fitted for the standardized t distribution, the one-step ahead 100(1 – p)% VaR for the t distribution with v degrees of freedom is given by [Tsay 2002]:

$$VaR_{Econ,t} = \hat{\mu}_{t+1} + (\, t_{p,v} \,/\, \sqrt{v / v - 2}) \sqrt{\hat{h}_{t+1}} \tag{7}$$

The $t_{p,v}$ is the pth lower quantile of the t distribution with v degrees of freedom. The parameter v and other model parameters are jointly estimated.

Conditional density methods. These methods deal with fitting distributions with parameters that are time-dependent [Jondeau et al. 2007]. The econometric methods are special cases of this class of procedures. Distributions used usually require more than two parameters, which would include shape parameters such as those that affect skewness and kurtosis. In this family, higher parameters are modeled with a time-varying structure to adapt to the concept dynamics in higher moments. Distributions that have been used in the literature are the skewed Student's *t* distribution (Hansen [1994]; Harvey and Siddique [1999]), Pearson Type IV distribution [Yan 2005], Johnson S_U distribution [Yan 2005], Edgeworth series densities [Rockinger and Jondeau 2001], and the Gram-Charlier densities [Jondeau and Rockinger 2001]. Each distribution has its caveats and advantages, such as those that directly influence the skewness and kurtosis but are computationally intensive to estimate the quantiles, e.g., the Gram-Charlier densities and Edgeworth series densities (Jondeau and Rockinger [2001]; Rockinger and Jondeau [2001]), and those which produce computationally and analytically derivable quantiles yet do not directly influence the coefficients of skewness and kurtosis, such as the Pearson IV and Johnson S_U distributions [Yan 2005]. Due to the computational ease of estimation for parameters using maximum likelihood estimation and analytically derivable quantiles after estimation [Yan 2005], the Johnson S_U distribution is considered in this paper for time-varying conditional density.

2. The Johnson S_U distribution

The Johnson S_U distribution is one of the distributions derived by Johnson [1949] by translating the normal distribution to certain functions. The cumulative distribution function of the JS_U distribution is shown below. If $Y \sim JS_U(\xi, \lambda, \gamma, \delta)$:

$$P(Y \le y) = F_Y(y; \xi, \lambda, \gamma, \delta) = \Phi\left[\gamma + \delta \sinh^{-1}\left((y - \xi)/\lambda\right)\right] \tag{13}$$

The function $\Phi(u)$ is the cumulative distribution function of the standard normal distribution. From the equation above, the quantile function F_Y^{-1} can be directly derived as:

$$F_Y^{-1}(p; \xi, \lambda, \gamma, \delta) = \xi + \lambda \sinh\left[\left(\Phi^{-1}(p) - \gamma\right)/\delta\right] \tag{14}$$

The quantile function depends on the quantiles of the standard normal distribution $\Phi^{-1}(p)$ which are easily tractable. The density of the JS_U distribution, which will be used for the estimation procedure, is equal to [Yan 2005]:

$$f_Y(y; \xi, \lambda, \gamma, \delta) = \delta / \left[\lambda \sqrt{1 + \left((x - \xi)/\lambda\right)^2}\right] \varphi\left[\gamma + \delta \sinh^{-1}\left((x - \xi)/\lambda\right)\right] \tag{15}$$

The function $\varphi(u)$ is the probability density function of the standard normal distribution. The parameters of the JS_U are $(\xi, \lambda, \gamma, \delta)'$ with each affecting the location, scale, skewness, and kurtosis of the distribution. The parameters are not the direct raw moments of the distribution. The first four moments, the mean, variance, third central moment, and fourth central moment, respectively, of the distribution are the following [Yan 2005]:

$$\mu = \xi + \lambda\omega^{1/2} \sinh \Omega \tag{16}$$

$$\sigma^2 = (\lambda^2/2)(\omega - 1)(\omega \cosh 2\Omega + 1) \tag{17}$$

$$\mu_3 = -(1/4)\omega^2(\omega^2 - 1)^2\left[\omega^2(\omega^2 + 2)\sinh 3\Omega + 3\sinh \Omega)\right] \tag{18}$$

$$\mu_4 = -(1/8)(\omega^2 - 1)^2\left[\omega^4(\omega^8 + 2\omega^6 + 3\omega^4 - 3)\cosh 4\Omega + 4\omega^4(\omega^2 + 2)\cosh 2\Omega + 3(2\omega^2 + 1)\right] \tag{19}$$

The quantities in the moment equations are $\Omega = \gamma / \delta$ and $\omega = \exp(\delta^{-2})$. The standard distribution for the JS_U exists when $\xi = 0$ and $\lambda = 1$, but the mean and the variance are not 0 and 1, respectively. To use the Johnson distribution as a standardized error distribution in econometric modeling (e.g., in AutoRegressive-Moving-Average-GARCH modeling), we set the parameters in the following manner [Yan 2005]:

$$\xi_s = -\omega^{1/2} \sinh \Omega \left[\sqrt{1/2(\omega - 1)(\omega \cosh 2\Omega + 1)} \right]^{-1} \quad (20)$$

$$\lambda_s = \left[\sqrt{1/2(\omega - 1)(\omega \cosh 2\Omega + 1)} \right]^{-1} \quad (21)$$

2.1.Joint estimation procedure for JS_U distribution

From the standardization of the distribution, mean-variance specifications can be introduced for econometric modeling with the JS_U distribution. For maximum likelihood estimation, the higher parameters can be modeled to have a time-varying structure, ultimately introducing dynamic properties to the skewness and kurtosis. In modeling using the JS_U density with joint estimation of parameters having time-varying structures, the following are sets of equations are defined:

Mean-Variance-Error Equation: $y_t = \mu_t + \sqrt{h_t} z_t$ (22)

Mean Specification: $\mu_t = g_\mu(t)$ (23)

Variance Specification: $h_t = g_h(t)$ (24)

Error Specification: $\begin{cases} E(z_t) = 0; \text{var}\,(z_t) = 1; \\ z_t \sim JS_U(\xi_{S,t}, \lambda_{S,t}, \gamma_t, \delta_t) \end{cases}$ (25)

Third Parameter Specification: $y_t = g_\gamma(t)$ (26)

Fourth Parameter Specification: $\delta_t = g_\delta(t)$ (27)

The functions g_μ, g_h, g_γ, and g_δ are time-dependent functions related to t, e.g., $g_\mu \equiv ARMA(p,q)$ process for the mean, $g_h \equiv GARCH(p_1,q_1)$ for the variance, $g_\gamma(t) = \beta_0 + \beta_1 x_{t-1}$ for the structure of the third parameter, and $g_\delta(t) = \delta_0$ a constant value for the fourth parameter. The location and scale parameters are functions of the time-varying third and fourth parameters due to standardization of the JS_U distribution, i.e.,

$$\xi_{S,t} = f_1(\gamma_t, \delta_t)\ ,\ \lambda_{S,t} = f_2(\gamma_t, \delta_t)$$

In this structure, it is implied that the skewness and kurtosis would have time-varying properties due to the structure of the third and fourth parameters.

The log-likelihood sum to be maximized for estimation is written below:

$$l(\mu_t, h_t, \gamma_t, \delta_t \mid \gamma_1, \ldots, \gamma_n) = \sum_{t=1}^{n}\left\{\log\left[f_Y\left(\left(y_t - \mu_t\right) \Big/ \sqrt{h_t}; \xi_{S,t}, \lambda_{S,t}, \gamma_t, \delta_t\right) - \tfrac{1}{2}\log[h_t]\right]\right\} \tag{28}$$

The use of the functions g_μ, g_h, g_γ, and g_δ as arguments in the log-likelihood function implies the estimation of the parameters inside these functions. The location and scale parameters should be substituted for the appropriate functional form based on g_γ and g_δ. If lagged values of the time series data are being used, the addends of the summation are reduced to adapt to the use of lags.

2.2. Two-step procedure for JS_U distribution

Another estimation procedure that introduces time-varying mean and variance specifications in the JS_U distribution is a two-step procedure, where first the return series r_t are fitted with the appropriate model for mean μ_t and variance h_t and estimation is carried out using quasi-maximum likelihood estimation (QMLE) [Bollerslev and Wooldridge 1992].

$$e_t = \left(r_t - \hat{\mu}_t\right) \Big/ \sqrt{\hat{h}_t} \tag{29}$$

From these residuals, they are fitted with the JS_U distribution with structures in the third and fourth parameters. The log-likelihood to be minimized would be of the form below:

$$l(\gamma_t, \delta_t \mid e_1, \ldots, e_n) = \sum_{t=1}^{n}\left\{\log\left[f_Y\left(y_t - \mu_t \Big/ \sqrt{h_t}; \xi_{S,t}, \lambda_{S,t}, \gamma_t, \delta_t\right)\right]\right\} \tag{30}$$

2.3. Value-at-Risk formula using Johnson S_U distribution

After the estimation of the parameters of the model, either through the joint estimation or the two-step procedure, the one-step ahead 100(1 – p)% long position VaR is equal to:

$$VaR_{JS_U} = \hat{\mu}_{t+!} + \sqrt{\hat{h}_{t+1}}\left(\hat{\xi}_{S,t+1} + \hat{\lambda}_{S,t+1}\sinh\left[\left(\Phi^{-1}(p) - \gamma_{t+1}\right) / \hat{\delta}_{t+1}\right]\right) \tag{31}$$

The paper then compares the performance of VaR of the time-varying JS_U with other VaR methods using Philippine financial time series datasets, applying the usual evaluation procedures associated with VaR.

3. The evaluation of VaR methods

3.1. Number of exceptions: Basel Committee on Banking Supervision requirements

A popular evaluation procedure of the VaR methods is done through the number of exceptions [Basel Committee on Banking Supervision 1996]. A VaR exception occurs when the actual loss exceeds the value of the anticipated VaR. Depending on the VaR probability level, a specific amount of VaR exceptions are allowed per year. For example, for the 99 percent VaR, it is expected and permitted that the number of exceptions be equal to 1 percent of the total number of periods in a year. In 250 time periods, a maximum of four exceptions are allowed. The number of exceptions of the VaR model is classified into three zones: (1) the green zone; (2) the yellow zone; and (3) the red zone. Depending on the number of exceptions for a given year, a penalty multiplier in introduced in the calculation of appropriate risk capital based on VaR. Table 1 displays the multipliers for each zone and number of exceptions.

TABLE 1. Classification zones based on number of exceptions and appropriate scaling factors for risk capital

Zone	Number of exceptions	Scaling factors for the market risk capital
	0	3.00
	1	3.00
Green zone	2	3.00
	3	3.00
	4	3.00
	5	3.40
	6	3.50
Yellow zone	7	3.65
	8	3.75
	9	3.85
Red zone	10 or more	4.00

Source: Basel Committee on Banking Supervision [1996]

3.2. Number of exceptions: likelihood ratio tests

Another method for assessing the performance of the VaR model is based on the method's adherence to the desired risk probability. Christoffersen [1998] introduced a system of successive Chi-Square tests to assess the VaR methods based on the number of exceptions within the forecast evaluation period. Three tests are conducted on the frequency of exceptions, done successively: (1) the unconditional coverage test, which tests whether the risk probability is fulfilled by the VaR model; (2) the independence test, which tests whether the probability of two successive exceptions is equal to the proportion of exceptions succeeded by non-exceptions; and (3) the conditional convergence test, which tests whether the probabilities of successive and non-successive exceptions are equal to the coverage probability. When an initial test leads to the acceptance of the null hypothesis, it is a favorable result for the VaR model and a succeeding test is conducted. If an initial test leads to rejection, then the succeeding test is not done and we conclude the given VaR procedure does not have a favorable property based on the test. The sequence of tests is listed in Table 2. The logarithms are all in base e.

3.3. Magnitudes of values

As risk capital is based on the value of VaR, the magnitude of the VaR methods are analyzed and compared. Three general features should be considered by an appropriate method: (1) conservatism, which indicates that it generally give a relatively higher VaR compared to other methods; (2) accuracy, in which the method is able to identify the level of loss with minimum error in the magnitude; and (3) efficiency, in which the method is able to compute the adequate level of risk capital such that risk is fully accounted yet not too high that the opportunity loss for other financial activity is constrained [Engel and Gizycki 1999]. Statistical measures are selected for each quality as a measure of their compliance with the desired feature. Table 3 shows the statistics and their intended analysis. Though only one measure of accuracy is shown, the analyses of the number of exceptions are also taken as accuracy measures by Engel and Gizycki [1999].

TABLE 2. Likelihood testing procedures for VaR assessment

Name of test	Hypotheses	Test statistic	Implication of rejection
Unconditional coverage	$H_0 : \pi = p$ $H_A : \pi > p$	$LR_{uc} = 2\log\left[\left((1-\hat{\pi}_t)(1-p)\right)^{T-T_1}\left(\hat{\pi}/p\right)^{T_1}\right] \sim \chi^2_{(1)}$ T = Number of data points in the forecast period T_1 = Number of VaR exceptions in the forecast period $\hat{\pi} = T_1 / T$ = Proportion of VaR exceptions	Method is not appropriate in the given level of risk probability. Adjust VaR model.
Independence	$H_0 : \pi_0 = p_1$ $H_A : \pi_0 > p_1$	$LR_{ind} = 2\log\left[\left((1-\hat{\pi}_0)^{T_{00}}\hat{\pi}_0^{T_{01}}(1-\hat{\pi}_1)^{T_{10}}\hat{\pi}_1^{T_{11}}\right) \Big/ (1-\hat{\pi}_0)^{T_{00}+T_{10}}\hat{\pi}_0^{T_{01}+T_{11}}\right] \sim \chi^2_{(1)}$ T_{00} = Number of two consecutive days with no exception T_{01} = Number of periods with no exceptions followed by an exception T_{10} = Number of periods with exceptions followed by no exceptions T_{11} = Number of two consecutive days with exceptions $\hat{\pi}_i = T_{i1} / T_{i1} + T_{i0}$	VaR model produces exception clustering. VaR model is not appropriate in mitigating risks in times of volatility clustering. Adjust model.
Conditional coverage	$H_0 : \pi_0 = \pi_1 = p$ $H_A : \pi_0 = \pi_1 > p$	$LR_{cc} = LR_{uc} + LR_{ind} \sim \chi^2_{(2)}$	Proportions are higher than desired risk. Adjust VaR model.

Source: Christoffersen [1998]

TABLE 3. Statistical measures for VaR comparisons

Desired quality	Statistical measure	Analysis of statistic
Conservatism	Mean Relative Bias (MRB) [Engel and Gizycki 1999] $MRB_i = 1/T \sum_{t=1}^{T} \left(VaR_{it} - \overline{VaR_t}\right) \Big/ \overline{VaR_t}\ ;\ \overline{VaR_t} = \sum_{i=1}^{N} VaR_{it}$ VaR_{it} = The VaR on time *t* based on method *i* T = Return series data length of evaluation period N = Number of VaR methods being compared	Higher MRB means more conservative relative to other models.
Accuracy	Average Quadratic Loss (AQL) [Engel and Gizycki 1999] $AQLF = 1/T \sum_{t=1}^{T} L\left(VaR_t, r_t\right)$ $L\left(VaR_t, r_t\right) = \begin{cases} 1 + \left(VaR_t - r_t\right)^2 \ if\ VaR_t > r_t \\ 0\ otherwise \end{cases}$	Lower AQL means more accurate VaR method in forecasting and accounting for possible loss.
Efficiency	Spearman Rank Correlation of Value-at-Risk Estimates and Absolute Returns (RKCORR) [Engel and Gizycki 1999] $r_{Spearman} = n \sum_{t=1}^{T} R_{\lvert r_t \rvert} R_{-VaR_t} - \sum_{t=1}^{T} R_{\lvert r_t \rvert} \sum_{t=1}^{T} R_{-VaR_t} \Big/ \sqrt{\left[n \sum_{t=1}^{T} R_{\lvert r_t \rvert} - \left(\sum_{t=1}^{T} R_{\lvert r_t \rvert}\right)^2\right]\left[n \sum_{t=1}^{T} R_{-VaR_t} - \left(\sum_{t=1}^{T} R_{-VaR_t}\right)^2\right]}$ $R_{\lvert r_t \rvert}$ = rank of $\lvert r_t \rvert$; R_{-VaR_t} = rank of $-VaR_t$	Higher correlation implies better ability of the model in following the asset exposure and volatility. Otherwise, lower correlation implies poor ability.
Efficiency	Average Market Risk Capital (AMRC) [Basel Committee on Banking Supervision 1996] $AMRC = 1/T \sum_{t=1}^{T} MRC_t$ $MRC_t = \max\left[-k/60 \sum_{k=t-1}^{t-60} VaR_t,\ VaR_{t-1}\right]$ k = The penalty multiplier based on the number of exceptions (see Table 1)	Lower AMRC means lower risk capital to be allocated on the average.

Using these evaluation procedures, the paper assessed the time-varying methods in estimating VaR using the JS_U distribution and compared it with other VaR methods suggested in the literature.

3.4. Empirical results

In the evaluation of the different VaR methodologies, the following financial time series data are used: (1) the Philippine Peso-US Dollar Exchange Rate (RUSD) from 4 January 1999 to 10 November 2011; (2) the Philippine Peso-Euro Exchange Rate (REUR) from 4 January 1999 to 18 November 2011; and (3) the Philippine Stock Exchange index (PSEi) from 3 January 2000 to 18 November 2011.

To generate out-of-sample forecasts, the last 250 data points of each series are used for forecast evaluation while the rest of the periods are used in the estimation.

The data series are evaluated with long position 99% one-step-ahead VaR values. The two VaR models based on the time-varying JS_U methods are evaluated and compared with six different VaR models: (1) GARCH(1,1) with normal distribution (using Quasi Maximum Likelihood Estimation or QMLE); (2) GARCH(1,1) with Student's distribution; (3) TARCH(1,1,1) with normal distribution (using QMLE); (4) TARCH(1,1,1) with Student's t distribution; (5) a rolling 250-period historical simulation quantile method; and (6) RiskMetrics method with $\lambda = 0.95$.

GARCH(1,1) is based on the model by Bollerslev [1986] with the form for the variance given below:

$$h_t = \alpha_0 + \alpha_1 h_{t-1} z_{t-1}^2 + \beta h_{t-1} \tag{32}$$

The argument z_{t-1}, is the standardized error of one period before, and the parameters (α_0, α_1, β) are estimated. The model assumes a symmetric effect of changes in the immediate past to the variance of current changes, i.e., it assumes no leverage effect. To account for asymmetric effect of past changes to current volatility, TARCH(1, 1, 1) [Zakoian 1994] adds a term on the volatility and models the conditional standard deviation. Thus in modeling the variance, the equation is modified as shown below:

$$h_t = \left(\alpha_0 + \alpha_1 \left| \sqrt{h_{t-1}} z_{t-1} \right| + \psi I_{(0,\infty)}\left(\sqrt{h_{t-1}} z_{t-1}\right) \left| \sqrt{h_{t-1}} z_{t-1} \right| + \beta \sqrt{h_{t-1}} \right)^2$$

$$I_{(0,\infty)}(u) = \begin{cases} 0 \; if \; u \leq 0 \\ 1 \; if \; u > 0 \end{cases} \tag{34}$$

For the joint estimation of parameters on the JS_U distribution, the following specification of the variance, third, and fourth parameters are shown below:

$$h_t = \exp\left\{\theta_0 + \theta_1 \left| \sqrt{h_{t-1}} z_{t-1} \right| + \theta_2 \sqrt{h_{t-1}} z_{t-1} + \theta_2 \log h_{t\text{-}1}^{(20)} \right\} \tag{35}$$

$$\gamma_t = \varphi_0 + \varphi_1 \left| \sqrt{h_{t-1}} z_{t-1} \right| + \varphi_2 \sqrt{h_{t-1}} z_{t-1} + \varphi_2 SK_{t\text{-}1}^{(20)} \tag{36}$$

$$\delta_t = \zeta_0 + \zeta_1 \left| \sqrt{h_{t-1}} z_{t-1} \right| + \zeta_2 \sqrt{h_{t-1}} z_{t-1} + \zeta_2 K_{t\text{-}1}^{(20)} \tag{37}$$

The terms $h_{t\text{-}1}^{(20)}$, $SK_{t\text{-}1}^{(20)}$, and $K_{t\text{-}1}^{(20)}$ are a rolling-window 20-lag variance, skewness, and kurtosis of returns modeled to affect changes in the variance, third, and fourth parameters of the JS_U distribution.

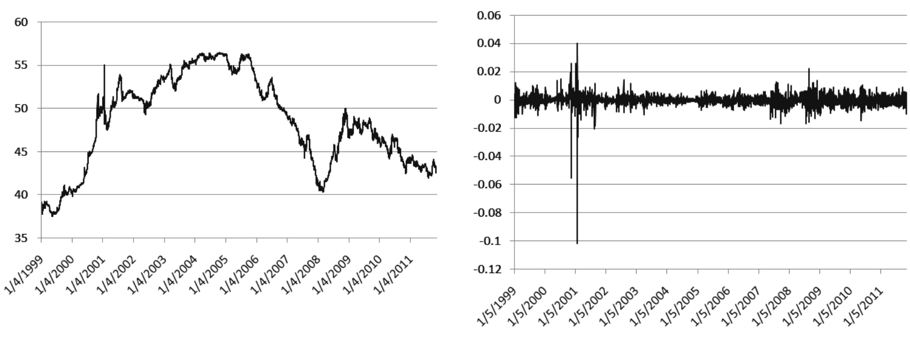
For the two-step procedure, the variance is estimated with the GARCH(1,1) model and the residuals were modeled with the JS_U using equations (36) and (37) with $\sqrt{h_{t-1}} = 1$ since the residuals have unit variance. The mean specification for all models was set to zero, i.e., $\mu_t = 0$ and $r_t = \sqrt{h_t z_t}$.

4. Descriptive analysis of the data

4.1. Graphical analysis of the levels and returns

Figure 1 displays the graphs of levels and returns of the RUSD series. A period of increase or depreciation of the Philippine peso occurred from before 1999 to 2001 primarily due to the effects of the Asian financial crisis and political crisis in the Philippines. Stability in the level of the exchange rate was achieved from 2002 to 2004, and the appreciation of the peso started in 2005. Another round of depreciation started in 2008 until 2009 when the peso again appreciated. In the return series, the occurrences of very large changes were at end of 2000 until early 2001, most likely due to political uncertainty during the impeachment and eventual ouster of president Joseph Estrada. After 2001, stability in the changes was observed until after 2008, when changes in the return series were showing wider ranges compared to the period 2002 to 2007.

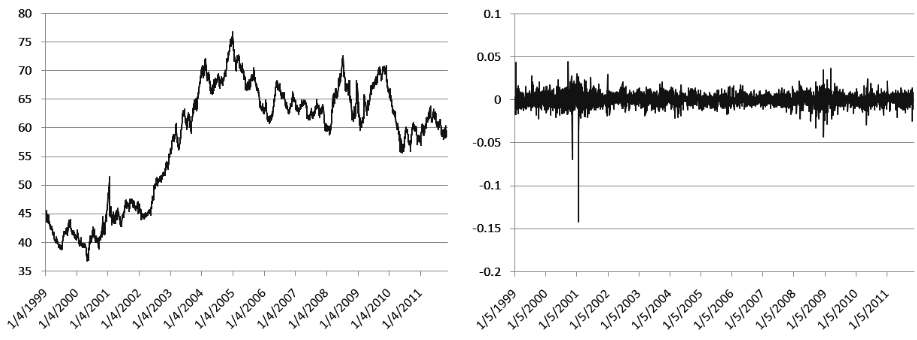
FIGURE 1. Time plots of US dollar exchange rate



Price level Returns

The graphs of the REUR are shown in Figure 2. With the euro, a surge of depreciation of the peso occurred from 2001 to 2005. Since then, the changes were relatively stable, except in 2008-2010 during which the euro had a wave of upturns and downturns with respect to the peso. In terms of volatility, large changes occurred during the latter part of 2000 up to early 2001. A period of stability in changes was observed in during the period 2002 to 2007, followed by large volatility during the global financial crisis of 2008 to 2009.

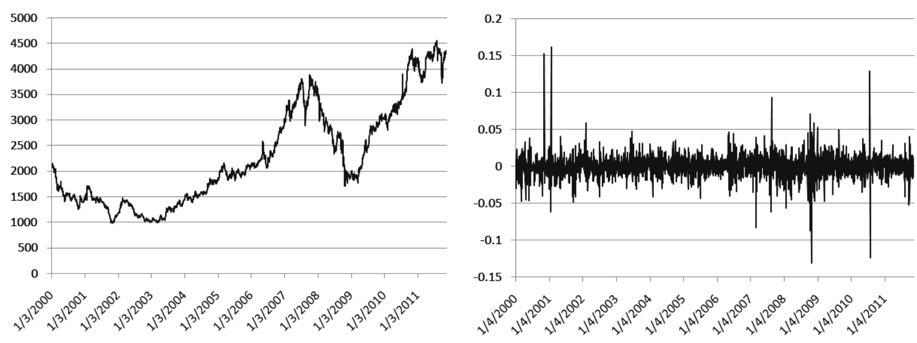
FIGURE 2. Time plots of euro exchange rate



Price level Returns

Figure 3 shows the time plots of the Philippine Stock Exchange Index (PSEi). The index experienced a surge in level in 2002 and the increase lasted until 2007. The index retreated during the late 2007 until early 2009, most likely due to the global economic crisis, before expanding during the second quarter of 2009. In terms of volatility, two sharp increases were observed at the end of 2000 and the start of 2001. Stable variance was observed during the period 2002 to 2007 followed by sharp downward changes at the end of 2008. Two spikes were observed in 2010.

FIGURE 3. Time plots of Philippine Stock Exchange index



Price level Returns

4.2. Summary statistics of returns

The values in Table 4 show the different summary statistics of the return series. The series have high kurtosis values, suggesting fat tails or non-normal distribution. Moreover, the return series associated with the currencies (RUSD and REUR) have negative skewness, while the return series on the PSEi is skewed to the right, perhaps due to the bullish performance of the stock market in the observation period. The return series on the PSEi is also the most volatile with a wider range between the minimum and the maximum values and high standard deviation.

TABLE 4. Summary statistics of the financial returns

	RUSD	REUR	PSEi
Obs	3192	3300	2990
Mean	2.950×10^{-5}	7.827×10^{-5}	2.332×10^{-4}
Standard deviation	4.290×10^{-3}	7.602×10^{-3}	1.441×10^{-2}
Skewness	-4.482	-2.070	0.462
Kurtosis (unadjusted)	115.221	43.303	20.662
Minimum	-.101	-.142	-.131
Maximum	.0402	.0441	.162

5. Results and discussion

5.1. Number of exceptions

Table 5 shows the results of statistics based on the number of exceptions and likelihood ratio tests for the different VaR methods. The econometric methods performed better in terms of the number exceptions compared to the JS_U procedures. The econometric methods had a maximum of four exceptions (green zone) compared to the JS_U and RiskMetrics methods that are in the yellow zone (5 to 9 exceptions). The historical quantile method is at the middle, in the yellow zone for the two assets.

The tests for independence and conditional coverage were not conducted in some cases because no VaR exception was observed for the econometric methods in all series, the JS_U, quantile, and RiskMetrics methods in RUSD and REUR return series. Other p-values in the PSEi return series from the JS_U, quantile, and Riskmetrics methods were still shown, even though in some results it may indicate rejection of initial desirable characteristics.

TABLE 5. Evaluation measures based on exceptions

Model	**Joint JS_U**			**Two-step JS_U**		
Time series	**RUSD**	**REUR**	**PSEi**	**RUSD**	**REUR**	**PSEi**
Number of exceptions	2	6	9	2	6	7
Likelihood ratio tests p-values						
Unconditional coverage	0.742		0.001	0.742		0.019
Independence	-	-	0.002	-	-	0.174
Conditional coverage	-	-	0.000	-	-	0.025
Model	**GARCH QMLE**			**GARCH t**		
Time series	**RUSD**	**REUR**	**PSEi**	**RUSD**	**REUR**	**PSEi**
Number of exceptions	1	4	2	1	2	2
Likelihood ratio tests p-values						
Unconditional coverage	0.278	0.381	0.742	0.278	0.742	0.742
Independence	-	-	-	-	-	-
Conditional coverage	-	-	-	-	-	-
Model	**TARCH QMLE**			**TARCH t**		
Time series	**RUSD**	**REUR**	**PSEi**	**RUSD**	**REUR**	**PSEi**
Number of exceptions	1	4	2	0	2	2
Likelihood ratio tests p-values						
Unconditional coverage	0.278	0.3815	0.742	-	0.742	0.742
Independence	-	-	-	-	-	-
Conditional coverage	-	-	-	-	-	-
Model	**Historical quantile**			**RiskMetrics**		
Time series	**RUSD**	**REUR**	**PSEi**	**RUSD**	**REUR**	**PSEi**
Number of exceptions	2	3	5	1	5	8
Likelihood ratio tests p-values						
Unconditional coverage	0.742	0.758	0.162	0.278	0.162	0.005
Independence	-	-	0.000	-	-	0.000
Conditional coverage	-	-	0.000	-	-	0.000

The likelihood ratio tests for the joint JS_U methods and RiskMetrics showed poor performance using the PSEi return series. Using these methods results in a low confidence of achieving the appropriate risk probability for the series, with higher risks that the joint JS_U and RiskMetrics estimates of VaR frequently and consecutively exceed by actual losses. The quantile method performed generally well in covering the appropriate risk levels, yet are very susceptible to exception clustering, risking portfolio to have frequent consecutive extreme losses. The two-step JS_U performed well using coverage and noexceptions clustering for the PSEi return series. This suggests that the two-step JS_U method may result in a better model compared to the historical quantile and RiskMetrics procedures.

5.2. Magnitudes of values

The results of the assessment using the magnitude-based statistics for comparison between different VaR methods are shown in Table 6. Using the Mean Relative Bias (MRB) criterion, the JS_U methods were generally less conservative compared to the econometric methods in accounting for risk, having larger negative MRB values compared to other methods. Using the Average Quadratic Loss (AQL) criterion, the JS_U methods were relatively poor in accuracy compared to the other methodologies, except the two-step JS_U using the PSEi return series, which is more accurate compared to the RiskMetrics procedure.

TABLE 6. Evaluation measures based on magnitudes

Model	**Joint JS_U**			**Two-step JS_U**		
Time series	**RUSD**	**REUR**	**PSEi**	**RUSD**	**REUR**	**PSEi**
MRB	-0.1317#	-0.0960	-0.2157#	-0.1061	-0.1289#	-0.2077
AQL	0.0080*	0.0241*	0.0361*	0.0080*	0.0241*	0.0281
RKCORR	0.0386	-0.0621	0.1925	0.0641	-0.0638	0.2417*
AMRC	0.0197#	0.0462	0.0843	0.0205	0.0445	0.0819#
Model	**GARCH QMLE**			**GARCH t**		
Time series	**RUSD**	**REUR**	**PSEi**	**RUSD**	**REUR**	**PSEi**
MRB	0.0111	0.0053	0.071	0.0938	0.0763	0.1595
AQL	0.0040	0.0161	0.0080#	0.0040	0.0080#	0.0080#
RKCORR	0.0805	-0.0386*	0.2122	0.0796	-0.0458	0.2125
AMRC	0.0231	0.0264#	0.0908	0.025	0.0472	0.0985
Model	**TARCH QMLE**			**TARCH t**		
Time series	**RUSD**	**REUR**	**PSEi**	**RUSD**	**REUR**	**PSEi**
MRB	0.0558	0.0105	0.1043	0.1504*	0.0799*	0.1902*
AQL	0.0040	0.0161	0.0080#	0.0000#	0.0080#	0.0080#
RKCORR	0.0881*	-0.1225	0.2308	0.0845	-0.0879	0.2245
AMRC	0.0240	0.0440	0.0933	0.0261*	0.0471	0.1007*
Model	**Historical quantile**			**RiskMetrics**		
Time series	**RUSD**	**REUR**	**PSEi**	**RUSD**	**REUR**	**PSEi**
MRB	-0.0283	0.1016	-0.0165	-0.0450	-0.0487	-0.0850
AQL	0.0080*	0.0120	0.0201	0.0040	0.0201	0.0321
RKCORR	-0.0830#	-0.0927	0.0320#	0.0656	-0.1308#	0.1145
AMRC	0.0227	0.0485	0.0923	0.0218	0.0519*	0.0979

Legend: * highest; #lowest

Using the efficiency-based rank correlation criterion (Spearman Rank Correlation), the historical quantile and RiskMetrics methods performed poorly in their ability to follow through the exposure of the financial asset, leading to inefficiently allocating appropriate risk capital based on their VaR estimates. The two-step JS_U method performed better in terms of efficiency in allocating proper risk capital in the PSEi return series. QMLE methods had better performance in efficiency for RUSD and REUR return series.

In assigning appropriate risk capital (Average Market Risk Capital), the two-step JS_U method allocates the lowest risk capital in RSGD and PSEi return series. The method is the most efficient for this evaluation procedure compared to other models in its ability to allocate appropriate capital. The lowest risk capital allocations for the RUSD and REUR return series are given by the joint JS_U and GARCH QMLE, respectively. The highest market risk capital allocations using the RUSD and PSEi return series are computed using the TARCH (*t*-distribution), while Riskmetrics method resulted in the highest market risk capital allocation for the REUR return series.

6. Conclusions

This paper introduced an alternative methodology of computing for Value-at-Risk (VaR) using a time-varying conditional JS_U density. The performance of this alternative procedure is compared with the existing methods of estimating VaR. The paper showed that JS_U methods were relatively less conservative and less accurate in capturing appropriate risk levels, but the two-step JS_U is more efficient in allocating proper risk capital for extreme losses compared to popular methods, such as the historical quantile and RiskMetrics. The results suggest the usefulness of the two-step JS_U method in estimating VaR. This paper's contribution is in the use of non-normal distributions in estimating VaR in asset portfolio. We hope this paper can stimulate other researchers to verify the performance of the JS_U VaR using other series or to introduce other non-normal distributions in estimating VaR.

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References

Bangko Sentral ng Pilipinas (BSP) [2006] *Circular no. 538: revised risk-based capital adequacy framework*, *BSP regulations*. Manila.

Basel Committee on Banking Supervision [1996] *Supervisory framework for the use of "backtesting" in conjunction with the internal models approach to market risk capital requirements*. Basel.

Basel Committee on Banking Supervision [2004] *International convergence of capital measurement and capital standards*. Basel.

Bollerslev, T. [1986] "Generalized autoregressive conditional heteroscedasticity", *Journal of Econometrics* **31**: 307-327.

Bollerslev, T. and J. Wooldridge [1992] "Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances", *Econometric Reviews* **11**: 143-172.

Box, G., G. Jenkins, and G. Reinsel [1994] *Time series analysis: forecasting and control*. 3rd edition, Englewood Cliffs, New Jersey: Prentice Hall.

Chatterjee, S., A. Hadi, and B. Price [2000] *Regression analysis by example*, 3rd edition. New York: John Wiley & Sons, Inc.

Christoffersen, P. [1998] "Evaluating interval forecast", *International Economic Review* **3**(4): 841-862.

Christoffersen, P. and D. Pelletier [2003] "Backtesting Value-at-Risk: a duration-based approach", *CIRANO Scientific Series*.

Danielsson, J. and C. de Vries [1997] "Tail index and quantile estimation with very high frequency data", *Journal of Empirical Finance* **4**: 241-257.

Engel, J. and M. Gizycki [1999] "Conservatism, accuracy and efficiency: comparing Value-at-Risk models", Australian Prudential Regulation Authority, Reserve Bank of Australia, Working Paper 2.

Engle, R [1982] "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation", *Econometrica* **50**(4): 987-1007.

Fallon, E. and J. Sarmiento-Sabogal [2003] "Is historical VaR a reliable tool for relative risk measurement in the Colombian stock market?: an empirical analysis using the coefficient of variation", Working Paper of the Pontificana Universidad Javeriana, Bogota, Colombia.

Hansen, B. [1994] "Autoregressive conditional density estimation", *International Economic Review* **35**(3): 705-730.

Harvey, C. and A. Siddique [1999] "Autoregressive conditional skewness", *Journal of Financial and Quantitative Analysis,* **34**(4): 465-487.

Johnson, N. [1949] "Systems of frequency curves generated by methods of translation", *Biometrika* **36**: 149–176.

Jondeau, E., S. Poon, and M. Rockinger [2007] *Financial modeling under non-Gaussian distributions.* London: Springer-Verlag.

Jondeau, E. and M. Rockinger [2001] "Gram-Charlier densities", *Journal of Economic Dynamics and Control* **25**: 1457-1483.

Jorion, P. [2007] *Value at risk: the new benchmark for managing financial risk.* 3rd edition. New York: McGraw-Hill.

Longerstaey, J. and M. Spencer [1996] *RiskMetrics*™ – *technical document*. USA: J. P. Morgan/Reuters.

Rockinger, M. and E. Jondeau [2001] "Entropy densities with applications to autoregressive skewness and kurtosis", Working Paper of the Direction Generale des Etudes et des Relations Internationales, Direction des Etudes Economiques et de la Recherche, NER # 79.

Tsay, R.S. [2002] *Analysis of financial time series*. New Jersey: John Wiley & Sons.

Yan, J. [2005] "Asymmetry, fat-tail, and autoregressive conditional density in financial return data with systems of frequency curves", Working Paper in Department of Statistics and Actuarial Science, University of Iowa, USA.

Zakoian, J. [1994] "Threshold heteroskedastic models", *Journal of Economic Dynamics and Control* **18**(5): 931-955.

APPENDIX: Model Results

	RUSD		REUR		PSEi	
Model parameters	**Coeff**	**p-value**	**Coeff**	**p-value**	**Coeff**	**p-value**
Joint JS_U						
Sigma_2						
theta_1	31.1347	0.0000	16.0325	0.0000	-2.7734	0.3250
theta_2	83.1788	0.0000	15.4491	0.0370	12.6526	0.0010
theta_3	0.6850	0.0000	0.5705	0.0000	0.4033	0.0000
theta_0	-3.7118	0.0000	-4.3001	0.0000	-5.1818	0.0000
Gamma						
phi_1	-10.3608	0.0000	2.8237	0.0180	-3.2173	0.0000
phi_2	-4.7802	0.0400	0.2707	0.8740	-0.3899	0.6080
phi_3	-0.0012	0.9200	0.0009	0.9470	0.0161	0.1120
phi_0	0.0041	0.7060	-0.0100	0.4340	-0.0093	0.4310
Delta						
zeta_1	-4.1759	0.5330	-39.4274	0.2250	0.3816	0.9280
zeta_2	-2.8375	0.6860	10.8124	0.7870	3.5070	0.4940
zeta_3	-0.0973	0.0000	0.0073	0.9320	-0.0416	0.0000
zeta_0	1.8614	0.0000	2.0866	0.0000	1.4060	0.0000
Two-step JS_U						
Gamma						
phi_1	-0.0773	0.0000	0.0161	0.0820	-0.0556	0.0000
phi_2	-0.0020	0.8890	0.0007	0.9640	-0.0037	0.7530
phi_3	0.0000	0.9970	0.0015	0.9090	0.0161	0.1320
phi_0	-0.0085	0.5410	-0.0074	0.5970	-0.0087	0.4770
Delta						
zeta_1	0.1633	0.3190	-0.0434	0.6430	-0.0602	0.2220
zeta_2	0.2140	0.2260	-0.0319	0.8060	0.0360	0.4780
zeta_3	-0.0898	0.0000	-0.0773	0.0000	-0.0335	0.0000
zeta_0	1.8096	0.0000	2.0547	0.0000	1.3717	0.0000
GARCH QMLE						
alpha_1	0.2239	0.0000	0.0647	0.0010	0.1153	0.0000
Beta	0.7925	0.0000	0.9312	0.0000	0.7425	0.0000
alpha_0	0.0000	0.0000	0.0000	0.1110	0.0000	0.0000
GARCH t						
alpha_1	0.2121	0.0000	0.0379	0.0000	0.1647	0.0000
Beta	0.7992	0.0000	0.9476	0.0000	0.7164	0.0000
alpha_0	0.0000	0.0000	0.0000	0.0020	0.0000	0.0000
Df	7.0380	0.0000	7.3951	0.0000	4.3797	0.0000
TARCH QMLE						
alpha_1	0.1888	0.0000	0.0533	0.0000	0.1261	0.0000
Psi	0.0391	0.0000	0.0460	0.0000	-0.0687	0.0000
Beta	0.8150	0.0000	0.9277	0.0000	0.8238	0.0000
alpha_0	0.0001	0.0000	0.0001	0.0000	0.0016	0.0000
TARCH t						
alpha_1	0.1690	0.0000	0.0402	0.0000	0.1750	0.0000
Psi	0.0257	0.0980	0.0173	0.0610	-0.0639	0.0030
Beta	0.8393	0.0000	0.9502	0.0000	0.7973	0.0000
alpha_0	0.0001	0.0000	0.0001	0.0030	0.0014	0.0000
Df	6.3864	0.0000	7.4785	0.0000	4.3345	0.0000