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A game of subsidization or taxation for new technology adoption in international duopoly

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The adoption of new technology by firms is important for the economic growth, particularly developing countries. However, depending on the values of various parameters about demand, production cost, and set-up cost, the adoption of new technology by firms may be insufficient or excessive in less competitive industries from the point of view of social welfare. Then, subsidization or taxation policies by the governments are necessary. In this paper we present an analysis of subsidization or taxation for new technology adoption in a framework of international duopoly with differentiated goods.

JEL classification: F13, D43, L13
Keywords: subsidy or tax, new technology adoption, international duopoly

1. Introduction

We consider the following story. There are two countries: one is a developed country; the other is a developing country. Across these countries, there is a duopolistic industry with one firm in each country. The firms produce differentiated goods, and the goods are substitutes or complements. They can use a common new production technology which is more efficient than the present technology. The production cost with the new technology is lower than that with the present technology. This new technology is developed by a laboratory or a firm in the developed country, and the government of the developed country wants to transfer the new technology to the developing country as a foreign aid free of charge. However, both the firm in the developed country and the firm in the developing country must expend some fixed set-up cost for the education of its staff to adopt and use the new technology.

We may consider a different story with two developing countries and an international organization which develops a new technology.

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The adoption of new technology by firms is important for economic growth of countries, particularly developing countries. However, depending on the values of various parameters about demand, production cost, and set-up cost, the adoption of new technology by firms may be insufficient or excessive in less competitive industries from the point of view of social welfare. Then, subsidization or taxation policies by the governments are necessary. In this paper, we present an analysis of subsidization or taxation for new technology adoption in a framework of international duopoly with differentiated goods.

There are many references about technology adoption or R & D investment in duopoly or oligopoly such as Boone [2001], Dermot and Neary [2009], Elberfeld and Nti [2004], Filippini [2005], Kabiraj [2005], Kamien and Tauman [1986], Katz and Shapiro [1985], La Manna [1993], Matsumura, Matsushima, and Cato [2013], Pal [2010], Sen and Tauman [2007], Wang and Yang [2004], Watanabe and Muto [2008], Zhang, Mei, and Zhong [2014], Hattori and Tanaka [2014] and [2015]. In particular, the model in Pal [2010] and the model of this paper are similar. However, in Pal [2010], the governments' policies are not analyzed.

We consider the following three-stage game. In the first stage, the government of each country determines a subsidy to or a tax on its firm for the adoption of new technology. In the second stage, the firms decide whether or not they adopt new technology. In the third stage, the firms determine their outputs.

At the sub-game perfect equilibrium after the second stage of the game, one or two or no firm adopts new technology depending on the values of the set-up cost and subsidy or tax. The social welfare in each country is defined to be the sum of consumers' surplus and the profit of the firm in that country, which is equal to consumers' utility minus production cost including the set-up cost of new technology. Subsidies to the firms are financed by lump-sum taxes on the consumers. Taxes on the firms are transferred to the consumers as a lump-sum manner. The lump-sum taxes and transfers are not related to the goods of the firms. Excluding income effects, they do not affect the demand for the goods, and they are canceled out with subsidies or taxes in the social welfare.

There are several types of equilibria. Many equilibria are states with subsidies to the firms or inaction of the governments. However, a part of equilibria are states with taxes on the firms.

In the case where the goods of the firms are substitutes, there is the following equilibrium. Without subsidy or tax, one country does not want the adoption of new technology by its firm, the other country wants, and the firm in the former country has an incentive to adopt new technology. Then the government of the former country should impose a tax on its firm to prevent the adoption of new technology. See 2-(3)-(ii) of Theorem 1.

In the case where the goods are complements, there is the following equilibrium. Without subsidy or tax both firms adopt new technology and no country wants adoption, then both governments should impose taxes on their firms to prevent the adoption of new technology. See 2-(3)-(ii) and 3-(4) of Theorem 2.

In the next section, we present the model of this paper. In Section 3, we analyze a game of subsidization between two countries when the goods of the firms are substitutes. In Section 4, we consider the case of complements.

2. The model

There are two firms in two countries: Firm A in Country A and Firm B in Country B. The firms supply differentiated goods to their domestic countries and export them to their foreign countries. The firms consider the adoption of new technology. Technology itself is free, but each firm must expend a fixed set-up cost, for example, the education cost of its staff.

The government of each country may subsidize or tax the firm in the country so as to maximize the social welfare, which is the sum of consumers' surplus, and the profit of the firm, which is equal to consumers' utility minus production cost including the set-up cost of new technology. Subsidies to the firms are financed by lump-sum taxes on the consumers, and the taxes on the firms are transferred to the consumers in a lump-sum manner. The lump-sum taxes or transfers are not related to the goods of the firms. Excluding income effects, they do not affect the demand for the goods, and they are canceled out with subsidies or taxes in the social welfare.

We denote the supplies of Firm A and B to Country A by x_A and x_B and their supplies to Country B by y_A and y_B, the prices of their goods in Country A by p_A and p_B and the prices in Country B by q_A and q_B. The utility functions of consumers in Country A and B are

$$u_A = a(x_A + x_B) - \frac{1}{2}x_A^2 - bx_A x_B - \frac{1}{2}x_B^2,$$

$$u_B = a(y_A + y_B) - \frac{1}{2}y_A^2 - by_A y_B - \frac{1}{2}y_B^2,$$

where $a > 0$. When the goods of the firms are substitutes, $0 < b < 1$, and when the goods are complements, $-1 < b < 0$. From these utility functions, the inverse demand functions in Country A and B are derived as follows.

$$p_A = a - x_A - bx_B,\ p_B = a - x_B - bx_A,$$

$$q_A = a - y_A - by_B,\ q_B = a - y_B - by_A.$$

The marginal cost before the adoption of new technology is c, and the marginal cost after the adoption of new technology is zero. They are common to both firms. A fixed set-up cost is e, which is also common. There is no fixed cost other than the set-up cost. The subsidy to or tax on Firm A by Country A for new technology adoption is denoted by s_A and that by Country B is s_B. If s_A or s_B is negative, it is a tax. The net set-up costs of Firm A and Firm B are

$$e_A = e - s_A, e_B = e - s_B.$$

We analyze a game of subsidy or tax policies for new technology adoption between two countries.

If the adoption of new technology and non-adoption are indifferent for a firm, it adopts new technology. Also, if the adoption of new technology and non-adoption are indifferent for a country, the government of the country selects the adoption of new technology.

We consider the following three-stage game. In the first stage, the government of each country determines a subsidy to or tax on its firm for the adoption of new technology. In the second stage, each firm decides whether or not it adopts new technology. In the third stage, the firms determine x_A, x_B, y_A and y_B.

Before the adoption of new technology, the social welfare of Country A and the social welfare of Country B are as follows:

$$W_A = a(x_A + x_B) - \frac{1}{2}x_A^2 - \frac{1}{2}x_B^2 - bx_Ax_B - p_Bx_B + q_Ay_A - c(x_A + y_A),$$

$$W_B = a(y_A + y_B) - \frac{1}{2}y_A^2 - \frac{1}{2}y_B^2 - by_Ay_B - q_Ay_A + p_Bx_B - c(x_B + y_B),$$

After the adoption of new technology by the firms they are

$$W_A = a(x_A + x_B) - \frac{1}{2}x_A^2 - \frac{1}{2}x_B^2 - bx_Ax_B - p_Bx_B + q_Ay_A - e,$$

$$W_B = a(y_A + y_B) - \frac{1}{2}y_A^2 - \frac{1}{2}y_B^2 - by_Ay_B - q_Ay_A + p_Bx_B - e.$$

3. Case of substitutes

3.1. Firm behavior

First, we consider the case of substitutes. We assume, without loss of generality, $e_i \leq e_j, i, j \in \{A, B\}, \ j \neq i$. The profits of Firm A and B before the adoption of new technology are

$$\pi_A = (a - x_A - bx_B)x_A + (a - y_A - by_B)y_A - c(x_A + y_A),$$

$$\pi_B = (a - x_B - bx_A)x_B + (a - y_B - by_A)y_B - c(x_B + y_B).$$

After the adoption of new technology, they are

$$\pi_A = (a - x_A - bx_B)x_A + (a - y_A - by_B)y_A - e_A,$$

$$\pi_B = (a - x_B - bx_A)x_B + (a - y_B - by_A)y_B - e_B.$$

We assume Cournot type behavior of the firms.

The conditions for profit maximization in Country A in the third stage of the game when both firms adopt new technology are

$$a - 2x_A - bx_B = 0, a - 2x_B - bx_A = 0.$$

The equilibrium outputs and prices are

$$x_A = x_B = \frac{a}{2+b}, p_A = p_B = \frac{a}{2+b}.$$

Similarly, the conditions for profit maximization in Country B are

$$a - 2y_A - by_B = 0, a - 2y_B - by_A = 0.$$

The equilibrium outputs and prices are

$$y_A = y_B = \frac{a}{2+b}, q_A = q_B = \frac{a}{2+b}.$$

The profits of Firm A and B are written as follows:

$$\hat{\pi}_A = \frac{2a^2}{(2+b)^2} - e_A, \pi_B = \frac{2a^2}{(2+b)^2} - e_B.$$

The conditions for profit maximization in Country A when only Firm A adopts the new technology are

$$a - 2x_A - bx_B = 0, a - 2x_B - bx_A - c = 0.$$

The equilibrium outputs are

$$x_A = \frac{(2-b)a + bc}{4-b^2}, x_B = \frac{(2-b)a - 2c}{4-b^2}.$$

The prices of the goods are

$$p_A = \frac{(2-b)a + bc}{4-b^2}, p_B = \frac{(2-b)a + (2-b^2)c}{4-b^2}.$$

Similarly, the conditions for profit maximization in Country B are

$$a - 2y_A - by_B = 0, a - 2y_B - by_A - c = 0.$$

The equilibrium outputs are

$$y_A = \frac{(2-b)a + bc}{4-b^2}, y_B = \frac{(2-b)a - 2c}{4-b^2}.$$

The prices of the goods are

$$q_A = \frac{(2-b)a + bc}{4-b^2}, q_B = \frac{(2-b)a + (2-b^2)c}{4-b^2}.$$

The profits of the firms are written as follows:

$$\pi_A^A = \frac{2[(2-b)a + bc]^2}{(4-b^2)^2} - e_A, \pi_B^A = \frac{2[(2-b)a - 2c]^2}{(4-b^2)^2}.$$

Similarly, the profits of the firms when only Firm B adopts new technology are written as follows:

$$\pi_A^B = \frac{2[(2-b)a - 2c]^2}{(4-b^2)^2}, \pi_B^B = \frac{2[(2-b)a + bc]^2}{(4-b^2)^2} - e_B.$$

The conditions for profit maximization in Country A when no firm adopts new technology are

$$a - 2x_A - bx_B - c = 0, a - 2x_B - bx_A - c = 0.$$

The equilibrium outputs and prices are

$$x_A = x_B = \frac{a-c}{2+b}, p_A = p_B = \frac{a + (1+b)c}{2+b}.$$

Similarly, the conditions for profit maximization in Country B are

$$a - 2y_A - by_B - c = 0, a - 2y_B - by_A - c = 0.$$

The equilibrium outputs and prices are

$$y_A = y_B = \frac{a-c}{2+b}, q_A = q_B = \frac{a + (1+b)c}{2+b}.$$

The profits of the firms are written as follows:

$$\pi_A^0 = \pi_B^0 = \frac{2(a-c)^2}{(2+b)^2}.$$

If

$$\hat{\pi}_A \geq \pi_A^B \text{ and } \pi_B \geq \pi_B^A,$$

that is

$$\frac{2a^2}{(2+b)^2} - e_i \geq \frac{2[(2-b)a-2c]^2}{(4-b^2)^2}, i \in \{A, B\},$$

the best response of Firm i (A or B) when the rival firm adopts new technology is the adoption of new technology. Then, we have

$$e_i \leq \frac{8c[(2-b)a-c]}{(4-b^2)^2}.$$

If

$$\pi_A^A \geq \pi_A^0 \text{ and } \pi_B^B \geq \pi_B^0,$$

that is

$$\frac{[2(2-b)a+bc]^2}{(4-b^2)^2} - e_i \geq \frac{2(a-c)^2}{(2+b)^2}, i \in \{A, B\},$$

the best response of Firm i when the rival firm does not adopt new technology is the adoption of new technology. Then, we have

$$e_i \leq \frac{8c[(2-b)a-(1-b)c]}{(4-b^2)^2}.$$

Let

$$e^0 = \frac{8c[(2-b)a-(1-b)c]}{(4-b^2)^2}, e^1 = \frac{8c[(2-b)a-c]}{(4-b^2)^2}$$

Since $0 < b < 1$, we have $e^0 > e^1$. Thus, we get the following lemma.

Lemma 1. When the goods are substitutes, the sub-game perfect equilibria after the second stage of the game are as follows.

1. If $e_i \leq e_j \leq e^1$, the sub-game perfect equilibrium is a state where both firms adopt new technology.
2. If $e_i \leq e^1 < e_j \leq e^0$, the sub-game perfect equilibrium is a state where only Firm i adopts new technology.

3. If $e_i \le e^1 < e^0 < e_j$, the sub-game perfect equilibrium is a state where only Firm i adopts new technology.
4. If $e^1 < e_i \le e^0 < e_j$, the sub-game perfect equilibrium is a state where only Firm i adopts new technology.
5. If $e^1 < e_i \le e_j \le e^0$, the sub-game perfect equilibrium is a state where only one firm (Firm i or j) adopts new technology.
6. If $e^0 < e_i \le e_j$, the sub-game perfect equilibrium is a state where no firm adopts new technology.

Proof

1. The adoption of new technology is a dominant strategy for both firms.
2. Adoption is a dominant strategy for Firm i, and non-adoption is the best response for Firm j to adoption by Firm i.
3. Adoption is a dominant strategy for Firm i, and non-adoption is a dominant strategy for Firm j.
4. Adoption is the best response for Firm i to non-adoption by Firm j, and non-adoption is a dominant strategy for Firm j.
5. Adoption is the best response for each firm to non-adoption by the rival firm, and non-adoption is the best response for each firm to adoption by the rival firm. Thus, only one firm (Firm i or j) adopts new technology.
6. Non-adoption is a dominant strategy for both firms.

3.2. Subsidy and tax policies

When both firms adopt new technology, the social welfare of Country A and that of Country B are

$$\hat{W}_A = W_B = \frac{(3+b)a^2}{(2+b)^2} - e.$$

When only Firm A adopts new technology, they are

$$W_A^A = \frac{24a^2 + 2a^2b^3 - 2a^2b^2 - 16a^2b - 2ab^3c - 2ab^2c + 16abc - 8ac + b^2c^2 + 4c^2}{2(4-b^2)^2} - e,$$

$$W_B^A = \frac{24a^2 + 2a^2b^3 - 2a^2b^2 - 16a^2b - 2ab^3c + 6ab^2c + 16abc - 40ac - 3b^2c^2 + 20c^2}{2(4-b^2)^2}.$$

When only Firm B adopts new technology, they are

$$W_A^B = \frac{24a^2 + 2a^2b^3 - 2a^2b^2 - 16a^2b - 2ab^3c + 6ab^2c + 16abc - 40ac - 3b^2c^2 + 20c^2}{2(4-b^2)^2},$$

$$W_B^B = \frac{24a^2 + 2a^2b^3 - 2a^2b^2 - 16a^2b - 2ab^3c - 2ab^2c + 16abc - 8ac + b^2c^2 + 4c^2}{2(4-b^2)^2} - e.$$

When no firm adopts new technology, they are

$$W_A^0 = W_B^0 = \frac{(3+b)(a-c)^2}{(2+b)^2}.$$

Let

$$e_w^1 = \hat{W}_A + e - W_A^B = W_B + e - W_B^A = \frac{(40a - 6ab^2 - 16ab + 3b^2c + 2ab^3 - 20c)c}{2(4-b^2)^2},$$

$$e_w^0 = W_A^A + e - W_A^0 = W_B^B + e - W_B^0 = \frac{(40a + 2ab^3 - 6ab^2 - 16ab - 2b^3c + 3b^2c + 16bc - 20c)c}{2(4 \quad {}^2)^2}.$$

Since $0 < b < 1$, we see

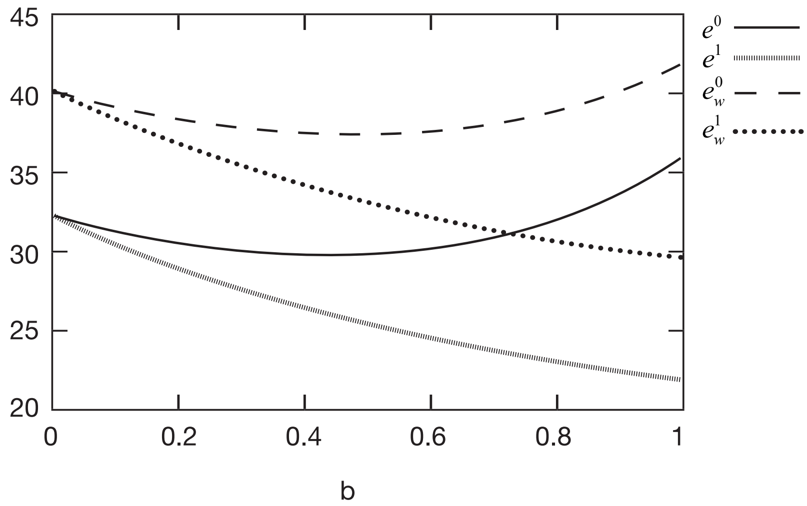
$$e_w^0 - e_w^1 = \frac{(8-b^2)bc^2}{(4-b^2)^2} > 0.$$

Then, we obtain the following lemma.

Lemma 2.

1. If $e \le e_w^1$, both countries want the adoption of new technology by their firms.
2. If $e_w^1 < e \le e_w^0$, each country wants the adoption of new technology by its firm when the firm in the other country does not adopt, and it wants non-adoption when the firm in the other country adopts.
3. If $e > e_w^0$, both countries want non-adoption of new technology by their firms.

FIGURE 1. The graphs of e^0, e^1, e_w^0 and e_w^1: $a=10$, $c=4$, $0<b<1$



For reasonable values of parameters, a, c and b, we can show (in Figure 1)

$$e_w^0 - e^0 > 0, e_w^0 - e^1 > 0, e_w^1 - e^1 > 0.$$

However,

$$e_w^1 - e^0 = \frac{(8a + 2ab^3 - 6ab^2 + 3b^2c - 16bc - 4c)c}{2(4-b^2)^2}$$

may be positive or negative. For example, if $a=10$, $c=4$, it is negative for $0.73 \le b < 1$. Thus, the more substitutable the goods of the firms are, the more likely negative the value of $e_w^1 - e^0$ is.

About the equilibrium policies we get the following theorem.

Theorem 1.

1. When $e_w^1 \ge e^0$, the equilibria of the game between two countries are as follows.
   (1) If $e \le e^1$, both governments do nothing.
   (2) If $e^1 < e \le e^0$, both countries give subsidies to their firms. The level of the subsidy in each country must not be smaller than $e - e^1$.
   (3) If $e^0 < e \le e_w^1$, both countries give subsidies to their firms. The level of the subsidy in each country must not be smaller than $e - e^1$.

(4) If $e_w^1 < e \le e_w^0$, one country, A or B, gives a subsidy to its firm. The level of the subsidy must not be smaller than $e - e^0$.

(5) If $e > e_w^0$, both governments do nothing.

2. When $e_w^1 < e^0$, the equilibria of the game between two countries are as follows.
   (1) If $e \le e^1$, both governments do nothing.
   (2) If $e^1 < e \le e_w^1$, both countries give subsidies to their firms. The level of the subsidy in each country must not be smaller than $e - e^1$.
   (3) If $e_w^1 < e \le e^0$, there are three types of equilibria.
      i. The government of each country should do nothing.
      ii. Only one country should impose a tax on its firm. The level of the tax must be larger than $e^0 - e$.
      iii. Only one country should give a subsidy to its firm. The level of the subsidy must not be smaller than $e - e^1$.
   (4) If $e^0 < e \le e_w^0$, one country, either A or B, gives a subsidy to its firm. The level of the subsidy must not be smaller than $e - e^0$.
   (5) If $e > e_w^0$, both governments do nothing.

Proof

1. (1) In this case, both firms adopt the new technology without subsidy or tax, and both countries want the adoption of new technology by their firms. Thus, each country should do nothing.
   (2) In this case, although both countries want the adoption of new technology by their firms, only one firm adopts new technology without subsidy or tax (Case 5 of Lemma 1). Then, both countries should give subsidies to their firms. If only one country gives a subsidy, we obtain Case 2 of Lemma 1 and only one firm adopts new technology. The level of the subsidy in each country must not be smaller than $e - e^1$.
   (3) In this case, although both countries want the adoption of new technology by their firms, no firm adopts new technology without subsidy or tax. Thus, both countries should give subsidies to their firms. The level of the subsidy in each country must not be smaller than $e - e^1$.
   (4) In this case, although one country (A or B) wants the adoption of new technology by its firm, no firm adopts new technology without subsidy or tax. Thus, one country gives a subsidy to its firm. The level of the subsidy must not be smaller than $e - e^0$. It makes Case 4 of Lemma 1 emerge. It is sufficient for one of the firms to adopt new technology.
   (5) In this case, no firm adopts new technology without subsidy or tax, and both countries do not want the adoption of new technology by their firms. Thus, each country should do nothing.

2. (1) Its proof is the same as the proof of (1) of 1.
   (2) Its proof is the same as the proof of (2) of 1.
   (3) In this case, at the equilibrium only one firm adopts new technology, and only one country wants the adoption of new technology by its firm. There are three types of equilibria.
      i. If Country A (or B) wants adoption and Firm A (or B) adopts, the equilibrium policy is to do nothing.
      ii. If Country A (or B) wants adoption and Firm B (or A) adopts, Country B (or A) imposes a tax on its firm to prevent the adoption of new technology by its firm. The level of the tax must be larger than $e^0 - e$. Then, Case 4 of Lemma 1 emerges, and only Firm A (or B) has an incentive to adopt.
      iii. If Country A (or B) wants adoption and Firm B (or A) adopts, Country A (or B) gives a subsidy to its firm. The level of the subsidy must not be smaller than $e - e^1$. Then, Case 2 of Lemma 1 emerges, and only Firm A (or B) adopts new technology.
   (4) Its proof is the same as the proof of (4) of 1.
   (5) Its proof is the same as the proof of (5) of 1.

4. A case of complements

4.1. Firm behavior

Next, we consider a case where the goods are complements. Assume $e_i \leq e_j, i, j \in \{A, B\}, j \neq i$. Since $b < 0$, we have $e^0 < e^1$. Thus, we obtain the following lemma.

Lemma 3. If the goods are complements, the sub-game perfect equilibria in the game after the second stage are as follows.

3. If $e_i \leq e_j \leq e^0$, the sub-game perfect equilibrium is a state where both firms adopt new technology.
4. If $e_i \leq e^0 < e_j \leq e^1$, the sub-game perfect equilibrium is a state where both firms adopt new technology.
5. If $e_i \leq e^0 < e^1 < e_j$, the sub-game perfect equilibrium is a state where only Firm i adopts new technology.
6. If $e^0 < e_i \leq e_j \leq e^1$, there are two sub-game perfect equilibria. One is a state where no firm adopts new technology, and the other is a state where both firms adopt new technology.
7. If $e^0 < e_i \leq e^1 < e_j$, the sub-game perfect equilibrium is a state where no firm adopts new technology.
8. If $e^1 < e_i \leq e_j$, the sub-game perfect equilibrium is a state where no firm adopts new technology.

Proof

1. The adoption of new technology is a dominant strategy for both firms.
2. Adoption is a dominant strategy for Firm i, and adoption is the best response for Firm j to adoption by Firm i.
3. Adoption is a dominant strategy for Firm i, and non-adoption is a dominant strategy for Firm j.
4. Adoption is the best response for each firm to adoption by the rival firm, and non-adoption is the best response for each firm to non-adoption by the rival firm. Thus, there are two subgame perfect equilibria. One is a state where no firm adopts new technology, and the other is a state where both firms adopt new technology.
5. Non-adoption is the best response for Firm i to non-adoption by Firm j, and non-adoption is a dominant strategy for Firm j.
6. Non-adoption is a dominant strategy for both firms.

4.2. Subsidy or tax policies

Since $b<0$, we have

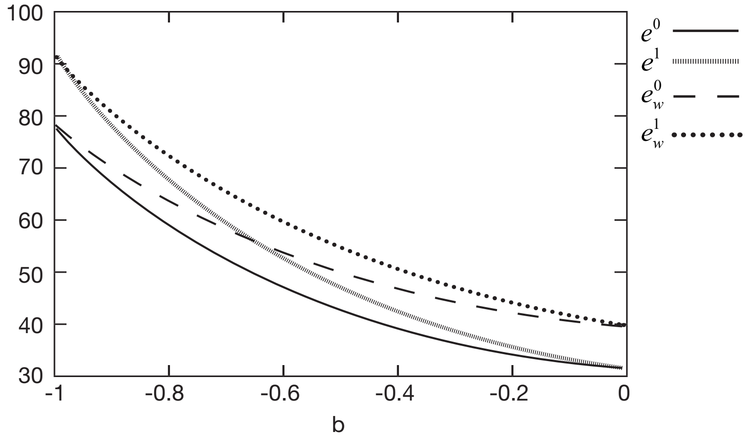
$$e_w^0 - e_w^1 = \frac{(8-b^2)bc^2}{(4-b^2)^2} < 0.$$

About the social welfare of the countries Lemma 2 is modified as follows:

Lemma 4. If the goods are complements,

1. If $e \le e_w^0$, both countries want the adoption of new technology by their firms.
2. If $e_w^0 < e \le e_w^1$, each country wants the adoption of new technology by its firm when the firm in the other country adopts, and it wants non-adoption when the firm in the other country does not adopt.
3. If $e > e_w^1$, both countries want non-adoption of new technology by their firms.

FIGURE 2. The graphs of e^0, e^1, e_w^0 and e_w^1: $a=10$, $c=4$, $-1<b<0$



For reasonable values of parameters, a, c and b, we can show (in Figure 2)

$$e_w^0 - e^0 > 0, e_w^1 - e^0 > 0.$$

However,

$$e_w^0 - e^1 = \frac{(8a + 2ab^3 - 6ab^2 - 2b^3c + 3b^2c + 16bc - 4c)c}{2(4 - b^2)^2}$$

may be positive or negative. For example, if $a = 10$, $c = 4$, it is negative for $-1 < b \leq -0.65$. Thus, the more complementary the goods of the firms are, the more likely negative the value of $e_w^0 - e^1$ is. Also

$$e_w^1 - e^1 = \frac{(8a + 3b^2c + 2ab^3 - 6ab^2a - 4c)c}{2(4 - b^2)^2}$$

may be positive or negative. If $a = 10$, $c = 4$, it is negative for $-1 < b \leq -0.974$. Thus, if the goods are extremely complementary, $e_w^1 - e^1$ may be negative. About the equilibrium policies we get the following theorem.

Theorem 2. If the goods are complements, the equilibrium of the game between two countries are as follows.

1. When $e_w^0 \geq e^1$ and $e_w^1 > e^1$, the equilibria of the game between two countries are as follows.
   (1) If $e \leq e^0$, both governments do nothing.

   (2) If $e^0 < e \leq e^1$, there are three sub-game perfect equilibria. The first is a state where the governments should do nothing. The second is a state where both countries give subsidies to their firms. And the third is a state where one country gives a subsidy to its firm. The level of the subsidy in each country in the latter two cases must not be smaller than $e - e^0$. Both firms adopt new technology even when only one country gives a subsidy to its firm.

   (3) If $e^1 < e \leq e_w^0$, both countries give subsidies to their firms. The level of the subsidy in each country must not be smaller than $e - e^0$.

   (4) If $e_w^0 < e \leq e_w^1$, there are two sub-game perfect equilibria. One is a state where both countries give subsidies to their firms, and the other is a state where both governments do nothing. The level of the subsidy in each country in the former case must not be smaller than $e - e^0$.

   (5) If $e > e_w^1$, both governments do nothing.

2. When $e_w^0 < e^1$ and $e_w^1 > e^1$, the equilibria of the game between two countries are as follows.
   (1) If $e \leq e^0$, the equilibrium is the same as that in (1) of 1.
   (2) If $e^0 < e \leq e_w^0$, the equilibrium is the same as that in (2) of 1.
   (3) If $e_w^0 < e \leq e^1$, there are three types of equilibria.
      i. Both governments should do nothing.
      ii. Both governments should impose taxes on their firms to prevent the adoption of new technology. The level of the tax in each country must be larger than $e^1 - e$.
      iii. Both governments give subsidies to their firms. The level of the subsidy in each country must not be smaller than $e - e^0$.
   (4) If $e^1 < e \leq e_w^1$, the equilibrium is the same as that in (4) of 1.
   (5) If $e > e_w^1$, the equilibrium is the same as that in (5) of 1.

3. When $e_w^0 < e^1$ and $e_w^1 \leq e^1$, the equilibria of the game between two countries are as follows.
   (1) If $e \leq e$, the equilibrium is the same as that in (1) of 1 and 2.
   (2) If $e^0 < e \leq e_w^0$, the equilibrium is the same as that in (2) of 1 and 2.
   (3) If $e_w^0 < e \leq e_w^1$, the equilibrium is the same as that in (3) of 2.
   (4) If $e_w^1 < e \leq e^1$, both governments do nothing, or both governments impose taxes on their firms.
   (5) If $e > e^1$, the equilibrium is the same as that in (5) of 1 and 2.

Proof

1. (1) In this case, both firms adopt new technology without subsidy or tax, and both countries want the adoption of new technology by their firms. Thus, each country should do nothing.
   (2) In this case, both countries want the adoption of new technology by their firms. On the other hand, there are two sub-game perfect equilibria in the game after the second stage without subsidy or tax. One is a state where both firms adopt new technology, and the other is a state where no firm adopts new technology (Case 4 of Lemma 3). In the first case, the governments should do nothing. Consider the latter case. If one country gives a subsidy to its firm so that it adopts new technology, the firm in the other country has an incentive to adopt new technology. Then, both firms adopt new technology even when only one country gives a subsidy to its firm. However, since both countries have incentives to give subsidies, subsidization by both countries and subsidization by only one country are sub-game perfect equilibria. The level of the subsidy in at least one of the countries must not be smaller than $e - e^0$.
   (3) In this case, although both countries want the adoption of new technology by their firms, no firm adopts new technology without subsidy or tax. Thus, both countries should give subsidies to their firms. The level of the subsidy in each country must not be smaller than $e - e^0$.

(4) In this case, no firm adopts new technology without subsidy or tax. On the other hand, both countries want the adoption of new technology by their firms, or no country wants adoption by its firm. Thus, there are two sub-game prefect equilibria. One is a state where both countries give subsidies to their firms, and the other is a state where no country gives a subsidy to its firm. The level of the subsidy in each country in the former case is[1] $e-e^0$.

(5) In this case, no firm adopts new technology without subsidy or tax, and both countries do not want the adoption of new technology by their firms. Thus, each country should do nothing.

2. (3) In this case, two firms adopt or no firm adopts new technology after the second stage of the game. Also, in the first stage, two countries want the adoption of new technology by their firms, or no country wants adoption. Thus, there are three types of equilibria.

   i. i. If both firms adopt new technology and both countries want adoption, or no firm adopts new technology and no country wants adoption. Then, both governments should do nothing.

   ii. ii. If both firms adopt new technology, and no country wants adoption. Then, both governments should impose taxes on their firms to prevent the adoption of new technology. The level of the tax in each country must be larger than e^1-e.

   iii. iii. If no firm adopts new technology, and both countries want adoption. Then, both governments give subsidies to their firms. The level of the subsidy in each country must not be smaller than $e-e^0$.

3. (4) In this case, no country wants the adoption of new technology. On the other hand, two firms adopt or no firm adopts after the second stage of the game. In the latter case both governments should do nothing. In the former case both governments should impose taxes on their firms. The level of the tax in each country must be larger than e^1-e.

4.2. Payoff dominance and correlated strategies in (4) of 1 in Theorem 2

In (4) of 1 in Theorem 2, there are two sub-game perfect equilibria. One is a state where both countries give subsidies to their firms, and the other is a state where no country gives a subsidy.

Let us compare the social welfare in each country between two equilibria. Comparing $\hat{W}_A(=W_B)$ and $W_A^0(=W_B^0)$,

$$\Theta(a) = \hat{W}_A - W_A^0 = \frac{(3+b)(2ac-c^2)}{(2+b)^2} - e.$$

Since e is not larger than e_w^1 we have

$$\Theta(a) \geq \frac{(8a-2b^3c-b^2c+16bc-4c+2ab^3+2ab^2-16ab)c}{2(4-b^2)^2}.$$

[1] In (2), (3), and (4) of 1 and 3-iii of 2, the subsidy by one of the governments may be $e-e^1$, which is smaller than $e-e^0$. However, the large subsidy does not change the social welfare because subsidies and lump-sum taxes are canceled out in the social welfare. In these cases, the subsidy by the other government must not be smaller than $e-e^0$.

When $a = c$, we get

$$\Theta(a) = \frac{(4+b^2)c^2}{2(4-b^2)^2} > 0.$$

Since $-1 < b < 0$ and $a > c$, we have

$$\frac{\partial \Theta(a)}{\partial a} = \frac{8 + 2b^3 + 2b^2 - 16b}{2(4-b^2)^2} > 0.$$

Therefore, $\Theta(a) > 0$, and the equilibrium of subsidization by both countries is payoff dominant to the equilibrium of subsidization by no country. If there are some correlated strategies, the former equilibrium can be realized.

Similarly, in (4) of 2 in Theorem 2, the equilibrium where both firms adopt is payoff dominant to the equilibrium where no firm adopts.

5. Concluding remarks

In this paper, we analyzed a game of subsidization for new technology adoption between two countries in an international duopoly. The optimal policies for each country and the equilibria of a game between two countries are not simple. They depend on the magnitude of the set-up costs and whether or not the goods of firms are substitutes or complements. Many equilibria are states with subsidies to the firms or inaction of the governments. However, a part of equilibria are states with taxes on the firms. The tax cases emerge when $e_w^1 < e^0$ or $e_w^0 < e^1$ or $e_w^1 < e^1$. Theorems 1 and 2 have shown that the more substitutable or more complementary the goods of the firms are, the more likely the equilibrium policies are taxation.

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