



UP School of Economics Discussion Papers

Discussion Paper No. 2014-13

September 2014

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Abstract

As a proxy for a Pareto-efficient market economy, we adopt the two-party Nash Bargaining model featuring a qualitative bias in the treatment of the contributions of the parties. The Picketty inequality here is the share in total welfare accruing to the richer party over total welfare attained at agreement point. We show that this inequality can never exceed the inequality in initial contributions if the qualitative bias is zero. The rising Picketty inequality requires that the qualitative bias exceed a positive threshold. The Picketty trajectory emerges if the qualitative bias oscillates around the threshold due to changing social and economic environment.

JEL Classification: C78, D31, D63, D 71

Key Words: Picketty, Inequality, Nash Bargaining

I. Introduction

The Picketty Thesis (Picketty, 2014) that income and wealth inequality measured by the income share of the top decile or percentile tends in normal times to rise monotonically in capitalist economies has taken the world by storm. The actual Picketty trajectory has income and wealth inequality rising from around 1810 to about 1910 when it starts a downward trajectory until about 1970, and then starts to rise again to the present. This runs in the face of the Kuznets Hypothesis (Kuznets, 1955; see also Ray, 1998; also Ray, 2014) that inequality will first, of itself, rise in the process of development as income grows, reach a peak and finally fall as the economy reaches maturity. The period studied by Kuznets, 1910-1948, indeed showed a declining inequality in the USA. Picketty argues that this interregnum of falling inequality was the exception due to the wealth and capital destruction brought about by the two world wars and the subsequent recovery, especially the Thirty Glorious Years of exceptionally high growth after World War II. The book is being subjected to close scrutiny but the weight of evidence thus far suggests that Picketty's empirics is overall robust and will stand further scrutiny (Krugman, May 2014; Picketty, 2014; Giles, 2014; Winship, 2014). Picketty argues further from the perspective of growth theory that this is due to the return to capital being systematically higher than the growth of income ($r > g$) and not due to any market failure. The adequacy of this explanation has been disputed by Ray (2014) who shows that the inequality $r > g$ is a staple of growth models satisfying dynamic efficiency and has little to do with inequality. Ray views the demonstrated capacity of capital to progressively substitute for labor as the main explanation for the growing Picketty inequality.

This paper departs from the growth theoretic and factorial share perspective and adopts instead a social bargaining contract perspective where the distribution of the economic surplus depends explicitly on the relative power of the actors. This tack is hinted in Piketty's (2014) Introduction, p. 20:

"The history of inequality is shaped by the way economic, social and political actors view what is just and what is not, as well as the relative powers of those actors and the collective choices that result."

For this purpose, the paper adopts as social contract the Nash Bargaining paradigm. Apart from explicitly recognizing the role of the parties' reservation utilities, it also doubles as a two-party proxy for the well-functioning market economy that Piketty favors. He makes it clear that the explanation he offers is not due to any market imperfection.¹ Note that the Nash bargaining solution satisfies Pareto efficiency, individual rationality and feasibility, which are also satisfied by a perfectly competitive market.

The paper additionally introduces a qualitative bias $q \geq 0$ in the treatment of initial reservation utilities of the parties to reflect the different capacities implicit in the initial contributions. Qualitative bias may be understood as issuing from the capacity of one party's contribution (say, capital) to substitute for the contribution of the other (say, labor) or from the imbalance in the absolute sizes of the contributions allowing thereby access to size-related scale economies or from the fact that one party's contribution is more mobile than the other party's thereby eliciting premium treatment.

In Section II we introduce the Nash bargaining paradigm and the concept of qualitative bias q . Two types of Nash social contract arise: unbiased with $q = 0$, and biased with $q > 0$. In Section III, we then define the Piketty inequality in terms of total welfare both at agreement and disagreement point. We show that the Piketty thesis of rising inequality in total welfare is impossible under the unbiased Nash bargaining contract ($q = 0$). A rising Piketty inequality emerges only when the qualitative bias in favor of one party's contribution is higher than a threshold value q^* . The actual Piketty inequality trajectory of rising and falling inequality can be reproduced by the oscillation over time of the qualitative bias above and below a threshold due to changes, as Piketty puts it, "in the way economic, social and political actors view what is just and what is not..." Both these results occur within the ambit of Nash Bargaining and thus not due to market imperfection.

II. The Nash-Bargained Social Contract

Consider a polity consisting of two non-intersecting groups: Patricians (P) exhibiting utility U and Plebeians (p) exhibiting utility V . The two groups form a partnership in some potentially welfare-increasing project. The Patricians bring initial resources equivalent to U^0 in utility terms to the project,

¹ Piketty (2014), Introduction, page 27

and the Plebeians bring the equivalent of V^0 . We assume that the parties have agreed to adopt the Nash bargaining solution to determine the allocation of the welfare harvest of the partnership.

Following Fabella (1991), we consider two players P and p coming to the exchange each with a basket of resources $T_p^0, T_p^0 \geq 0$. Let T be the set of all possible combinations including convex combinations that can be formed out of the initial contributions. Endow each player with a Von Neumann-Morgenstern utility which is unique up to linear transformation. Each point in T then maps into a point in the set of utility profiles (the utility possibility set, for short), $S \in R_+^2$, S is closed, bounded and convex. (U^0, V^0) is the image in S of (T_p^0, T_p^0) in T . We call U^0 the maximin value of the game to player 1 and V^0 the maximin value to player 2. The maximin value is the minimum value that a player will accept since he can realize this unilaterally. We assume that there is a point $(U, V) \in S$, with $U > U^0$, and $V > V^0$. A concise summary of the game is (S, U^0, V^0) . As is well-known, the Nash Bargaining solution satisfies the following Nash axioms:

N1 (*Individual Rationality*): $(U^*, V^*) \geq (U^0, V^0)$;

N2 (*Feasibility*): $(U^*, V^*) \in S$;

N3 (*Pareto Optimality*): If $(U, V) \in S$ and $(U, V) \geq (U^*, V^*)$, then $(U, V) = (U^*, V^*)$;

N4 (*Independence of Irrelevant Alternatives*): If $(U^*, V^*) \in F \subset S$ and (U^*, V^*) solves (S, U^0, V^0) , then (U^*, V^*) solves (F, U^0, V^0) ;

N5 (*Independence of Linear Transformation*): Let H be obtained from S by the linear transformation: $U' = a_1 U + b_1, V' = a_2 V + b_2, a_i, b_i \in R^1, i = 1, 2$. Let (U^*, V^*) solve (S, U^0, V^0) . Then $(a_1 U^* + b_1, a_2 V^* + b_2)$ solves $(H, a_1 U^0 + b_1, a_2 V^0 + b_2)$.

N6 (*Symmetry*): If S is symmetric, i.e., $(U, V) \in S$ and $(V, U) \in S$, and $U^0 = V^0$, then $U^* = V^*$.

The Nash bargaining solution is a two-player proxy for the competitive market because both satisfy Individual Rationality, Pareto Optimality and Feasibility. The Nash Bargaining solution satisfies additional conditions which together insure uniqueness. The competitive market equilibrium also satisfies uniqueness only under an additional strong assumption regarding the Jacobian matrix. In the foregoing we will introduce an additional asymmetry in the form of a qualitative bias accorded the original contributions of the players.

The Nash bargaining solution is

$$(U^*, V^*) = \operatorname{argmax} \left(U - U^0(1+q) \right) \left(V - (V^0 - qU^0) \right), q \geq 0. \quad (1)$$

(U^*, V^*) uniquely satisfies the Nash axioms above. Assuming a smooth and differentiable Utility Possibility Frontier ($UPF \in S$), which is representable by equation $V(U)$, with $V' < 0$. (U^*, V^*) solves the 1^o necessary condition:

$$(V^* - (V^0 - qU^0)) = (U^* - (1+q)U^0)(-V'). \quad (2)$$

For convenience, let $a = (-V') > 0$. U^0 and V^0 as the utility equivalent of the original resource contribution of P and p , respectively, to the partnership endeavor. But P 's bargaining position within the partnership is $(1+q)U^0$. It has two parts: the original resource contribution part U^0 and the internally settled qualitative bias q , $0 \leq q < 1$. Any $q > 0$ improves the bargaining power of P in the agreement point; but P takes out just its original contribution U^0 if no agreement results. The same goes for p . Note that if $q = 0$, we are back to the original or unbiased Nash bargaining solution. If $q > 0$, it implies that P 's resource contribution is considered more valuable than p 's resource contribution within the partnership. P may be contributing more mobile and more scarce capital and p contributing immobile and more abundant labor. P 's resource contribution may also be larger and may thus allow access to valuable scale-economies. In Cincinnatus' Rome circa 200 BC, the more affluent Patricians contributed armor, weaponry, logistics, horses and pack animals, and the Plebeians contributed warm bodies to the collective projects such as the war effort. Even then, capital already had the capacity to substitute for labor. Now, of course, the use of armed drones completely eliminates live pilots in aerial military operations.

Notice that if the UPF is symmetric and $(1+q)U^0 = V^0 - qU^0$, then $(-V') = 1$ and $U^* = V^*$ as required by the symmetry axiom. We now rewrite equation (2) as

$$U^* = [V^* - (V^0 - qU^0)]a^{-1} + (1+q)U^0, \quad (3)$$

which gives U^* in terms of V^* . We let $V^0 - U^0 / m$, where $m \geq 1$ and $m > 1$ means that P is more affluent than p . We also assume that m is large enough so that $am > 1$. In the case of an agreement, total Nash-bargained welfare is $(U^* + V^*)$. The total welfare at disagreement point is $(U^0 + (U^0 / m))$. Re-writing (3) we get

$$U^* = [V^* - (U^0 / m)(1 - qm)]a^{-1} + (1+q)U^0. \quad (4)$$

III. The Piketty Inequality in Welfare Terms

We define the Piketty inequality as the share of P in total welfare.

Definition 1: The Piketty Inequality in welfare terms at

$$(i) \quad \text{Agreement point } (U^*, V^*) \text{ is } S^P = U^*(U^* + V^*)^{-1} \quad (5)$$

$$(ii) \quad \text{Disagreement (Resource Contribution) point } (U^0, V^0) \text{ is } S^0 = U^0(U^0 + V^0)^{-1}. \quad (6)$$

Definition 2: We say that the Piketty Inequality in welfare terms rises if $S^P > S^0$.

We have the following:

Lemma: A necessary condition for $S^P > S^0$ is that

$$(U^0 / m)(V^*)^{-1} > (am - 1)[(am - 1) + qm(a - 1)]^{-1}. \quad (7)$$

Proof: We now rewrite equation 5 in terms of (4) as:

$$S^P = \left\{ \left[V^* - (U^0 / m)(1 - qm) \right] a^{-1} + (1 + q)U^0 \right\} \left\{ V^*(1 + a)a^{-1} - U^0(1 - qm)(am)^{-1} + (1 + q)U^0 \right\}^{-1}. \quad (8)$$

We now rewrite equation (6) as:

$$S^0 = m(1 + m)^{-1}. \quad (9)$$

Thus, $S^P > S^0$ implies:

$$\left\{ \left[V^*a^{-1} - U^0(1 - qm) \right] (ma)^{-1} + (1 + q)U^0 \right\} \left\{ V^*(1 + a)a^{-1} - U^0(1 - qm)(ma)^{-1} + (1 + q)U^0 \right\}^{-1} > m(1 + m)^{-1}$$

or

$$V^*a^{-1} - (1 + m)U^0(1 - qm)(am)^{-1} + (1 + m)(1 + q)U^0 > mV^*(1 + a)a^{-1} - U^0(1 - qm)(a)^{-1} + m(1 + m)U^0.$$

And furthermore,

$$(1 + m)V^*a^{-1} - (1 + m)U^0(1 - qm)(am)^{-1} + (1 + m)(1 + q)U^0 >$$

$$mV^*(1 + a)a^{-1} - U^0a^{-1}(1 - qm) + m(1 + q)U^0.$$

Gathering U^0 -associated expressions on the left-hand side and V^0 -associated ones on the right-hand side of the inequality, we get

$$(U^0 / am) \left[(1 - qm)m - am(1 + q)m - (1 + m)(1 - qm) + (1 + m)(1 + q)am \right] > (V^* / a)(ma - 1).$$

Simplifying,

$$(U^0 / am) [(1+q)am - (1-qm)] > (V^* / a)(ma-1),$$

or simplifying further,

$$\left[(U^0 / m) / V^* \right] > (ma-1) / [(ma-1) + qm(1+a)], \quad (10)$$

which is as claimed. Q.E.D.

Since $(1-qm) > 0$, we have

$$\left[(U^0 / m)(1-qm) / V^* \right] > (ma-1)(1-qm) / [(mq-1) + qm(1+a)]. \quad (11)$$

Note that the left hand side of the inequality (11) is the ratio of the (biased or diminished) maximin and the utility at agreement point. We show that a rising Piketty inequality is impossible under an unbiased Nash Bargaining contract.

Proposition 1: It is impossible for the Piketty inequality in welfare terms to rise under the unbiased ($q = 0$) Nash-bargained social contract. In fact, it always falls.

Proof: If $(q = 0)$, (10) reduces to $(U^0 / m) / V^* > (ma-1) / (ma-1) = 1$. But this violates the Nash axiom of Individual Rationality. Thus, impossible. Individual Rationality and $(q = 0)$ together is consistent only with $S^P < S^0$ or the Piketty inequality in welfare terms always falls. QED

For initial inequality only to be maintained, it is necessary that q be strictly positive.

Definition: The qualitative bias $q > 0$ is inequality-neutral if $S^P = S^0$.

The following is obvious from (10).

Corollary 2: The inequality-neutral level of q , q^* , equals

$$q^* = \left[V^* (U^0 / m)^{-1} - 1 \right] \left[(am-1)(1+a)m \right]^{-1}. \quad (12)$$

Proof: Setting $S^P = S^0$ results in equation (10) being an equality and solving for $q = q^*$ gives

$$q^* = \left\{ V^* (U^0 / m)^{-1} - 1 \right\} (am-1) (m(1+a))^{-1}. \quad \text{Q.E.D.} \quad (13)$$

We have the following :

Proposition 2: Under the biased Nash bargained social contract, the necessary and sufficient condition for a rising Piketty inequality in welfare terms ($S^P = S^0$) is $q > q^*$.

Proof: (Sufficiency): Corollary 1 shows that ($S^P = S^0$) implies inequality (11) which, by the definition of q^* , further implies that $q > q^*$.

(Necessity): Suppose $q > q^*$. Then inequality (11) follows from *Corollary 2* and working backwards we get ($S^P > S^0$). QED

The rise in the Piketty inequality requires a biased treatment of the contributions of the parties in favor of the Patrician. But not any bias ($q > 0$) will suffice to bring a rise. The bias must be large enough ($q > q^*$). If such biased treatment of resource contribution is absent ($q = 0$ or $q < q^*$), the Piketty inequality cannot rise and indeed must fall at agreement point.

IV. Discussion

Within the context of the bargaining game itself, the qualitative bias can emerge in a pre-bargaining phase where the rules of negotiation are adopted. Aside from agreeing to adopt the Nash axioms, the parties must also agree on the qualitative bias q to be assigned – effectively the social valuation of the maximin positions. This latter seems born out today in the disagreement on the parameters of negotiation in the Israel-Gaza conflict or on how to value the maximin positions of the parties.

Our sense is that how the maximin positions are valued is intimately related to earlier alluded to proposed economic explanations for the rising Piketty inequality. The capacity of the first party's contribution (say capital) to progressively substitute for the second party's contribution (say labor) is one factor identified (Ray, 2014). This leads to a decreasing bargaining power of the second party. Likewise, that the Patrician party brings to the table a larger absolute contribution can mean that the partnership is then enabled to access economies of scale otherwise out of reach. The Patricians may also be able to hold out longer in case of an impasse. Society may thus accord it premium treatment just as banks treat large premium borrowers more kindly. It is also possible that the Patrician's contribution say capital is more mobile and thus can play one jurisdiction against another in its choice of partner.

Finally and no less importantly, the bias may fluctuate depending on the overall political sentiment of the collective over time. In the interwar- and post-World War II period into the 1970s, the apparent success of the Soviet Union sold as the workers' paradise and its challenge to the capitalist world forced western governments towards policies that favored labor claims and limited the capitalists'

reach such as the nationalization of the “commanding heights” in Great Britain and elsewhere (Yergin and Stanislaw, 1998). Economic sentiment also shifted mightily after J. M. Keynes (1936), responding to the Great Depression and meaning to save Capitalism from its own excesses, made employment creation the prime duty of the state, a doctrine that gripped the world and started to wane only after the 1970s. Despite their work being a paean to markets and globalization, Yergin and Stanislaw’s expressed concern for the need to close the inequality gap in market economies anticipated Piketty’s:

“The market also requires something else: legitimacy. But here it faces an ethical conundrum. It is based upon contracts, rules, and choice – in short, on self-restraint – which contrasts mightily with other ways of organizing economic activity. Yet a system that takes the pursuit of self-interest and profit as its guiding light does not necessarily satisfy the yearning in the human soul for belief and some higher meaning beyond materialism.”

In the course of time, the Piketty inequality may rise or fall if q goes from being above to being below q^* . That is, there may be economic and political environments which result in $q > q^*$ and others which result in $q < q^*$. The period from 1910 to 1970 hosted a great many of the military and political upheavals that rendered $q < q^*$, resulting in a decreasing Piketty inequality. The period from 1970 to the present reversed the previous weights accorded the contributions of workers and capitalists and inequality rose.

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