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Abstract

J. Harsanyi introduced structural polymorphism in game theory, that is, there are many possible agent types such as “low productivity” or high productivity” with corresponding probability but all operating under one behavioral type, strict rationality. In this paper, we introduce behavioral polymorphism into Bayesian games. The multiplicity of behavioral types have become increasingly recognized and studied. Agents ascribe to each other a probability distribution across the possible types. They then choose the appropriate type as response to the possible type of the others which type determines the choice of strategy. We show in a dimorphic game model with the two types being strict rationality (*SR*) and utilitarian altruist (*UA*) that there always is a high enough assignment such that cooperation is the dominant strategy for both players in initially social dilemma games. Thus, the strategy set is endogenous in games with behavioral polymorphism. We argue that the assignment is based on some heuristics such as the counter-parties’ membership in some groups.

JEL Classification: C70, C72

Key words: behavioral polymorphism, Bayesian games, cooperation, dominant strategy

I. Introduction

Harsanyi (1967, 1968) introduced multiple possible player types. Each player's type is private information which is known to others only up to a probability distribution. This probability distribution is as it were derived from nature. These different types, however, all come under the rubric of one behavioral type, strict rationality (SR); a player may be either "low cost" or "high cost" but strict rationality is maintained across types. The polymorphism that was introduced by Harsanyi was thus structural rather than behavioral.

Our interest here is the introduction of multiple behavioral types of which SR is just one. Many other behavioral types have emerged as objects of scholarly curiosity in evolutionary and experimental game theory (see, e.g., Ostrom, 2000): altruism, reciprocal altruism, conditional cooperation, spitefulness, etc. Each player's behavioral type is private information known to others only up to a probability distribution. This probability distribution is subjective and is ascribed to each player by his/her counterparties. How this impacts on the behavior of players in strategic games is of interest and may shed additional insights on the enduring social science question of the emergence of cooperation.

II. The Dimorphic Model

In this introductory note, the original game G is a social dilemma game where the two players are (A, B) each facing a common strategy set $(\text{Cooperate}, \text{Defect}) = (C, D)$. We assume behavioral dimorphism, that is, there are only two behavioral types in the set of types $T = \{UA, SR\}$ where SR stands for strict rationality and UA stands for utilitarian altruism. Each player is either an SR type or UA type and not both. Player A ascribes to B the probability p that B is UA and $(1 - p)$ that B is SR; B , in turn, ascribes to A the probability g that A is UA and $(1 - g)$ that A is SR. Letting $p = g = 0$ gives the familiar monomorphic game theory. The ascriptions are fixed only throughout the unfolding of the game.

The difference from the usual Bayesian paradigm is that the (p, g) do not emanate from "Nature" but are arbitrary assignments by the players. Thus, this assignment may be erroneous in which case A may be disappointed and a process of reassessment follows. The Bayesian Game G is summarized as follows:

$$G = [\{A, B\}, \{C, D\}, \{U_A, U_B\}, T = \{UA, SR\}, P = \{p, q\}]$$

The payoff table of G in normal form is given in **Table 1**:

Table 1. Payoff Table of G for SR

		Player B	
		C	D
Player A	C	a_{cc}, b_{cc}	a_{cd}, b_{cd}
	D	a_{dc}, b_{dc}	a_{dd}, b_{dd}

Table 1 also gives the payoff table for SR players. The following specifications are assumed: $a_{cc} > a_{dd}$, $b_{cc} > b_{dd}$, $b_{cc} < b_{cd}$, $a_{cc} < a_{dc}$, $a_{dd} > a_{cd}$, $b_{dd} > b_{dc}$. Thus, for an SR, G is a social dilemma game in the form of a Prisoner's Dilemma Game (PDG), specifically a dominant strategy game, with D as the dominant strategy and (D, D) as the dominant strategy equilibrium. Likewise, let the following hold: $(a_{cc} + b_{cc}) > (a_{cd} + b_{cd})$, $(a_{dc} + b_{dc}) > (a_{dd} + b_{dd})$; furthermore, let $(a_{dd} + b_{dd}) > (a_{dc} + b_{dc})$, $(a_{cd} + b_{cd})$.

Definition 1: Player i , $i = A, B$, is a Utilitarian Altruist (UA) if i^{th} best reply to any j^{th} strategy choice by $\sim i$ maximizes the sum of the utilities or expected utilities of the two players.

For a UA, the payoff table is given in **Table 2**:

Table 2. Payoff Table of G for UA

		Player B	
		C	D
Player A	C	$a_{cc} + b_{cc}$	$a_{cd} + b_{cd}$
	D	$a_{dc} + b_{dc}$	$a_{dd} + b_{dd}$

The definition of a UA player is demonstrated using **Table 2**: If B chooses C , A chooses C since, as assumed, $(a_{cc} + b_{cc}) > (a_{dc} + b_{dc})$ and if B chooses D , A chooses D since as assumed, $(a_{dd} + b_{dd}) > (a_{cd} + b_{cd})$.

Since there are two types, dominance has to be specified for each of the types. We now define dominance in this dimorphic player game. Let $EU_{ijl}(h)$, $i = A, B$; $j = UA, SR$, be the expected utility of agent i of type j when choosing strategy $h = C, D$.

Definition2: (i) C dominates D for SR player A if the expected utility of A , $EU_{A/SR}(C) > EU_{A/SR}(D)$, given p ; (ii) C dominates D for UA player A if $EU_{A/UA}(C) > EU_{A/UA}(D)$ or $p[a_{cc} + b_{cc}] + (1 - p)[a_{dd} + b_{dd}] \geq p[a_{dc} + b_{dc}] + (1 - p)[a_{cd} + b_{cd}]$. (iii) If both (i) and (ii) hold, we say that C dominates D for A regardless of type or the dominance of C over D is type-invariant. Ditto for player B .

Suppose A is SR. Now $EU_A(C) = pa_{cc} + (1 - p)a_{cd}$ while $EU_A(D) = pa_{dd} + (1 - p)a_{dd} = a_{dd}$. The latter is because if B is UA with probability $(1 - p)$, his/her best reply to D is D since $(a_{dd} + b_{dd}) > (a_{dc} + b_{dc})$. If A is an SR, he/she is indifferent between C and D when $EU_{A/SR}(C) = EU_{A/SR}(D)$ or:

$$pa_{cc} + (1 - p)a_{cd} = a_{dd}$$

which gives

$$p^* = (a_{dd} - a_{cd}) / (a_{cc} - a_{cd}).$$

Thus, C dominates D for an SR A , i.e., $EU_A(C) > EU_A(D)$, if $p > p^*$. Note that $0 < p^* < 1$ since $(a_{dd} - a_{cd}) < (a_{cc} - a_{cd})$. By the same token, C dominates D for SR B if g is greater than

$$g^* = (b_{dd} - b_{cd}) / (b_{cc} - b_{cd}),$$

where again $0 < g^* < 1$. (p^*, g^*) are probability ascriptions at which C and D are indifferent for SR players.

Suppose A is UA . The expected payoff of a UA player if B chooses C is $p[a_{cc} + b_{cc}] + (1 - p)[a_{dd} + b_{dd}]$; the expected payoff of A if he chooses D is $p[a_{dc} + b_{dc}] + (1 - p)[a_{dd} + b_{dd}]$. Thus, C dominates D for A being a UA if $p[a_{cc} + b_{cc}] \geq p[a_{dc} + b_{dc}]$ which is true by assumption on **Table 1**. That is, a UA will always choose C over D in G independently of p .

We have shown the following:

Claim 1: Suppose $T = \{UA, SR\}$ in game G with $P = (p, g)$, $0 < p, g < 1$. (i) Suppose $(p, g) > (p^*, g^*)$. Then (C, C) is a Bayesian equilibrium of G , or there always exists high enough probability assignment (p, g) so that C is best reply to C regardless of type (i.e., (C, C) is a type-invariant Nash Equilibrium of G); (ii) This high enough (p, g) is a consistent belief equilibrium.

The condition $(p, g) > (p^*, g^*)$ ensures the cooperative outcome is the Bayesian equilibrium of G : A believes that B will play C by a high enough p ; B believes A will play C by high enough g ; they each play C thus confirming each other's expectations by their actions.

A 's assigned probability as observed may be erroneous and B may turn out to be SR , or $p = 0$. This means that A will be disappointed since B will play D in answer to C and A will realize only $a_{cd} < a_{cc}$ if SR and only $(a_{cd} + b_{dc}) < (a_{cc} + b_{cc})$ if UA . A can do better by playing D in which case he realizes $a_{dd} > a_{cd}$ if SR and $(a_{dd} + b_{dd}) > (a_{cd} + b_{dc})$ if UA . Thus in subsequent plays, p will be adjusted downwards to some $p' < p^*$ and A now chooses D as the dominant strategy. By contrast, there is no adjustment process if $p > p^*$ and $g > g^*$ in which case C is the dominant strategy for both players and their belief is confirmed by outcomes.

The dominance relation between C and D will sometimes reverse by type. In which case we say that the dominance relation among the options is *type-dependent*. We have:

Corollary: Suppose $(p, g) < (p^*, g^*)$. Then the dominance relation between C and D is type-dependent; that is, D dominates C for $A(B)$ if $A(B)$ is SR but C dominates D for $A(B)$ if $A(B)$ is UA .

Note that in the monomorphic case where there is only one type, SR , it is impossible for C to ever dominate D . Let $EU_{ij}(C)$ be the expected utility of option C to i as type $j = SR$ or UA . Now $EU_{A/SR}(C) = pa_{cc} + (1 - p)a_{cd}$ while $EU_{A/SR}(D) = pa_{dc} + (1 - p)a_{dd}$. Suppose we let $EU_{A/SR}(C) = EU_{A/SR}(D)$, that is, A as SR is indifferent between C and D . Then $p(a_{cc} - a_{cd}) = (1 - p)(a_{dd} - a_{dc})$, which is impossible for any $p \geq 0$ since,

by assumption, $(a_{cc} - a_{dc}) < 0$ while $(a_{dd} - a_{cd}) > 0$. The same holds for B . The following serves as the benchmark of *Claim 1*:

Corollary: Suppose only one behavioral type in G , $T = \{SR\}$, or $p = 0$, $g = 0$. Then it is impossible for C to dominate D for either player.

This is clearly what we should expect since G is just a dominant strategy game with (D, D) as the DSE and the Bayesian version does not alter the situation. The ordinary version of the game has $p = g = 0$.

Example: A game that satisfies all the stipulated conditions for G is given by **Table 3**. For convenience of treatment, we have defined G to be symmetric.

Table 3. Payoff Table of G

		Player B	
		C	D
Player A	C	5, 5	1, 6
	D	6, 1	4, 4

For a UA player the same payoff table resolves into in **Table 4**:

Table 4. Payoff Table of G

		Player B	
		C	D
Player A	C	10	7
	D	7	8

It is easy to show that C is indifferent to D for A at $p^* = (3/4)$ and by symmetry C is indifferent to D for B if $g^* = (3/4)$. Any $(p, g) > (3/4, 3/4)$ constitutes a consistent belief equilibrium which supports (C, C) as a Bayesian DSE of G .

At $p = p^*$, A realizes $EU_A(C) = (3/4)5 + (1/4)6 = EU_A(D) = 4$ realized if A is SR . If A is UA , he/she realizes $(5 + 5) = 10$ if he plays C while he realizes 7 if he plays D . Thus, C dominates D if UA . By symmetry, C dominates D for B . If $p > (3/4)$, C dominates D for A who is SR . Thus, C dominates D regardless of A 's type. By symmetry, the claims are also true for B . Any $(p, g) > (3/4, 3/4)$ constitutes a consistent belief equilibrium supporting (C, C) as a Bayesian equilibrium.

If $(p, g) < (3/4, 3/4)$, D dominates C only for SR type but C dominates D for UA type. The dominance relation is divergent.

Suppose A 's assignment p is too high and the true p is lower than $(3/4)$. Then A will be disappointed since B 's reply to C is D , in which case A gets 1 if SR and 7 if UA . A can do better with D where he realizes 4 if SR and 8 if UA . Thus, an adjustment process ensues.

III. Group Membership as Heuristic

Clearly, the level of ascribed probabilities are crucial for the emergence of cooperation in dimorphic game G . How do players go about ascribing probabilities to each other? There is a growing evidence that decision makers use *heuristics* – simple rules that simplify complex tasks characterized by incomplete information by focusing on a few or a single aspect of these problems. This was the original position of Herbert Simon (1955) and made compelling decades later by Kahneman and Tversky (1979; 1981) and Kahneman, Slovic and Tversky (1982), Kahneman (2003), Gigerenzer and Selten (2002). One heuristic that may play a crucial role in the assignment of type probability in workaday interactions is group membership. Members of a coherent group tend to assign high probability of type UA to fellow members of the group. That is, when interacting with fellow members in a dilemma game, they choose type UA and thus choose C as dominant strategy. For non-members, they find salient characteristics that are readily observed and use those as heuristics to calculate p . While this decision routine is subject to Cognitive Psychology's "availability bias", it works most of the time.

III. Conclusion

In the behaviorally polymorphic Bayesian Game model, agents first size up their game partners as to type (friendly or unfriendly), choose the appropriate behavioral type to don (like a hat), and from there the course of action follows. This re-game stage or meta-game stage means that certain options in the strategy set becomes irrelevant. If the counterparty is a member of "family", the option " D " does not apply and may be dropped. A detailed examination of the payoff structure may be dispensed with in the choice of strategy when there is reason to believe that the other party is a "friend". The orthodox monomorphic game theory assumes that the type of the counterparty is a "fiend" with probability one. The monomorphic game theory leads to many market failures such M. Olson's (1965) "zero contribution hypothesis" or G. Hardin's (1968) the "tragedy of the commons". This in turn creates the need for an outside intervention. E. Ostrom's (1990, 2000, 2009) documentation of common property management successes among small communities points to a much stronger capacity for cooperation among agents who are better modeled as "conditional cooperators" than as "rational egoists". We argue that the Ostrom communities are better modelled by polymorphic game theory.

We analyze a dimorphic Bayesian game with SR (strict rationality) and UA (utilitarian altruism) as the two possible types and A and B as the two players. The original game is a dominant strategy Prisoner's Dilemma game. We show that cooperation is the dominant strategy for the two players if the two player ascribe a high enough probability of each being UA to each other, that is, if $(p, g) > (p^*, g^*)$ where p^* and g^* are probability ascriptions that make players indifferent between C and D . High (p, g) may result from the players recognizing each other to be members of the same cohesive group. Group membership here acts as a heuristic for the assignment of high p, g . Interacting agents first size up each

other as to type (*UA* or *SR*) and choose the action appropriate to the assignments. Some actions may be dropped from the strategy set as inappropriate. Interacting agents in real life may not go into a detailed examination of the payoff structures the way game analysts do. Many conundra in monomorphic game theory such as the divergences between dilemma games predictions and experimental outcomes will cease to surprise.

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