



UP School of Economics

Discussion Papers

Discussion Paper No. 2016-09

September 2016

Procurement Auctions with Interdependent Values and Affiliated Signals

by

RM Balanquit and MJM Kimwell

University of the Philippines School of Economics

UPSE Discussion Papers are preliminary versions circulated privately to elicit critical comments. They are protected by Republic Act No. 8293 and are not for quotation or reprinting without prior approval.

Procurement Auctions with Interdependent Values and Affiliated Signals[☆]

RM Balanquit and MJM Kimwell

School of Economics, University of the Philippines, Diliman, Quezon City

Abstract

Procurement auctions that assume independent private values (IPV) provide a benchmark for analysis that is readily demonstrated but often unrealistic. Firms who compete for exclusive selling rights normally derive outputs from a highly similar set of inputs which, in turn, allows them to obtain some knowledge on how others would price their goods. In this paper, we incorporate this assumption by showing how affiliated signals and interdependent values can possibly affect the expected quantities sold and selling prices of some endogenous-quantity procurement auction formats. The resulting equilibrium bidding strategies no longer give credence to the typical equivalence result which holds under IPV. In this environment, the second-price auction yields both higher expected prices and lower expected quantities than the first-price auction. This result is consistent with similar studies showing suboptimality of auction mechanisms that allow for winning bids of less-than-the-highest willingness to pay, when values are not fully independent.

Keywords: procurement auctions, affiliated signals, interdependent values, first price vs. second price

JEL Classification: D44, H57

1. Introduction

For the large-scale procurement of both goods and services, auction is seen as one efficient means of generating mutually beneficial contracts. The most basic formulation of the procurement (or reverse) auction features the assumption that bidders maintain independent, private values (IPV)[1]. Standard results include quantity and revenue equivalence between the first-price (FPA) and second-price (SPA) formats, and that expected prices in the FPA are also strictly lower than those in the SPA [2].

However, contrary to the standard assumption, relative homogeneity and symmetry among bidders in a procurement auction may be viewed as the rule rather than the exception [3, 4]. In the case where firms bid for the right to sell batches of an identical good to a single buyer, such firms may be considered to essentially

[☆]Working paper, for comment

Email address: mmkimwell@up.edu.ph (RM Balanquit and MJM Kimwell)

compete in the same market, where exogenous factors such as prevailing market prices and costs of input (such as transportation, raw materials, or fees such as duties and other taxes) affect each competing firm in a highly similar, if not uniform, manner.

The competitive benefits of the auction as a revenue-maximizing, efficient allocation mechanism have previously been shown to depend almost exclusively on key primitives, one of which is the assumption that bidder values are independent, private, and distributed according to regularity rules [5]. Relaxation of these assumptions leads to sub-optimality of the auction mechanism, although a segment of the literature aims to clarify whether these results derive from the interdependence of values themselves or the assumed statistical interdependence of bidder information asymmetry [5–7]. In particular, Campbell and Levin [5] showed that the conditions under which the auction can be an optimal selling mechanism when values are interdependent are highly constrained, so much so that other, simpler mechanisms will outperform the auction in terms of allocative efficiency and revenue maximization. Empirically, correlation among private values has been demonstrated in data from various auctions, where lower expected profits, reserve prices, and bidder surplus were attributed directly to the dependence among valuations [8].

In this note, we consider the auction where bidders compete for exclusive selling rights to an auctioneer whose primary interest lies in obtaining the greatest quantity of goods at the lowest cost possible. As in the standard procurement auction literature, we assume that bidding firms are constrained by their respective marginal costs, such that profit to the bidder is made possible only when the price at which the bidder wins the contract exceeds its marginal cost of fulfilling the contract.

We use as benchmark the endogenous-quantity models of Hansen (1988) and Dastidar (2008) in incorporating the assumptions of interdependence and affiliation in our study. More precisely, we apply the structural analysis approach used in Milgrom and Weber’s (1982) treatment of the general affiliated values model to the procurement auction, deriving Bayesian equilibrium bidding strategies for both the FPA and SPA. We show that, when bidder values are interdependent and their signals are affiliated, expected prices in the SPA are shown to be at least as high as expected prices in the FPA, while the opposite is true in terms of their expected quantities.

2. Assumptions of the Model

2.1. General Rules

The model assumes an auctioneer who seeks to procure an undetermined number q of identical objects. Sealed bids are elicited from n risk-neutral bidders ($i = 1, \dots, n$), who are thus competing for rights to sell to the auctioneer. Each bid b_i is equal to the unit price p at which the bidder is willing to sell the goods, so that the final quantity of good sold to the winning bidder is endogenously determined. In contrast to Dastidar’s treatment [2], we do not assume any particular functional form for quantity sold, adopting instead the more

general function

$$q = q(b_i). \quad (2.1)$$

In the first price auction (FPA), the contract to sell is awarded to the bidder with the lowest bid, at p equal to the winning bid. In the second-price auction (SPA), the winner is again the bidder with the lowest bid, but with a contract price p equal to the second lowest bid.

2.2. Value Functions

In this procurement auction, the value of the contract to each bidder is equivalent to the marginal cost c_i of producing each unit of good for sale. Interdependence of values is shown in the cost valuation function

$$\mathcal{C}(\mathbf{X}) \equiv \mathcal{C}(X_i, \mathbf{X}_{-i}) \quad \text{for all } i, \quad (2.2)$$

where $\mathbf{X} = (X_1, X_2, \dots, X_n)$ refers to the vector of real-valued internal signals, or informational variables, received by all bidders, and $\mathcal{C}$ is nonnegative, continuous, nondecreasing in all its arguments, and symmetric in its last $n - 1$ arguments.

Each X_i is to be understood as a random variable with domain normalized within the interval $[0, 1]$, where x_i is the realization of X_i (i.e., the value of the particular signal received by bidder i). If we let Y_i denote the i^{th} lowest among the $n - 1$ elements of $\mathbf{Y}$, where $\mathbf{Y} = (Y_1, Y_2, \dots, Y_{n-1})$ is the $(n - 1)$ -tuple of real-valued internal signals received by all bidders excluding bidder i , and $Y_1 \leq Y_2 \leq \dots \leq Y_{n-1}$, then (2.2) can be written for bidder i as

$$\mathcal{C}(X = x_i, \mathbf{X}_{-i} = \mathbf{y}) = c_i(x_i, y_1, y_2, \dots, y_{n-1}).$$

The signal variables $X_1, X_2, \dots, X_n$ are further assumed to be affiliated. The strong degree of correlation thus established among signals is captured in the technical definition of affiliation [9]: given n random variables jointly distributed in $\mathbb{R}^n$ according to a density f , any two “points” $\mathbf{x}' = (x'_1, x'_2, \dots, x'_n)$ and $\mathbf{x}'' = (x''_1, x''_2, \dots, x''_n)$ in $\mathbb{R}^n$ must satisfy the condition

$$f(\mathbf{x}' \wedge \mathbf{x}'') \cdot f(\mathbf{x}' \vee \mathbf{x}'') \geq f(\mathbf{x}') f(\mathbf{x}''), \quad (2.3)$$

where

$$\begin{aligned} \mathbf{x}' \wedge \mathbf{x}'' &= [\min(x'_1, x''_1), \min(x'_2, x''_2), \dots, \min(x'_n, x''_n)] \quad \text{and} \\ \mathbf{x}' \vee \mathbf{x}'' &= [\max(x'_1, x''_1), \max(x'_2, x''_2), \dots, \max(x'_n, x''_n)]. \end{aligned}$$

In addition, (2.3) can also be considered as a log-supermodular function whose key property allows any subset of a set of affiliated variables to be affiliated as well. The fact that the signals $X_1, X_2, \dots, X_n$ are affiliated shows that the variables X_1 and Y_1 , in particular, are also affiliated. This implies that, for

any $x' \geq x$, the distribution $F(\cdot | x')$ stochastically dominates $F(\cdot | x)$. This is typically expressed as $F(y | x') \leq F(y | x)$, or sometimes in terms of the hazard rate [10], *i.e.*

$$\frac{f(y | x')}{1 - F(y | x')} \leq \frac{f(y | x)}{1 - F(y | x)}.$$

The assumption of affiliation has numerous other implications, thoroughly established in the seminal papers on the subject [9, 11]; other properties relevant to the proofs in this paper will be included in the discussion elsewhere.

2.3. Bid Functions

In contrast to Dastidar's [2] IPV procurement auction model where bids are direct functions of marginal costs (*i.e.* $b_i(c_i)$), in this paper it is the informational signals that determine bid values. Each bidder i is assumed to receive a signal x_i but bids as if his signal were z_i , so that the bid function is

$$\beta(x_i) \equiv b(z_i)$$

where β is continuous and differentiable in x , and $\beta(\cdot) : [0, 1] \rightarrow \mathbb{R}_+$.

2.4. Payoff Functions

All bidders are assumed to be risk-neutral, so that they seek to maximize expected payoffs, and their payoff functions are linear in payments [9, 10]. For both auction mechanisms, bidder i wins if b_i is the lowest among all the bids submitted. Letting $\beta^*(Y_1)$ denote the lowest-order bid among all bidders other than i , bidder i wins if $b_i < \beta^*(Y_1)$. In addition, the quantity of goods sold is endogenous and is determined by the winning price that is an outcome of the auction. The general profit function for each bidder i who receives signal x_i and bids $b(z_i)$ thus becomes

$$\pi(b(z_i)) = \begin{cases} (b(z_i) - c_i) q(b(z_i)) & , \text{ if } b(z_i) < \beta^*(Y_1), \\ 0 & , \text{ otherwise.} \end{cases} \quad (2.4)$$

It should be noted that the above general payoff function is practically identical to that of the IPV case.

3. Equilibrium in the FPA

The equilibrium bidding strategy is derived through an adaptation of the heuristic-first approach used by Wilson (1977) and followed by Milgrom and Weber (1982). Since the value and bid functions are symmetric, the case below for bidder i applies identically to all bidders.

Let the n -tuple of bids $(b_i^*, \mathbf{b}_{-i}^*)$ denote an equilibrium strategy, where $b_i^* \equiv \beta^*(x_i)$ is the equilibrium bid of bidder i , and $(\mathbf{b}_{-i}^*)$ is the equilibrium profile for all the rest. If any bidder i receives a signal x_i , but

bids as if her signal were z_i , then her expected payoff is given by

$$E[\pi(b(z_i), x_i)] = E[(b(z_i) - c_i)q(b(z_i)) \cdot I_{b_i < \beta^*(Y_1)} \mid X_i = x_i], \quad (3.1)$$

where I is an indicator function equal to 1 if $b_i < \beta^*(Y_1)$ and equal to 0 otherwise. Since X_i and Y_1 are affiliated, the above expression can be expanded to

$$E[\pi(b(z_i), x_i)] = E[E[(b(z_i) - c_i)q(b(z_i)) \cdot I_{b_i < \beta^*(Y_1)} \mid X_i, Y_1] \mid X_i = x_i]. \quad (3.2)$$

Define the conditional expectation function

$$v(x_i, y) \equiv E[c_i \mid X_i = x_i, Y_1 = y]$$

as the expected cost to bidder i when $X_i = x_i$ and $Y_1 = y$, where v is nondecreasing in y and strictly increasing in x_i . Using this, (3.2) can be rewritten as

$$E[\pi(b(z_i), x_i)] = \int_{z_i}^1 [b(z_i) - v(x_i, y)]q(b(z_i))f(y \mid x_i)dy. \quad (3.3)$$

In the previous expression, the conditional density function $f(y \mid x_i)$ is derived from the distribution of Y_1 , denoted by $F(y \mid x_i)$, which is the next lowest signal to bidder i 's $X_i = x_i$. The bid function $b(z_i)$ is assumed to be increasing and differentiable within the interval $[b(0), b(1)]$.

The expected payoff in (3.3) can likewise be expressed as the difference between the expected revenue and expected cost of bidder i in winning the auction, *i.e.*

$$E[\pi(b(z_i), x_i)] = \int_{z_i}^1 b(z_i)q(b(z_i))f(y \mid x_i)dy - \int_{z_i}^1 v(x_i, y)q(b(z_i))f(y \mid x_i)dy. \quad (3.4)$$

Then, since $F(1 \mid x_i) = 1$,

$$E[\pi(b(z_i), x_i)] = b(z_i)q(b(z_i)) \left(1 - F(z_i \mid x_i) - q(b(z_i))\right) \int_{z_i}^1 v(x_i, y)f(y \mid x_i)dy. \quad (3.5)$$

Using the Leibniz formula to obtain the first-order condition for the maximum of $E[\pi(b(z_i), x_i)]$ and by some compression of notation, we have

$$\begin{aligned} \frac{\partial E[\pi(b(z_i), x_i)]}{\partial z_i} &= -bqf + (1 - F)(bb'q' + qb') + vqf - q'b' \int_{z_i}^1 v(x_i, y)f(y \mid x_i)dy = 0 \\ \implies (b - v)qf &= (1 - F)(bq' + q)b' - q'b' \int_{z_i}^1 v(x_i, y)f(y \mid x_i)dy. \end{aligned} \quad (3.6)$$

Thus, we obtain the equilibrium $b^*(z_i)$ for any bidder as the solution to the following differential equation:

$$b'(z_i) = \frac{(b(z_i) - v(x_i, z_i))q(b(z_i))f(z_i \mid x_i)}{(1 - F(z_i \mid x_i))[b(z_i)q'(b(z_i)) + q(b(z_i))] - q'(b(z_i)) \int_{z_i}^1 v(x_i, y)f(y \mid x_i)dy}. \quad (3.7)$$

This result is a generalization of Hansen's (1988) equilibrium procurement strategy, such that if values are private, *i.e.* $v(x_i, y) = v(x_i)$, and if signals are not affiliated with x , *i.e.* $F(\cdot | x_i) = F(\cdot)$, then

$$\int_{z_i}^1 v(x_i, y) f(y | x_i) dy = v(x_i) (1 - F(z_i)), \quad (3.8)$$

from which we can obtain Hansen's result:

$$b'(z) = \frac{(b(z) - v(x_i)) q(b(z))}{[b(z) - v(x_i)] q'(b(z)) + q(b(z))} \cdot \frac{f(z)}{(1 - F(z))}. \quad (3.9)$$

In the following proposition, we formally state that (3.7) solves the equilibrium bid for any bidder i and that this occurs when $z_i = x_i$. Moreover, since no bidder can bid below her valuation, we have $b(x) - v(x, x) \geq 0$, for all x . Also, since by assumption $v(0, 0) = 0$, we have the boundary condition $b(0) = 0$ forming part of the equilibrium solution.

Proposition 1. *In an endogenous-quantity procurement auction, the n -tuple $\mathbf{b}^*$ is a symmetric equilibrium of the FPA that solves the differential equation*

$$b'(x) = \frac{(b(x) - v(x, x)) q(b(x)) f(x | x)}{(1 - F(x | x)) [b(x) q'(b(x)) + q(b(x))] - q'(b(x)) \int_x^1 v(x, y) f(y | x) dy} \quad (3.10)$$

PROOF OF PROPOSITION 1. If $z < x$, then by affiliation, $F(\cdot | x)$ stochastically dominates $F(\cdot | z)$; that is, $F(\cdot | z) > F(\cdot | x)$, or in terms of the hazard rate,

$$\frac{f(z | x)}{1 - F(z | x)} < \frac{f(z | z)}{1 - F(z | z)}.$$

By rewriting equation (3.6) and applying the property of affiliation, we have

$$\begin{aligned} \frac{\partial E[\pi(\cdot)]}{\partial z} &= (1 - F(z | x)) \left[- (b(z) - v(x, z)) q \frac{f(z | x)}{1 - F(z | x)} + qb'(z) + q'b'(z)b(z) \right. \\ &\quad \left. - \frac{q'b'(z)}{1 - F(z | x)} \int_z^1 v(x, y) f(y | x) dy \right] \\ &> (1 - F(z | x)) \left[- (b(z) - v(z, z)) q \frac{f(z | z)}{1 - F(z | z)} + qb'(z) + q'b'(z)b(z) \right. \\ &\quad \left. - \frac{q'b'(z)}{1 - F(z | z)} \int_z^1 v(z, y) f(y | x_i) dy \right]. \end{aligned}$$

Denote the right hand side of the inequality as ξ . Because $v(z, z) < v(x, z)$, and by using (3.7),

$$\begin{aligned}\xi &= \frac{1 - F(z | x)}{1 - F(z | z)} \left[(1 - F(z | z)) (q + q' b(z)) - q' \int_z^1 v(z, y) f(y | x_i) dy \right] (-b'(z) + b'(z)) \\ &= 0,\end{aligned}$$

so that for all $z < x$ we have $\frac{\partial E(\pi(\cdot))}{\partial z} > 0$. We can show in a similar fashion that if $z > x$, then $\frac{\partial E(\pi(\cdot))}{\partial z} < 0$. Thus, $E(\pi(\cdot))$ is maximized by choosing $z = x$. $\square$

From the preceding result, we observe that when $q(b(x))$ is fixed, we have $q'(\cdot) = 0$. Then,

$$b'(x) = [b(x) - v(x, x)] \frac{f(x | x)}{1 - F(x | x)}.^1 \quad (3.11)$$

In the following corollary, following closely the method used by Krishna [10], we show how the above differential equation can be solved in a more concrete way along with the associated boundary condition $b(0) = 0$.

Corollary. *For procurement auctions with fixed quantity, the equilibrium bid for any player in the FPA is given by*

$$b^*(x) = \int_x^1 v(y, y) dM(y | x),$$

where

$$M(y | x) = \exp \left(\int_x^y \frac{-f(k | k)}{1 - F(k | k)} dk \right). \quad (3.12)$$

PROOF OF COROLLARY. Rearranging (3.11), we obtain

$$\begin{aligned}v(x, x) f(x | x) &= b(x) f(x | x) + b'(x) F(x | x) - b'(x) \\ &= \frac{d}{dx} (b(x) F(x | x) - b(x)).\end{aligned}$$

Integrating both sides for all $y > x$, we have

$$\begin{aligned}b(x) &= \frac{-\int_x^1 v(y, y) f(y | y) dy}{1 - F(x | x)} \\ &= \int_x^1 v(y, y) dM(y | x),\end{aligned}$$

¹The analog of this under the standard auction that grants the prize to the highest bidder is given by $b'(x) = [v(x, x) - b(x)] \frac{f(x | x)}{F(x | x)}$ (see Krishna [10], p.93).

where

$$\begin{aligned}
M(y | x) &\equiv \frac{1 - F(y | y)}{1 - F(x | x)} \\
&= \exp \left(\ln \left(\frac{1 - F(y | y)}{1 - F(x | x)} \right) \right) \\
&= \exp \left(\int_x^y \frac{d}{dk} \ln (1 - F(k | k)) dk \right) \\
&= \exp \left(\int_x^y \frac{-f(k | k)}{1 - F(k | k)} dk \right). \quad \square
\end{aligned}$$

Remark. $M(y | x)$ can be considered a distribution function with support $[x, 1]$. To see this, note that by affiliation, we have for all $k \geq x$,

$$\frac{f(k | k)}{1 - F(k | k)} \leq \frac{f(k | x)}{1 - F(k | x)}.$$

Then,

$$\int_x^1 \frac{-f(k | k)}{1 - F(k | k)} dk \geq \int_x^1 \frac{-f(k | x)}{1 - F(k | x)} dk,$$

which implies that, for all $k \geq x$, $M(\cdot | x)$ is nondecreasing over $[x, 1]$.

4. Equilibrium in the SPA

Let β^* denote the n -tuple equilibrium point, where $\beta^*(x_i) = b_i^*$ represents bidder i 's best (payoff-maximizing) response to the equilibrium strategies of the other $(n - 1)$ bidders. Again following the heuristic approach [9, 11], suppose first that all bidders $j \neq i$ follow the bidding strategy β_j^* and that the lowest bid among them is $\beta^*(Y_1) \equiv \min_{j \neq i} \beta^*(X_j)$, where Y_1 is a random variable with realization y .

According to the rules of the SPA, bidder i will win the auction if $\beta(X_i) < \beta^*(Y_1)$, and will be rewarded a contract price equal to $b^*(y)$. Bidder i 's decision problem, then, is to choose $b_i^* = \beta^*(x_i)$ that maximizes

$$E[\pi(b_i, c_i)] = E[(b(y) - c_i) q(b(y)) \cdot I_{\{b(y) > b_i\}} | X_i = x_i], \quad (4.1)$$

where I is an indicator function equal to 1 if $b_i < b^*(y)$, and equal to 0 otherwise.

Since bidder i bids as if she receives a signal $x_i = y$, and since the cost is also dependent on the second-lowest signal, we have $v(y, y) \equiv E[b_i | X_i = Y_1 = y]$ and $v(x_i, y) \equiv E[c_i | X_i = x_i, Y_1 = y]$, respectively. Equation (4.1) can then be rewritten in terms of the conditional expected payoff to bidder i as

$$E[\pi(b_i, x_i)] = \int_z^1 [v(y, y) - v(x_i, y)] q(v(y, y)) f(y | x_i) dy, \quad \text{where } z = \beta^{-1}(b_i). \quad (4.2)$$

The following proposition formally states the equilibrium strategy for each bidder under the SPA.

Proposition 2. *In an endogenous-quantity procurement auction, the n -tuple $\mathbf{b}^*$ is an equilibrium of the SPA when*

$$b^*(x) = v(x, x) \quad (4.3)$$

This result shows that each firm will bid at the level of its valuation cost, just as when the SPA is conducted in an IPV environment.

PROOF OF PROPOSITION 2. Optimizing equation (4.3) with respect to z and applying the Leibniz rule, we obtain

$$\begin{aligned} \frac{\partial E[\pi(b(y), x_i)]}{\partial z} &= \int_z^1 \frac{\partial}{\partial z} [v(y, y) - v(x_i, y)] q(v(y, y)) f(y | x_i) dy \\ &= -[v(z, z) - v(x_i, z)] q(v(z, z)) f(z | x_i). \end{aligned} \quad (4.4)$$

If $z > x_i$, then $v(z, z) > v(x_i, z)$ since v is an increasing function of the first argument. Consequently, $\frac{\partial E[\pi(\cdot)]}{\partial z} < 0$ since $q(\cdot)$ and $f(z | x_i)$ are nonnegative. On the other hand, if $z < x_i$ then $v(z, z) < v(x_i, z)$, thereby making $\frac{\partial E[\pi(\cdot)]}{\partial z} > 0$. Thus, $E(\pi(\cdot))$ is optimized by choosing $z = x$ so that the equilibrium bid is $b(x) = v(x, x)$. $\square$

While our goal in this paper is to determine the expected prices under the variable-quantity procurement auction, it is necessary to note how these expected prices compare under the fixed-quantity setup. We show this in the following lemma, which will be used in the proceeding section.

Lemma. *For fixed-quantity procurement auctions, the expected contract price in SPA is at least as high as the one in FPA, that is,*

$$E_{\bar{q}}^{SPA}[P] \geq E_{\bar{q}}^{FPA}[P]. \quad (4.5)$$

PROOF OF LEMMA. The result of Proposition 2 can likewise be expressed in the following manner:

$$\begin{aligned} E[b^{SPA}(y) | X_i = x, y > x] &= E[v(y, y) | X_i = x, y > x] \\ &= \int_x^1 v(y, y) dN(y | x) \end{aligned} \quad (4.6)$$

where, for all $y > x$,

$$N(y | x) \equiv \frac{1 - F(y | x)}{1 - F(x | x)}.$$

Now, as we have remarked in the previous corollary that $M(y | x)$ is a distribution with support $[x, 1]$, the same can also be argued for $N(y | x)$. Therefore, by affiliation, we say that for all $k > x$, $F(\cdot | k)$ stochastically dominates $F(\cdot | x)$ in terms of the hazard rate if

$$\frac{f(k | k)}{1 - F(k | k)} \leq \frac{f(k | x)}{1 - F(k | x)}.$$

Recalling equation (3.12), we have

$$\begin{aligned} M(y | x) &= \exp \left(\int_x^y \frac{-f(k | k)}{1 - F(k | k)} dk \right) \\ &\geq \exp \left(\int_x^y \frac{-f(k | x)}{1 - F(k | x)} dk \right) \\ &= \exp \left(\int_x^y \frac{d}{dk} (1 - F(k | x)) dk \right) \\ &= \exp \left(\ln \left(\frac{1 - F(y | x)}{1 - F(x | x)} \right) \right) \\ &= N(y | x). \end{aligned}$$

Thus, for all $y > x$, $N(y | x)$ stochastically dominates $M(y | x)$ i.e., $N(y | x) \leq M(y | x)$, which completes the proof. $\square$

5. Expected Prices and Expected Quantities

The assertion in Proposition 2 implies that whether the quantity of goods being auctioned is fixed or contingent to the winning price, bidders in the SPA will behave in the same way at the equilibrium. More precisely, we say that $E_q^{SPA}[P] = b^*(x) = E_q^{SPA}[P]$, where $E_q^{SPA}[P]$ refers to the expected price where the quantity of goods is fixed. However, this equivalence fails to hold when SPA is compared with the FPA, regardless of whether the quantity is set as fixed or variable. In what follows, we show in general that under an endogenous-quantity procurement auction, the expected price of an SPA winner is at least as high as that of the FPA winner.

Proposition 3 (Expected Prices). *When signals are affiliated and cost valuations are interdependent in an endogenous-quantity procurement auction, the expected contract price paid in SPA is at least as high as the one paid in the FPA; that is,*

$$E_q^{SPA}[P] \geq E_q^{FPA}[P]. \quad (5.1)$$

PROOF OF PROPOSITION 3. The proof here follows closely the method used by Hansen for the IPV procurement model. First, we rearrange equation (3.10) as follows:

$$b'_q(x) = \frac{(b(x) - v(x, x)) q(b(x)) f(x|x)}{(1 - F(x|x)) q(b(x)) + q'(b(x)) \left[(1 - F(x|x)) b(x) - \int_x^1 v(x, y) f(y|x) dy \right]}. \quad (5.2)$$

Then, observe that when $q(\cdot) = \bar{q}$ is fixed, which implies that $q'(\cdot) = 0$, we obtain equation (3.11):

$$b'_{\bar{q}}(x) = (b(x) - v(x, x)) \frac{f(x|x)}{1 - F(x|x)}.$$

Because $q'(b(x)) < 0$ and the expected benefit from winning should be always greater than expected cost, i.e., $(1 - F(x|x)b(x)) > \int_x^1 v(x, y) f(y|x) dy$, then we have for any $x \in (0, 1)$,

$$b'_q(x) > b'_{\bar{q}}(x). \quad (5.3)$$

Note that when $x = 1$, the probability of winning to any firm becomes zero, whether the quantity is variable or fixed. Thus, we have payoffs becoming zero and $b'_q(1) = b'_{\bar{q}}(1) = 0$. Now suppose $b_q(x) > b_{\bar{q}}(x)$, for all $x < 1$. Since $b(\cdot)$ is concave, the mean value theorem implies that there must exist a $\tilde{x} \in (x, 1)$ where $b'_q(\tilde{x}) \leq b'_{\bar{q}}(\tilde{x})$. This contradicts (5.3). It must be then that for any $x \in (0, 1)$, $b_q(x) \leq b_{\bar{q}}(x)$, which further implies that

$$E_{\bar{q}}^{FPA}[P] \geq E_q^{FPA}[P]. \quad (5.4)$$

Finally, since Proposition 2 argues that $E_q^{SPA}[P] = E_{\bar{q}}^{SPA}[P]$, and the lemma asserts that $E_{\bar{q}}^{SPA}[P] \geq E_{\bar{q}}^{FPA}[P]$, we now obtain the desired result. $\square$

From comparing the resulting equilibrium contract prices under the FPA and SPA, we turn now to their respective expected quantities. We show in the following proposition that the FPA offers a bigger bulk of purchase to the auctioneer than the SPA.

Proposition 4 (Expected Quantities). *The expected quantity sold in the FPA is at least as high as the expected quantity sold in the SPA; that is,*

$$E^{FPA}[q] \geq E^{SPA}[q]. \quad (5.5)$$

PROOF OF PROPOSITION 4. Recall that in any type of procurement auction that grants the prize to the lowest bidder, the expected profit to the winner is given by the following general equation:

$$E[\pi(P(z_i), x_i)] = \int_{z_i}^1 [P(z_i) - v(x_i, y)] q(P(z_i)) f(y | x_i) dy, \quad (5.6)$$

where $P(z_i)$ is the price paid by the winning bidder who receives a signal x_i but bids as if it were z_i . We see from Propositions 1 and 2 that expected profit is maximized when $z_i = x_i$. Thus, exercising the necessary condition $\frac{\delta E(\pi(P(z_i), x_i))}{\delta z_i} = 0$ leads us to:

$$P'(x) = \frac{(P(x) - v(x, x)) q(P(x)) f(x | x)}{(1 - F(x | x)) [P(x) q'(P(x)) + q(P(x))] - q'(P(x)) \int_x^1 v(x, y) f(y | x) dy}. \quad (5.7)$$

Rearranging the terms, we can express this simply as:

$$q'(P(x)) = q(P(x)) \mathcal{A}, \quad (5.8)$$

where

$$\mathcal{A} \equiv \frac{[P(x) - v(x, x)] f(x | x) - P'(x) (1 - F(x | x))}{P'(x) [(1 - F(x | x)) P(x) - \int_x^1 v(x, y) f(y | x) dy]}.$$

Note that $q(P) > 0$, and by the downward-sloping demand function, we have $q'(P) < 0$ which makes $\mathcal{A} < 0$. Solving now the differential equation leads us to the following:

$$\begin{aligned} \int \frac{q'(P)}{q(P)} dq(P) &= \ln q(P) = \int \mathcal{A} dP \\ e^{\ln q(P)} &= e^{\int \mathcal{A} dP + C} \\ q(P) &= k e^{\int \mathcal{A} dP}, \quad \text{where } k \equiv e^C. \end{aligned} \quad (5.9)$$

Finally, we show that the expected quantity $q(P)$ is decreasing in P *i.e.*, because $\mathcal{A} < 0$,

$$\frac{\delta q(P)}{\delta P} = \mathcal{A} k e^{\int \mathcal{A} dP} < 0. \quad (5.10)$$

This, with the result of Proposition 3, we have P in SPA greater than or equal to P in FPA which therefore leads us to the desired result. $\square$

6. Conclusion

In procurement auctions, where bidders compete for rights to sell identical goods to an auctioneer, it is not unreasonable to assume that the factors governing production of the same good among different firms

within the same market are highly similar, if not identical. In addition, it is also unlikely that these factors are unknown from one firm to the other. In this manner, the assumption of symmetrically affiliated signals and interdependent values is not trivial.

Together, Propositions 3 and 4 show that the SPA does no better than the FPA at protecting the auctioneer’s interests of gaining the largest quantity of good for the lowest price. This result is closely related to that obtained in the optimality analysis of Campbell and Levin [5], which can be seen as deriving directly from the interdependence of values. In the FPA, the winning bidder’s value can emerge as the lowest only against a background of high signals from the rest of the bidders. But, interdependence and affiliation will create a strong bias for low signals in the bid distribution, in favor of bidders with lower signals and therefore lower values.² On the other hand, the SPA mechanism allows for a win by a bidder who does not have the lowest revealed value. In the presence of both interdependent values and affiliated signals, adverse selection for higher values is created in the bid profile.

In such an environment, the analysis here shows that the auctioneer’s interest to obtain the greatest amount of good for the least price is protected by the first-price auction mechanism. This result affirms the analysis that auctions that allow for wins of less-than-the-highest willingness to pay, will likely fail to be optimal allocation mechanisms in the setting of interdependent values.

References

- [1] R. G. Hansen, Auctions with Endogenous Quantity, *The RAND Journal of Economics* (1988) 44–58.
- [2] K. G. Dastidar, On Procurement Auctions with Fixed Budgets, *Research in Economics* 62 (2) (2008) 72–91.
- [3] T. Li, I. Perrigne, Q. Vuong, Structural Estimation of the Affiliated Private Value Auction Model, *The RAND Journal of Economics* 33 (2) (2002) 171–193.
- [4] S. Campo, I. Perrigne, Q. Vuong, Asymmetry in First-Price Auctions with Affiliated Private Values, *Journal of Applied Econometrics* 18 (2) (2003) 179–207.
- [5] C. M. Campbell, D. Levin, When and Why Not to Auction, *Economic Theory* 27 (2006) 583–596.
- [6] J. Cremer, R. P. McLean, Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions, *Econometrica* 56 (6) (1988) 1247–1257.
- [7] R. P. McAfee, P. J. Reny, Correlated Information and Mechanism Design, *Econometrica* 60 (2) (1992) 395–421.
- [8] A. Aradillas-Lopez, A. Gandhi, D. Quint, Identification and Inference in Ascending Auctions with Correlated Private Values, *Econometrica* 81 (2) (2013) 489–534.
- [9] P. R. Milgrom, R. J. Weber, A Theory of Auctions and Competitive Bidding, *Econometrica* 50 (5) (1982) 1089–1122.
- [10] V. Krishna, *Auction Theory*, Academic Press, Elsevier Inc., 2010.
- [11] R. Wilson, A Bidding Model of Perfect Competition, *The Review of Economic Studies* 44 (3) (1977) 511–518.

²High signals in standard auction correspond to having high willingness to pay for the object being sold. Conversely, in the procurement auction, low signals may be interpreted as having a higher willingness to pay; that is, to bear the cost of providing a commodity to the auctioneer at a lower price.