A Theoretical Derivation of the Laffer Curve and the Effect of the Tax on Wage and Employment

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ABSTRACT

The Laffer curve showing tax revenue as a function of the tax on the prevailing money wage is derived from the aggregate supply of labor function. Consequently, the possible trade-off between tax revenue and employment (or total output) is considered. Then the tax is shown to result in wage rigidity downward even under competitive condition. Some policy implications and relevant theoretical issues are noted.

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> *I adhered scrupulously to the precept of that brilliant theoretical physicist L. Boltzmann, according to whom matters of elegance ought to be left to the tailor and to the cobbler.*
>
> **Albert Einstein**

I. Introduction

The controversy surrounding the Laffer curve, a strictly macroeconomic concept that relates total tax revenue to the tax on wage income, lies in the problem of determining the actual tax rate that maximizes tax revenue and consequently where the economy is on the curve. While the curve is of the right shape as deduced from a description of the macroeconomic behavior of workers (taxpayers) that affects total tax revenue due to changes in the tax rate on wage income, the tax discourages work and encourages tax avoidance only at the downhill side of the curve up to the point where the confiscatory tax rate is 100.0 percent. In this regard, we quote at length from Edward Meadows.[1]

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[1]Edward Meadows, "The Ideas of Arthur Laffer," Economic Impact, No. 35, 1981/3. International Communication Agency, Washington D.C. pp. 20-21.

"Where are we on the curve?

....the curve itself,...diagrams one of the most basic propositions in economics. At the left-hand side of the scale, the government imposes no income taxes on income and so gets no income tax revenues. Moving right along the scale, as the tax rate increases, so do revenues. But when the rates get high enough to discourage work and encourage tax avoidance, ever-higher rates produce less revenue (the down side of the hill). If the tax rate reaches 100 percent, it yields zero revenues--no one would bother to earn taxable income if the government confiscated all of it.

What's controversial is Laffer's assertion about where we are on the curve. He says we are descending the right-hand slope....

Laffer's friends...don't make such daring claims... Paul Craig Roberts who helped write the Kemp-Roth tax-cutting bill...thinks we are still on the left-hand slope of the curve..."

This controversy about the Laffer curve provides the rationale for the objective of this work which is to derive the curve more precisely from a basic theoretically and empirically received concept – the upward sloping aggregate supply of labor function. It will be shown that the shape of the Laffer curve, particularly its peak point, depends on the shape of the supply of labor function from which it is derived. The peak point of the curve, of course, determines the tax rate on income that maximizes total revenue. Under alternative shapes of the supply of labor function, the peak points of the corresponding Laffer curves vary with the tax rate that maximizes revenue. As a corollary, the possible trade-off between revenue maximization and the level of employment (or total output) from the start of taxation and as the tax increases, along the uphill side of the Laffer curve is considered. As to what the confiscatory tax rate is, this question is left to empirical investigation.

While the derivation is made to provide a firm theoretical and empirical bases for the curve, the more interesting offshoot with important policy implications is the effect of the tax on the level of employment and hence on total output brought about by the behavior of wage and changes in labor's supply decision due to the income tax. These are the essential concern of this work.

II.   Derivation of the Laffer Curve

Consider Fig. A below the upper panel of which graphically presents the labor market. Nominal or money wage w is measured on the vertical axis, the quantity of labor L on the horizontal axis. D_L and S_L are the aggregate demand for labor function and the aggregate supply of labor function, respectively. Both D_L and S_L are, for purposes of theoretical simplification, linear functions of nominal or money wage without tax. Nominal wage is used instead of real wage as an independent variable since it is what is actually being taxed. Real wage however, maybe used and gives the same result provided the price level is fixed otherwise everytime the price level changes the real wage must be recalculated and the corresponding tax rate revised. This is messy because the tax collector and the taxpayers must agree on the method and result of estimation of the price level.

In the upper panel of Fig. A, the prevailing equilibrium money wage without tax is w_0 and the equilibrium level of employment is L_0. At the equilibrium wage w_0 the total income of the total employed labor L_0 is $(w_0)(L_0) =$ the rectangle Ow_0BL_0. Thus tax revenue from wage income at this initial position is zero.
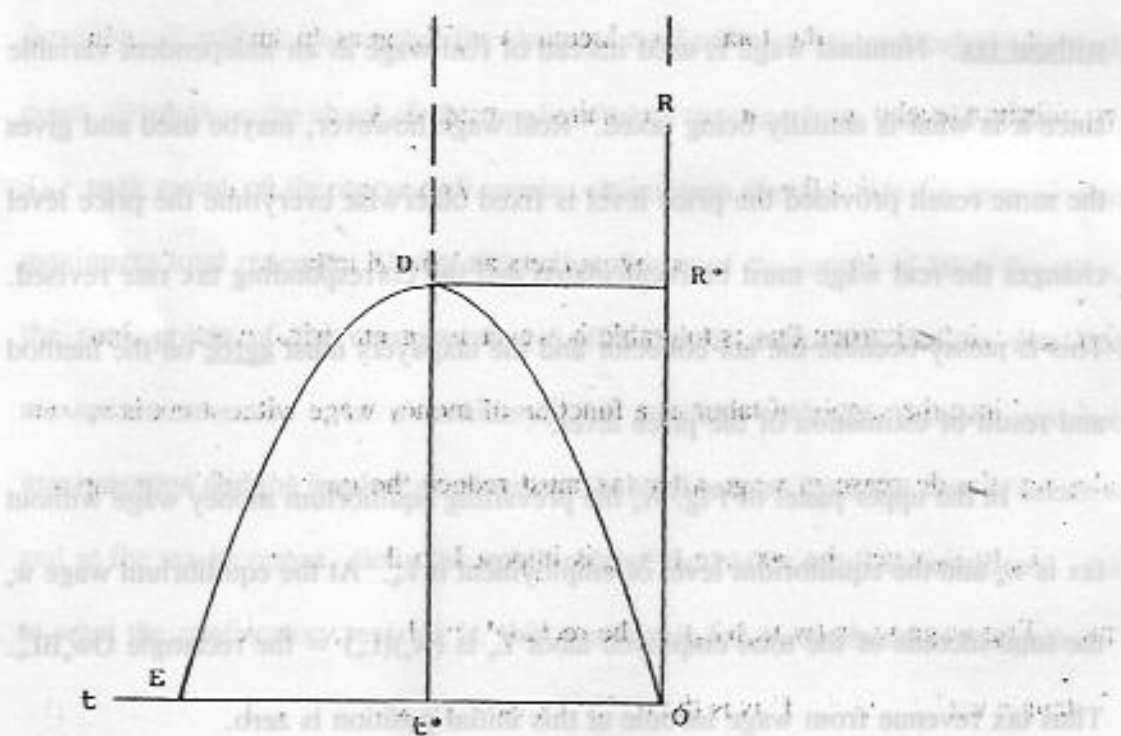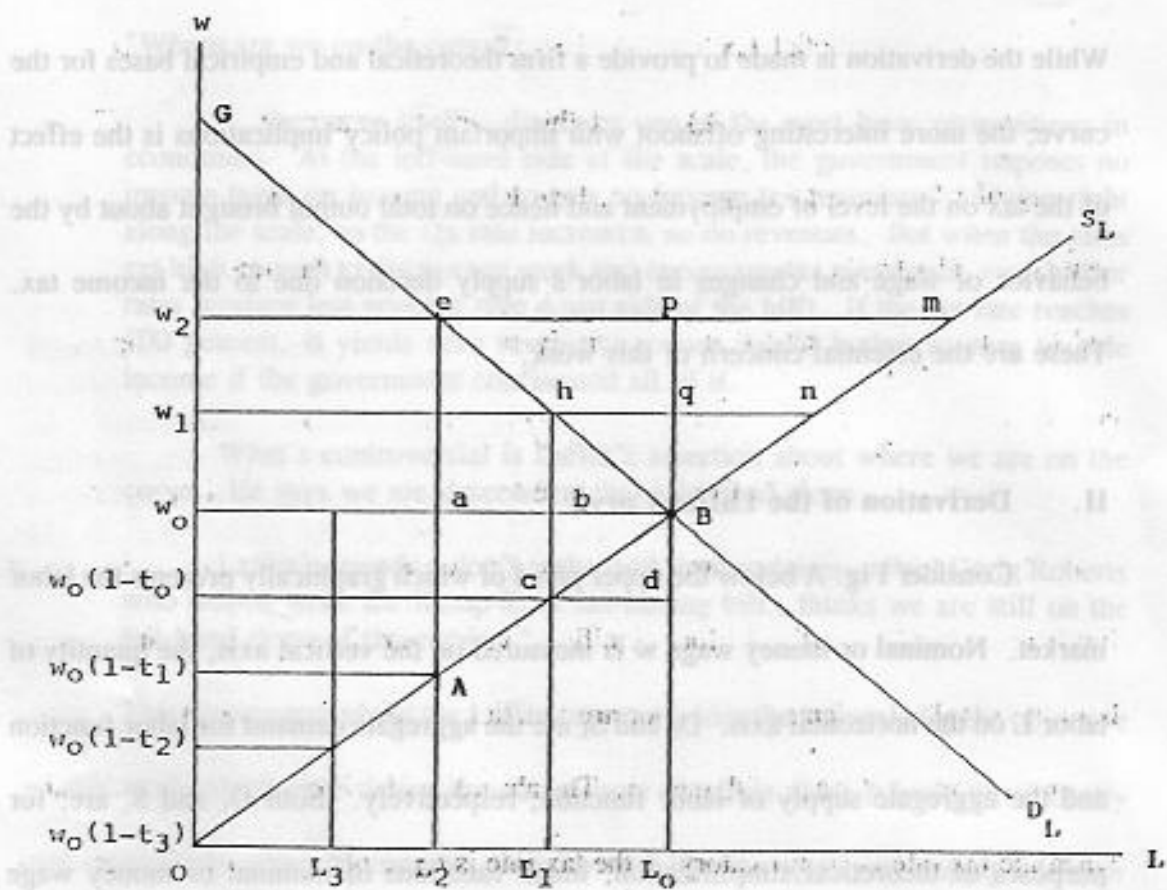
Fig. A

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Now let the initial tax rate on the prevailing wage w_o be t_o whence wage income after tax is now $w_o(1-t_o)$. If at this lower wage after tax the equilibrium quantity of labor L_o <u>continues</u> to work, then total tax revenue is $(w_o)(t_o)(L_o) =$ the rectangle $w_o(1-t_o)w_oBd$. The same result is obtained for any tax rate t such as t_1, t_2, and t_3 (where $t_o < t_1 < t_3$) that successively reduces wage after tax as long as the equilibrium level of employment remains unchanged at L_o. This highly unrealistic case will not produce the Laffer curve since the tax revenue is simply proportional to the tax rate t. Also, this runs counter to the workers' response to the rising tax burden as described earlier. More significantly, it contradicts the received idea that both the supply and demand for labor are functions of the <u>wage without tax</u> whether nominal or real, not <u>wage with the tax</u>. The absurdity of this case can be seen clearly when the tax rate is "confiscatory" – the tax rate is close or equal to 100.0 percent – so that almost all of the total wage income Ow_oBL_o goes to tax revenue but the equilibrium level of employment L_o remains unchanged. Of course, this does not rule out the possibility that the quantity of labor L_o may choose to remain employed if the tax is <u>low</u> enough to be considered by workers as "tolerable tax burden." What is a <u>low enough tax burden that is tolerable</u> however is for empirical work to find out.

Since the supply of labor as a function of money wage without tax is upward sloping, the decrease in wage after tax must reduce the quantity of labor supplied from L_o to L_1 when the tax rate $t = t_o$ is imposed so that the tax revenue from the prevailing wage w_o is $(w_o)(t_o)(L_1) =$ the rectangle $w_o(1-t_o)w_obc$ which is less than the rectangle $w_o(1-t_o)w_oBd$. This is the realistic case - workers L_1L_o <u>quit</u> the labor market

when their wage income is reduced by the tax - which is in keeping with the behavior of workers(taxpayers) behind the Laffer curve and the well-known distortionary effect of the tax.

As the tax rate t increases, tax revenue increases up to the tax rate $t = t_1' >$ t, at which it reaches its maximum since at point A of the segment OB of the linear supply of labor function OS_L, AB = OA so that AB/OA = 1.0. Thus at point A, the maximum tax revenue from wage income is $(w_o)(t_1)(L_2) =$ the rectangle $w_o(1-t_1)w_o aA$ at which the quantity of labor supplied - still at the prevailing wage w_o that is being taxed - decreases further from L_1 to L_2. Thereafter as the tax rate increases, revenue starts to decrease until it is zero at the confiscatory tax rate say, $t = t_3$ since at the wage after tax $w_o(1-t_3)$ which may still be positive, all workers withdraw from the labor market. It should be noted that as shown by the supply of labor curve, not all workers (taxpayers) react in the same way to changes in the tax rate, which is to be expected because not all of them have the same perception of what is a tolerable or intolerable tax burden. However, as the tax rate increases, there is an increasingly unanimous reaction of workers to what would be the confiscatory tax.

At this juncture, it is important to note that the confiscatory tax rate which in the example shown in the upper panel of Fig. A is t_3, need not be 100.0 percent. The confiscatory tax rate at which all workers withdraw from the labor market will depend on the workers' perception of the tax burden hence it can be any rate, perhaps much lower than 100.0 percent. For example, a 70.0 percent tax on wage income or even non-wage income maybe considered confiscatory by taxpayers for being a very heavy

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tax burden. The British tax on tea that sparked the American war of independence must have been much lower than 100.0 percent. Thus a confiscatory tax rate maybe defined as that tax rate which workers(taxpayers) feel as too onerous so that they either avoid paying the tax or, all or most of them quit the labor market altogether. Whatever is the confiscatory tax however, the shape of the curve that emerges from this discussion is shown in the lower panel of Fig. A.

The perfectly symmetrical curve ODE in the lower panel of Fig. A - a result of the linear labor supply function OS, in the upper panel - which shows the tax revenue function $R = f(t)$ is the Laffer curve. The Laffer curve can also be generated whatever is the prevailing wage that is taxed. If instead of the equilibrium wage w_o the prevailing wage that is taxed is w_4 at a tax rate that yields the tax revenue equal to the rectangle $w_o(1-t_i)chw_1$, and then w_2 at a tax rate that yields the maximum revenue shown by the rectangle $w_o(1-t_i)Aew_2$, and so on, a curve of similar shape as ODE in the lower panel of Fig. A is generated. OBG being a triangle, maximum revenue is the rectangle $w_o(1-t_i)Aew_2$ since $AB/QA = eB/Ge = 1.0$. Thereafter as the tax rate increases, revenue decreases.

The Laffer curve need not be perfectly symmetrical since if it is, given the range of the tax rate, it will simply be the rate at the middle of the range that will
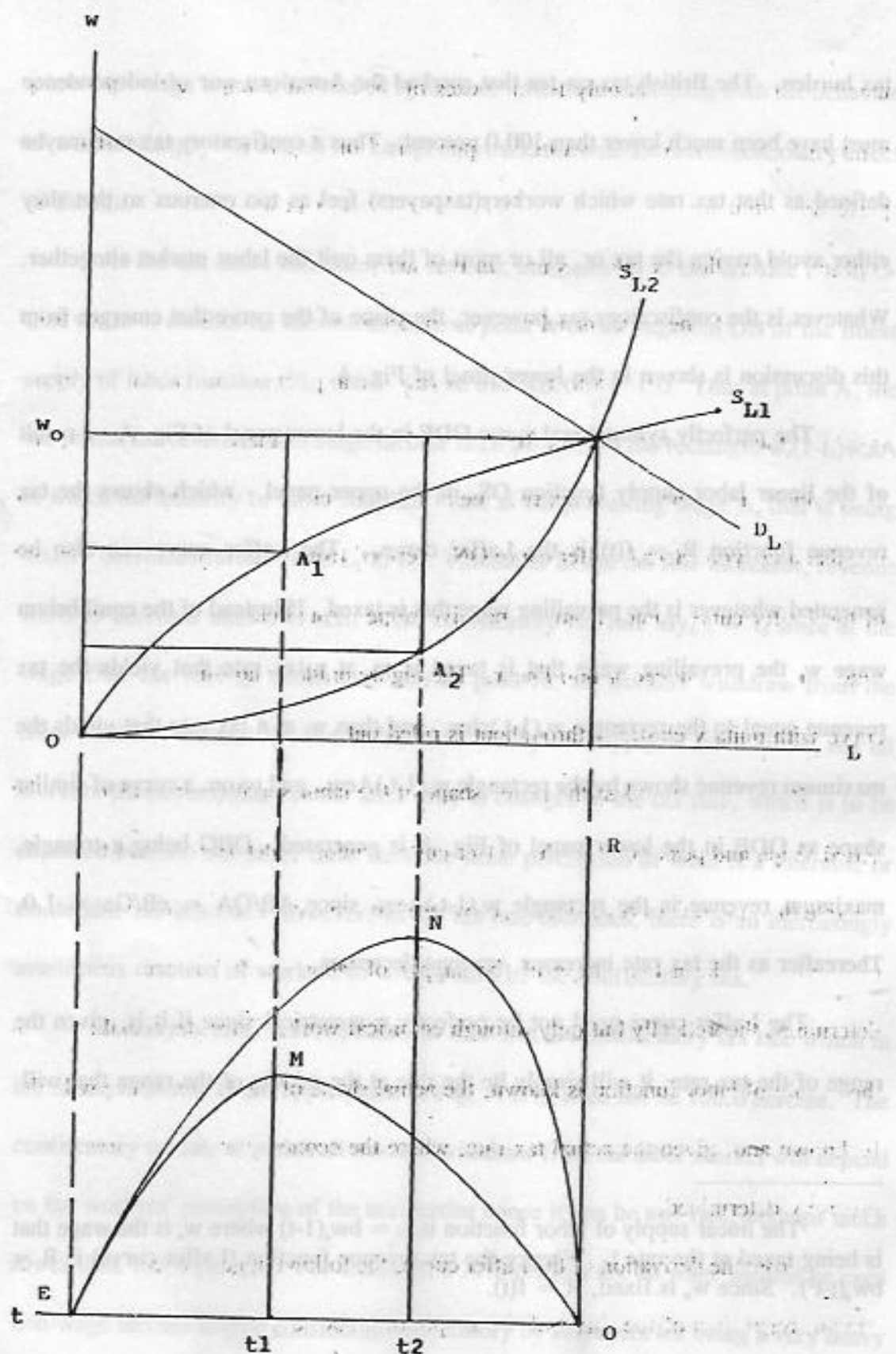
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Fig. B

maximize revenue. It is only for purposes of theoretical simplicity that the linear supply of labor function shown in the upper panel of Fig. A which yields the perfectly symmetrical Laffer curve in the lower panel whose maximum tax revenue is at point D at the tax rate t* is used in this study.

In Fig. B, the upper panel shows two examples of non-linear supply of labor functions, OS_{L1} which yields maximum tax revenue at point A_1 or, at point M at tax rate t1 of the non-symmetrical Laffer curve OME (lower panel) derived from it, and OS_{L2} which yields maximum tax revenue at point A_2 or, at point N at tax rate t2 of the also non-symmetrical Laffer curve ONE derived from it. The single-peak feature of the Laffer curve comes from the positive slope throughout of the labor supply function, whether linear or non-linear. The highly unlikely non-linear labor supply curve with unitary elasticity throughout is ruled out.

It is clear that unless the actual shape of the labor supply curve is known, the actual shape and peak point of the Laffer curve cannot be determined and hence the tax rate that maximizes revenue. Also, where the economy is on the curve cannot be determined. The actual shape of the supply of labor curve however, cannot be determined theoretically but only through empirical work. Once the actual shape of the supply of labor function is known, the actual shape of the Laffer curve will also be known and, given the actual tax rate, where the economy is on the curve can be precisely determined.

From the derivation of the Laffer curve, the following general conclusions are drawn, regardless of the shape of the supply of labor curve:

1) As the tax increases, the level of employment decreases.

2) On the upward sloping side of the Laffer curve up to its peak point where tax revenue is maximum, there is a trade-off between revenue and employment (or total output), that is, as the revenue increases (decreaces) employment (or total output) decreases(increases), while on the downward sloping side of the curve, as the tax revenue decreases(increases) employment (or total output) correspondingly decreases(increases).

The first of these two general conclusions is no more than the result of the behavior of workers in response to the increasing tax as described earlier that yields the Laffer curve. It has, however, serious implications on the behavior of wage and employment or total output since the wage that is being taxed cannot remain unchanged because it must give way as the tax erodes the level of employment.

With regards to the second, the trade-off between revenue and employment along the uphill side of the curve makes it difficult, if not impossible, to maximize tax revenue without increasing unemployment and consequently, decreasing the level of total output.

The actual shape of the Laffer curve as derived from the actual shape of the supply of labor function, given the prevailing wage, has however, certain important implications on the presence and extent of the trade-off between revenue and the level of employment. If the actual shape of the curve is ONE in the lower panel of Fig. B so that the tax rate that maximizes revenue is very low, the trade-off may not exist at all or at most it maybe very weak making revenue maximization possible with little

or no decrease in the level of employment hence with little or no decrease also in the level of total output. Indeed, if the true maximum point of the Laffer curve is very close to the vertical (revenue R) axis, the tax rate that maximizes revenue maybe considered by workers to be low and tolerable enough tax burden so that the trade-off is non-existent - tax revenue is maximized and the full employment level of total output is intact. There is no distortion in the labor market. On the other hand and in sharp contrast to this, the trade-off between revenue and employment becomes effective and more pronounced as the peak point of the Laffer curve occurs farther away from the vertical axis along with the revenue-maximizing tax rate. In all of these cases, the crucial variable is the revenue-maximizing tax rate - the higher this is, the greater will be the extent of the trade-off.

Both of the general conclusions, however, have very significant implications with respect to fiscal and/or monetary policies when linked with the behavior of nominal wage and the level of employment due to the income tax. This aspect of the theoretical derivation of the Laffer curve will be the highlight of the following section of this work.

III.   The Tax and the Behavior of Wage and Employment

The focal point of this section is the more significant first general conclusion of the preceding section that the quantity of labor supplied must fall as the tax on the prevailing money wage increases because the wage after tax to which the supply of labor responds decreases, and as this takes place the wage offer (without tax) of

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employers increases. With this result from the derivation of the Laffer curve, an attempt is now made to explain the impact of the tax on the behavior of wage and the level of aggregate employment (or unemployment) which is perhaps, the most important concern of macroeconomic policy as it affects the level of total output.

Consider the upper panel of Fig. A once more. It is clear that the equilibrium wage w_o that is being taxed cannot keep on prevailing under the distortionary impact of the tax which is, to decrease the level of employment as workers who consider the tax burden increasingly onerous <u>withdraw</u> from the labor market thus pushing the demand price for labor upward beyond w_o. When money wage after tax falls to $w_o(1-t_o)$, the withdrawal of labor L_1L_o from the labor market shrinks the quantity of labor supplied to L_1 thus raising the demand price for labor to w_1, the money wage (without tax) that employers are willing to pay L_1 of labor. Concomitantly, an excess of supply over the demand for labor at the demand price w_1 equal to the horizontal line segment hn opens up. Under competitive condition the adjustment process towards the full employment equilibrium wage w_o cannot take place because the situation is brought about by the exogenously determined tax, not by any transitory disequilibrating endogenous change in the labor market.

The questions closely related to this are: (a) what happens to the workers L_1L_o who withdrew from the labor market when money wage after tax decreases to $w_o(1-t_o)$ from the prevailing money wage before tax w_o? (b) will they remain unemployed and go on welfare or receive unemployment benefits which are all tax free for the rest of their lives? To answer these questions, consider the workers' supply of labor decision

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in response to the tax which reinforces further the downward wage inflexibility. The amount of labor L_1L_0 = the line segment $hq_1 = cd$ that withdrew from the labor market due to the decrease in their wage income after the tax may rejoin the labor market if the money wage offer of employers is high enough to offset the tax w_0t_0 or a large enough part of it. That is, these workers revise their supply price $\bar{w}$ such that $\bar{w} \geq w_0 - w_0t_0$ which must be matched by employers' demand price.

Suppose $\bar{w}_1 - w_1$ in Fig. A, that is employers' demand price w_1 matches labor's revised supply price $\bar{w}_1$ hence all workers who are willing to work will get the wage w_1. At this now prevailing wage w_1 however, only L_{f1} can be employed while the quantity of labor that is willing to work is w_1n hence there is involuntary unemployment equal to the horizontal line segment hn - the excess of the supply of labor over the demand for labor at the prevailing wage $\bar{w}_1 - w_1$. The line segment hn includes not only L_1L_0 = hq = cd, the quantity of labor that withdrew due to the tax but are now willing to rejoin the labor market at the wage w_1 but also those who will join the labor market qn at the same wage.

The prevailing wage w_1 is inflexible downward due to the income tax that workers offset by raising their supply price that is matched by employers' demand price as the level of employment is eroded by the tax. Thus, unless the tax is removed or perhaps drastically reduced to such rate or amount that all workers consider as tolerable tax burden relative to whatever wage will prevail, the adjustment process towards the previous full employment equilibrium wage (without tax) w_0 will not take place. (If a reduction of the tax is made, the consequence of this on revenue will depend on where the economy is on the Laffer curve and the extent of the trade-

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off between revenue and employment)[?] In effect, when workers raised their supply price to $\bar{w}_1 - w_1$ in response to the tax, the downward rigidity of money wage nullifies the segment On of the supply of labor function OS, so that it now becomes the curve $w_1 n S_L$ giving rise to underemployment equilibrium where the prevailing equilibrium nominal wage is now w_1 and the equilibrium level of employment is L_1. With the decrease in the level of employment, total output must fall.

The tax on wage income has thus created unemployment by preventing the money wage to fall freely to the full employment equilibrium wage w_o and for total output to increase correspondingly. The extent of the response of workers to the tax through changes in their supply price and the consequent size of unemployment will depend on the tax rate. An increase in the tax rate, for example from t_o to t_1, will raise their supply price $\bar{w}$ to $\bar{w} - w_2$ the demand price for labor of employers when labor supply shrinks further from L_1 to L_2 thus shifting the horizontal segment of the supply of labor function upward resulting in increased unemployment (em $>$ hn). The new wage w_2 that is established is again inflexible downward. Thus, as the tax increases and employment decreases, labor's counteracting wage increases, further raising unemployment and reducing total output.

The same result is obtained whatever is the prevailing nominal or money wage that is being taxed. For example, if w_2 is the prevailing underemployment equilibrium wage that is taxed, then labor's supply price $\bar{w}$ will be such that $\bar{w} - w_2 - w_2 t$. That is, workers keep on countering the tax that they consider unbearable, by raising their supply price hence shifting upward the horizontal segment at the higher wage that will then be established. In general, whatever is the

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prevailing money wage w, that is being taxed; as long as workers counter the tax and pass it on to employers - a perfectly rational behavior no different from what firms do to the tax on their income or output which they pass on to consumers - by raising their supply price $\bar{w}$ so that $\bar{w} - w_p \sim w_p t$, there will be unemployment at the wage that will then be established that is inflexible downward even under competitive condition. In this regard, it is important to note that since labor's supply price is such that $\bar{w} - w_p \sim w_p t$ or $\bar{w} \sim w_p + w_p t$ which shows $\bar{w}$ as an increasing function of the tax rate t, it is clear that when the tax is removed (t=0), $\bar{w} - w_p - w_o$ the full employment equilibrium wage. That is, when t=0, wage becomes fully flexible and under competitive condition the adjustment towards the full employment equilibrium wage shall take place.

A theoretical rationale for the well-known Keynesian wage stickiness downward and its consequent less than full employment equilibrium level of total output is thus provided namely, that wage rigidity downward is simply due to the tax that raises the demand and supply price of labor above its full employment level and prevents its adjustment towards full employment. Simply put, wage rigidity downward is nothing more than the distortionary effect of the income tax on the labor market. Accordingly, involuntary unemployment maybe redefined as follows: workers consider themselves involuntarily unemployed when they are looking for work but cannot find work at the prevailing money wage that employers are willing to pay and that workers consider will offset the income tax.

By way of concluding this section, the effects of the income tax and the imposition of a minimum wage on the labor market are briefly compared. The slight

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difference lies in the causal sequence of the changes in wage and employment. While in the case of the income tax it is the decrease in the level of employment as workers withdraw from the labor market due to the tax that drives up the wage, in the case of the minimum wage imposition, it is the compulsory wage hike that decreases the level of employment. Both the income tax and the minimum wage are, however, exogenously determined and have the same distortionary effect on the labor market - they increase wage above the equilibrium level and make it inflexible downwards creating unemployment even under competitive condition and concomitantly reducing the level of total output.

IV.   Conclusion

Since a more precise formulation of the Laffer curve entailed the use of the supply of labor function, when the demand for labor is brought into the scenario to complete the ingredients of the labor market, serious problems emerge namely, (1) the possible trade-off between tax revenue maximization - if the economy is on the uphill side of the Laffer curve - and the level of employment (or total output) and, (2) the adverse effect of the tax on wage on the full employment level of total output as the tax could result in wage inflexibility downward preventing adjustment towards the full employment level even under competitive condition.

These problems may bear on some of the views and/or ideas involving the short-run and long-run effects of fiscal and/or monetary policies, among others, on the level of employment and total output. These views and/or ideas are:

a) The Keynesian underemployment equilibrium level of total output and its closely allied tax-based fiscal policy as wage stickiness downward may not be a short-run phenomenon.

b) The short-run and long-run effects of changes in money supply and its closely allied rational expectations hypothesis as they bear on changes in the level of employment and total output.

c) The Phillips curve as the income tax raises money wage, prevents its downward adjustment towards the equilibrium, and creates unemployment.

d) The definition of such types of unemployment as natural unemployment, frictional unemployment, cyclical unemployment, and structural unemployment, all of which are inter-related and highly relevant especially to developed economies. Natural unemployment may not at all be "natural" but man-made - it is due to the exogenously determined income tax.

e) Alternative less distortionary form of taxation and its corresponding less burdensome tax structure.

f) The welfare and unemployment programs of most developed countries.

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