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Dynamics Once Again*

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On Sluggish Output and Exchange Rate Dynamics Once Again

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Abstract

This paper examines a model of exchange rate dynamics which incorporates sluggish output adjustment into the Dornbusch variable output model. In this model where both the price level and output cannot jump, the interest rate must decline in response to a monetary expansion so as to maintain money market equilibrium. As the interest rate declines, because of uncovered interest rate parity, there must be an expectation of a subsequent appreciation. However, the exchange rate need not necessarily overshoot initially and yet an expectation of a subsequent appreciation is created because expectations depend not only on the initial exchange rate deviation, as in the Dornbusch model, but also on the initial price deviation and these two deviations are now different sources of information for rational speculators. Furthermore, by explicitly deriving the time paths of the exchange rate and other variables, it is shown that indeed, consistent with perfect foresight, whatever is the initial exchange rate response, such is subsequently followed by actual appreciation.

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1. Introduction

Dornbusch (1976), in his seminal paper on exchange rate dynamics, has shown that the combination of sticky prices, fixed output, continuous asset market equilibrium, and rational expectations unambiguously causes the interest rate to decline and the exchange rate to overshoot its new long-run equilibrium value in response to monetary expansion. This result is modified when output is variable and can adjust instantaneously in the short-run (Dornbusch, appendix) because the interest rate may decline, remain the same, or rise, and correspondingly the exchange rate may overshoot, neither overshoot nor undershoot, or undershoot its new long-run equilibrium value. Thus, even when output can adjust instantaneously, the possibility of overshooting, though moderated, still remains.¹

This paper analyzes a model of exchange rate dynamics which incorporates sluggish output adjustment into the Dornbusch variable output model. While in other papers a perfect foresight consistent expectations scheme is assumed, in this paper perfect foresight is imposed directly and the model is solved using Dixit's (1980) method; in addition, the time paths of the exchange rate and the other variables are derived explicitly.²

This paper is organized as follows. Section 2 reviews the Dornbusch variable output model. Section 3 incorporates sluggish output adjustment into the Dornbusch model; specifically, it is assumed that short-run aggregate demand or output, like the price level, is also sticky and cannot jump. Given a monetary expansion, this section yields the following results: (1) an additional constraint, sluggish output adjustment, tends to reinforce (dampen) overshooting (undershooting); (2) the interest rate decline and thus the expectation of a subsequent appreciation is not necessarily associated with

exchange rate overshooting; and, (3) an expectation of a subsequent appreciation, given the assumption of perfect foresight, is indeed a subsequent actual appreciation. Finally, Section 4 summarizes the results and conclusions.

2. The Dornbusch Variable Output Model

The Dornbusch variable output model can be summarized by the following set of relationships:

$$y_t = y_t^d \equiv u + \tau y_t + \sigma i_t + \delta(e_t - p_t + p_f), \quad 0 < \tau < 1 \quad (1.1)$$

$$dp/dt = \pi(y_t - y^*), \quad (1.2)$$

$$m_t - p_t = \phi y_t - \beta i_t, \quad (1.3)$$

$$i_t = i_f + E(de/dt), \quad (1.4)$$

$$E(de/dt) = de/dt, \quad (1.5)$$

where e = log of exchange rate measured in units of domestic currency per unit of foreign currency; $e - p + p_f$ = log of real exchange rate; i , i_f = domestic and foreign interest rates; p , p_f = logs of domestic and foreign price levels; y = log of short-run income or output, y^d = log of aggregate demand, u = an exogenous fiscal variable, and all parameters are positive. "*" denotes a long-run equilibrium value while $E()$ is an expectations operator.

Equations (1.1) and (1.2) represent the goods market. Aggregate demand (1.1), which depends on output, the interest rate, and the real exchange rate, determines output in the short-run. The Phillips curve (1.2) shows price adjustment as a function of the gap between short-run and fixed natural output. The asset market, on the other hand, is described by (1.3) to (1.5). Money market equilibrium (1.3) obtains and money demand is a function of output and the interest rate. The money market is linked to the foreign exchange market by the uncovered interest rate parity condition (1.4) that

interest differentials must equal expected exchange rate changes. Equation (1.5) imposes perfect foresight on exchange-rate expectations.

The steady-state of the model, attained when $dp/dt = de/dt = E(de/dt) = 0$, is described by:

$$e^* = p^* - p_f + ((1-\tau)/\delta)y^* + (\alpha/\delta)i^*, \quad (2.1)$$

$$p^* = m - \phi y^* + \beta i^*, \quad (2.2)$$

$$i^* = i_f, \quad (2.3)$$

where $y^* = y^*$ and y^* is assumed to be exogenously fixed at its natural level. In the long-run, money is neutral, i.e., $de^*/dm = dp^*/dm = 1$ and $d(e^* - p^* + p_f)/dm = dy^*/dm = 0$.

The dynamics of the system can be represented by two short-run static equations and a system of differential equations:

$$y_t - y^* = (\beta\delta/V)(e_t - e^*) - [(\alpha + \beta\delta)/V](p_t - p^*), \quad (3.1.i)$$

$$i_t - i^* = (\phi\delta/V)(e_t - e^*) - [(\phi\delta - (1-\tau))/V](p_t - p^*), \quad (3.1.ii)$$

$$\begin{aligned} |de/dt| &= \begin{vmatrix} \phi\delta/V & -(\phi\delta - (1-\tau))/V \end{vmatrix} (e_t - e^*) \\ |dp/dt| &= \begin{vmatrix} \pi(\beta\delta)/V & -\pi(\alpha + \beta\delta)/V \end{vmatrix} (p_t - p^*), \end{aligned} \quad (3.2)$$

where $V = (1-\tau)\beta + \alpha\phi > 0$.³ The determinant of the coefficient matrix in (3.2) is negative, implying that the two roots, r_1 and r_2 , are real and opposite in sign and that the system yields a saddlepoint equilibrium.⁴ Given an initial steady-state where $e = e^*_0$ and $p = p^*_0$, any disturbance which affects the equilibrium price level will yield a new steady-state where $e = e^*$ and $p = p^*$. To ensure that the system will converge toward this new steady-state, there must be boundary conditions on the values, e_0 and p_0 , that the state variables will take following some disturbance.

Since the price level is sticky, its initial value p_0 is predetermined

and is equal to p^*_0 . It follows that the boundary condition for the price level is

$$p_0 - p^* = p^*_0 - p^*, \quad (4.1)$$

where $p^*_0 - p^* = -dp^*$.

The boundary condition for the free variable, the exchange rate, is standard in linear rational expectations models: the coefficient associated with the unstable root must equal zero. If r_2 is the unstable root, this transversality condition implies that

$$e_0 - e^* = [(\phi\delta - (1-\tau))/(\phi\delta + V(-r_1))](p_0 - p^*), \quad (4.2)$$

where $r_1 < 0$, $r_2 < 0$, and e_0 is the value of the exchange rate following some disturbance.⁵

Since the price level cannot jump, the exchange rate must jump to place the system on the stable arm of the saddlepoint. This unique, stabilizing jump is given by (4.2). After the jump, the system moves along the stable path where

$$de/dt = B(de/dt) = r_1(e_0 - e^*), \quad (5.1)$$

$$e_t - e^* = [(\phi\delta - (1-\tau))/(\phi\delta + V(-r_1))](p_t - p^*), \quad (5.2)$$

and $-r_1$ is the system's speed of adjustment.

Now consider a monetary expansion. The impact effects are $d(p_0 - p^*)/dm = -dp^* = -1$, and

$$d(y_0 - y^*)/dm = (\alpha/V) + (\beta\delta/V)\{1 - [(\phi\delta - (1-\tau))/(\phi\delta + V(-r_1))]\}, \quad (6.1)$$

$$d(i_0 - i^*)/dm = -((1-\tau)/V) + (\phi\delta/V)\{1 - [(\phi\delta - (1-\tau))/(\phi\delta + V(-r_1))]\}, \quad (6.2)$$

$$d(e_0 - e^*)/dm = -(\phi\delta - (1-\tau))/(\phi\delta + V(-r_1)), \quad (6.3)$$

where $1 - [(\phi\delta - (1-\tau))/(\phi\delta + V(-r_1))] > 0$, implying that the open economy monetary policy multiplier (6.1) exceeds the closed economy monetary policy multiplier (σ/V) and that $de_0/dm > 0$. From (6.3), the Dornbusch condition for the neither overshooting nor undershooting of the exchange rate is

$$\phi\delta - (1-\tau) = 0. \quad (7)$$

With instantaneous output adjustment, a monetary expansion causes a nominal depreciation and a real depreciation as well since the price level is sticky and the foreign price level is fixed. A real depreciation, in turn, causes exports and therefore output to increase (6.1). In the money market, money demand increases due to the increase in output; if at the initial interest rate there is an excess supply (equilibrium; excess demand), i.e., $(\phi\delta - (1-\tau)) < 0$ ($= 0$; > 0), then the interest rate must fall (remain the same; rise) to re-equilibrate the money market (6.2).⁶ Since the foreign interest rate remains the same and the interest rate declines, asset market equilibrium requires that there must be an expectation of a subsequent appreciation (constant exchange rate; depreciation), i.e., that $E(de/dt)$ be negative (zero; positive), which will hold only if initially the exchange rate overshoots (neither overshoots nor undershoots; undershoots) its new long-run equilibrium value (6.3).

Thus, even when output can adjust instantaneously, the possibility of overshooting, though moderated, still remains. In contrast, when output is fixed even in the short-run, effectively $\phi = 0$ and $V = (1-\tau)\beta$, and (6.2) and (6.3) imply respectively that $d(i_0 - i^*)/dm = -1/\beta < 0$ and $d(e_0 - e^*)/dm = 1/\beta(-r_1') > 0$, where r_1' is the corresponding negative root; that is, in response to monetary expansion, unambiguously, the interest rate declines and the exchange rate overshoots its new long-run equilibrium value.

3. Sluggish Output Adjustment

The Model

This paper analyzes a model of exchange rate dynamics which incorporates sluggish output adjustment into the Dornbusch variable output model. It is described by, as before, (1.2) to (1.5), and the following relationships (Bhandari (1982, Ch. 5), 1983) and Gandolfo (1981, pp. 10-12)):

$$y^d L_t = u + \tau y_t + \sigma i_t + \delta(e_t - p_t + p_f), \quad (8.1.i)$$

$$y^d S_t = u + \int_{-\infty}^t \tau_0 \exp^{-\alpha(t-T)} y_T dT - \int_{-\infty}^t \sigma_0 \exp^{-\alpha(t-T)} i_T dT \\ + \int_{-\infty}^t \delta_0 \exp^{-\alpha(t-T)} (e_T - p_T + p_f) dT, \quad (8.1.ii)$$

$$y_t = y^d S_t \quad (8.2.i)$$

$$dy/dt = \alpha(y^d L_t - y^d S_t) = \alpha(y^d L_t - y_t), \quad (0 < \alpha < \infty) \quad (8.2.ii)$$

where the long-run aggregate demand elasticities are such that $\tau = \tau_0/\alpha$, $\sigma = \sigma_0/\alpha$, and $\delta = \delta_0/\alpha$. Equation (8.1.i) defines long-run or fully-adjusted aggregate demand. Equations (8.1.ii) and (8.2.i) indicate that short-run aggregate demand, which determines short-run output, depends on the past values of output, interest rate and real exchange rate; this means that short-run aggregate demand or output is fixed at time t and cannot jump. Given (8.1.i) and (8.1.ii), output adjustment is given by equation (8.2.ii), which shows that output adjusts at the rate α based on the gap between long-run aggregate demand and short-run aggregate demand or output (8.3).⁷

The steady-state, which now occurs when $dy/dt = dp/dt = E(de/dt) = de/dt = 0$, is also described by (2.1) to (2.3), where $y^* = y^{dL*}$.

Dynamic Properties

The model's state-space representation is now given by:

$$i_t - i^* = (1/\beta)(p_t - p^*) + (\phi/\beta)(y_t - y^*), \quad (9.1)$$

$$\begin{aligned} \dot{e}/dt &= \begin{vmatrix} 0 & a_{12} & a_{13} \end{vmatrix} (e_t - e^*), \\ \dot{p}/dt &= \begin{vmatrix} 0 & 0 & \pi \end{vmatrix} (p_t - p^*), \\ \dot{y}/dt &= \begin{vmatrix} a_{31} & a_{32} & a_{33} \end{vmatrix} (y_t - y^*), \end{aligned} \quad (9.2)$$

where

$$\begin{aligned} a_{12} &= 1/\beta, & a_{13} &= \phi/\beta, & a_{31} &= \alpha(\beta\delta)/\beta, \\ a_{32} &= -\alpha(\sigma + \beta\delta)/\beta, & a_{33} &= -\alpha V/\beta, & V &= (1-\tau)\beta + \sigma\phi. \end{aligned}$$

Equations (9.1) and (9.2) fully describe the system's motion over time.⁸ The characteristic equation associated with (9.2) is

$$R^3 + A_1 R^2 + A_2 R + A_3 = 0, \quad (10)$$

where

$$\begin{aligned} A_1 &= -(R_1 + R_2 + R_3) = -a_{33} = \alpha(V/\beta) > 0, \\ A_2 &= R_1 R_2 + (R_1 + R_2) R_3 = \pi(-a_{32}) - a_{13} a_{31} = \alpha(\pi(\sigma + \beta\delta) - \phi\delta)/\beta, \\ A_3 &= -(R_1 R_2 R_3) = -\pi(a_{12} a_{31}) = -\alpha\pi\delta/\beta < 0, \end{aligned}$$

and R is a root. Since the discriminant, A_3 , is unambiguously negative, the system has either three positive roots implying total instability or one positive root implying saddlepoint instability. A_2 can be either positive or negative but in either case, there is only one variation in the sign of coefficients since the coefficient of R^3 is positive, $A_1 > 0$, and $A_3 < 0$; by Descartes' rule of signs, there exists only one positive root, associated with the rationally expected exchange rate.⁹

The initial steady-state is now described by $e = e^*_0$, $p = p^*_0$, and $y = y^*_0$ and any disturbance that affects the equilibrium price level will yield a new steady-state where by $e = e^*$, $p = p^*$, and $y = y^*$. Since there are now three state variables, there must also be three linearly independent boundary

conditions on the values - e_0 , p_0 , and y_0 - that the state variables will take following some disturbance.

As before, the price level cannot jump. However, now output also cannot jump. Thus both are now predetermined variables and the boundary conditions are such that their values at $t = 0$, p_0 and y_0 , are each equal to their respective initial steady-state values, p^*_0 and y^*_0 . Thus,

$$p_0 - p^* = p^*_0 - p^*, \quad (11.1)$$

$$y_0 - y^* = y^*_0 - y^*, \quad (11.2)$$

where $y^*_0 - y^* = -dy^* = 0$, since y^* is fixed.

The boundary condition for the exchange rate is the same as before: the product of the row eigenvector associated with the unstable root and the column vector of the initial values of the state variables measured as deviations from the new long-run equilibrium must equal zero (Dixit (1980)).¹⁰ If R_3 is the positive root, and since $y_0 - y^* = 0$, this transversality condition implies that (see Appendix):

$$\begin{aligned} e_0 - e^* &= - (c_{32}/c_{31})(p_0 - p^*), \\ &= \{[-(\delta - (1-\tau)) + (1/\alpha)(\delta/(\sigma + \beta\delta))]/C_1\}(p_0 - p^*), \end{aligned} \quad (11.3)$$

where

$$C_1 = (\beta/\alpha)[R_3^2 + [(\delta/(\sigma + \beta\delta)) + (\alpha V/\beta)]R_3 + \alpha((\sigma + \beta\delta)/\beta)] > 0$$

and c_{32}/c_{31} can be positive, zero, or negative but less than one in absolute value, implying that perverse exchange rate response is not possible.¹¹

Since both the price level and output are sticky and cannot jump, the exchange rate must jump to place the system onto the path converging toward the new steady-state. Such a jump (11.3) is unique and ensures that the

system will be dynamically stable. After the jump, the system moves along the stable path characterized by

$$de/dt = E(de/dt) = -\theta_1(e_t - e^*) + \theta_2(p_t - p^*), \quad (12.1)$$

$$(e_t - e^*) = - (c_{32}/c_{31})(p_t - p^*) - (c_{33}/c_{31})(y_t - y^*), \quad (12.2)$$

where¹²

$$\theta_1 = (\phi/\beta)(C_1/C_3) > 0,$$

$$\theta_2 = (\phi/\beta)[(\phi\delta - (1-\tau)) + (1/\alpha)(\delta/(\sigma + \beta\delta)) + (1/\alpha)((\sigma + \beta\delta)R_3^2 + (\phi/\beta)R_3 + (\pi/\beta))]/C_3,$$

$$C_3 = (\beta/\alpha)[(1/\alpha)(\sigma + \beta\delta)R_3^2 + (\phi/\beta)R_3 + (\pi/\beta)] > 0,$$

and $\theta_1 > 0$ since $C_1 > 0$ and $C_3 > 0$ while θ_2 is ambiguous.¹³

The Impact Effects of Monetary Expansion

Again, consider a monetary expansion. The impact effects are

$$d(i_0 - i^*)/dm = - (1/\beta) < 0, \quad (13.1)$$

$$d(e_0 - e^*)/dm = (c_{32}/c_{31}) = ((1/\beta) - \theta_2)/\theta_1, \\ = [- (\phi\delta - (1-\tau)) + (1/\alpha)(\delta/(\sigma + \beta\delta))]/C_1, \quad (13.2)$$

and, because both output and the price level cannot jump, $d(p_0 - p^*)/dm = -dp^*/dm = -1$ and $d(y_0 - y^*)/dm = -dy^*/dm = 0$. Given a monetary expansion, the interest rate must decline initially to maintain money market equilibrium (13.1); since the foreign interest rate is unaffected, asset market equilibrium requires that there must be an expectation of a subsequent appreciation, i.e., $E(de/dt) < 0$. However, the decline in the interest rate and thus the expectation of a subsequent appreciation is no longer necessarily associated with overshooting, as in the Dornbusch model. Instead, the domestic currency will depreciate and may overshoot, neither overshoot nor

undershoot, or undershoot its new long-run equilibrium value (13.2).

The condition for the neither overshooting nor undershooting of the exchange rate is now given by $c_{32}/c_{31} = 0$ or, equivalently,

$$\phi\delta - (1-\tau) = (1/\alpha)(\delta/(\sigma + \beta\delta)) > 0, \quad (14.1)$$

$$1/\beta = \theta_2 > 0. \quad (14.2)$$

Thus, when $\phi\delta - (1-\tau) \leq 0$ or $\theta_2 \leq 0$, the exchange rate will overshoot, and when $\phi\delta - (1-\tau) > 0$ and $(1/\alpha)(\delta/(\sigma + \beta\delta)) > (=; <) \phi\delta - (1-\tau)$ or $\theta_2 > 0$ and $1/\beta > (=; <) \theta_2$, the exchange rate will overshoot (neither overshoot nor undershoot; undershoot) its new long-run equilibrium value.

If output can adjust instantaneously ($\alpha = \infty$), then (14.1) reduces to the Dornbusch's condition, $\phi\delta - (1-\tau) = 0$.¹⁴ Clearly, when output adjustment is sluggish ($\alpha < \infty$), the condition is now more stringent, i.e., the likelihood of overshooting is greater, than when output adjustment is instantaneous.¹⁵ Thus, if with instantaneous output adjustment and sluggish price adjustment the interest rate declines (remains the same; increases) and the exchange rate overshoots (neither overshoots nor undershoots; undershoots), then constraints on both output adjustment, as reflected by the term $(1/\alpha)(\delta/(\sigma + \beta\delta))$, and price adjustment will cause the interest rate to decline and the exchange rate to overshoot by a greater extent (overshoot; overshoot, neither overshoot nor undershoot, or undershoot); thus an additional constraint tends to reinforce overshooting (produce overshooting; dampen undershooting). This result is not surprising because when the adjustment of a variable is constrained, the variables which can freely adjust will have to change by a greater amount. This is simply the Le Chatelier's principle applied to a dynamic general equilibrium system.

Interest Rate Decline and Expectation of A Subsequent Appreciation

In this model with sluggish output adjustment, as in (both versions of) the Dornbusch model, a decline in the interest rate implies, given uncovered interest rate parity and a fixed foreign interest rate, that there must be an expectation of a subsequent appreciation ($E(de/dt) < 0$).

In the Dornbusch model, since $E(de/dt)$ depends only on the exchange rate deviation $e_t - e^*$, or the price deviation $p_t - p^*$ but $p_t - p^*$ and $e_t - e^*$ are directly proportional and are the same sources of information ((5.1) and (5.2)) and if $i - i^* = E(de/dt) < 0$, then initially the exchange rate must depreciate and overshoot its new long-run equilibrium value. This implies that an interest rate decline and thus an expectation of a subsequent appreciation must be associated with initial overshooting and therefore $e_t - e^*$ can be inferred from the sign of $i - i^*$ or $E(de/dt)$. Thus in this model there can be an expectation of a subsequent appreciation only if the exchange rate overshoots initially.

Now when output adjustment is sluggish, $E(de/dt)$ depends not only on the exchange rate deviation $e_t - e^*$ but also on the price deviation $p_t - p^*$ (or the output deviation $y_t - y^*$), and the two are no longer directly proportional and therefore are different sources of information for rational speculators ((12.1) and (12.2)).¹⁸ This implies that exchange rate overshooting is no longer necessarily associated with an interest rate decline and an expectation of subsequent appreciation. That is, $e_t - e^*$ can no longer be inferred from $i - i^*$ or $E(de/dt)$ and hence there can be an expectation of a subsequent appreciation even when the exchange rate does not overshoot initially. With perfect foresight, an expectation of a subsequent appreciation means that following the initial response there will indeed be an actual subsequent appreciation, and this is examined next.

The Dynamics of Monetary Expansion

After the jump, the convergence towards the new steady-state, because of the underlying second-order dynamics, would no longer be necessarily monotonic nor unidirectional, as in the Dornbusch model. The analysis below assumes that the negative roots R_1 and R_2 are real and distinct.

The bounded solution to (3.2) is of the following form (see Appendix):

$$e_t - e^* = K_1[\exp(R_1 t) - (b_{21}/b_{22})(R_1/R_2)\exp(R_2 t)], \quad (15.1)$$

$$p_t - p^* = b_{21}K_1[\exp(R_1 t) - (R_1/R_2)\exp(R_2 t)], \quad (15.2)$$

$$y_t - y^* = b_{21}K_1(R_1/\pi)[\exp(R_1 t) - \exp(R_2 t)]; \quad (15.3)$$

and the solution to (3.1) is

$$i_t - i^* = R_1 K_1[\exp(R_1 t) - (b_{21}/b_{22})\exp(R_2 t)], \quad (15.4)$$

where

$$b_{21}K_1 = - (R_2/R_1)b_{22}K_2, \quad (16.1)$$

$$K_1 = (1/b_{21})[R_2/(R_1 - R_2)]dp^*, \quad (16.2)$$

$$R_1 K_2 = (a_{12} + a_{13}(R_1/\pi))b_{22}K_1 = (\phi/\pi\beta)(\pi/\phi + R_1)b_{22}K_1. \quad (16.3)$$

and (16.1) and (16.2) must hold because the price level cannot jump but $dp^* > 0$, output cannot jump and $dy^* = 0$, and, from (1.2), $dp/dt = \pi(y_t - y^*)$.

If $|R_2| > |R_1|$ ($|R_1| > |R_2|$), then $b_{21}K_1 < 0$ (> 0) and $b_{22}K_2 > 0$ (< 0); since $b_{21}K_1$ is associated with R_1 and $b_{22}K_2$ is associated with R_2 , then in either case, the negative constant is associated with the root that is smaller in absolute value and thus the implied paths are the same. Since it is immaterial which root is greater in absolute value, it is assumed here that

$$|R_2| > |R_1| \text{ or } b_{21}K_1 < 0, \quad (17)$$

implying that $R_1 - R_2 > 0$ or $R_1/R_2 < 1$, $b_{22}K_2 > 0$, and $|b_{21}K_1| > b_{22}K_2$.

Given (17) and (1.2), the price and output paths can be derived from (15.2) and (15.3). Inspection of (15.2) and (15.3) shows, respectively, that the t for which $p_t = p^*$ and the t for which $dp/dt = 0$ and thus $y_t = y^*$ do not exist, implying that the price path is unidirectional, i.e., over the interval $(0, \infty)$, the price level is monotonically rising and $y_t > y^*$.¹⁷ On the other hand, the output path resembles an inverted U, with the maximum occurring at

$$t_1 = \ln(R_2/R_1)/(R_1 - R_2) > 0, \quad (18.1)$$

where $0 < t_1 < \infty$ and t_1 is derived by using (15.3) and setting $dy/dt = 0$. This means that over the interval $(0, t_1)$, both the price level and output are rising; at t_1 , output reaches maximum; and, over the interval (t_1, ∞) , the price level is rising but output is declining.

The exchange rate and interest rate paths can be described using the uncovered interest rate parity condition (1.4), the perfect foresight assumption (1.5), (15.1), (15.4), and the following:

$$t_2 = \ln[(b_{21}/b_{22})(R_1/R_2)]/(R_1 - R_2), \quad (18.2)$$

$$t_3 = \ln(b_{21}/b_{22})/(R_1 - R_2), \quad (18.3)$$

$$t_4 = \ln[(b_{21}/b_{22})(R_2/R_1)]/(R_1 - R_2). \quad (18.4)$$

where t_2 is derived from (15.1) by setting e_t equal to e^* ; t_3 , from ((15.1) or (15.4)) by setting de/dt or $i_t - i^*$ equal to zero; and, t_4 , from (15.4) by setting di/dt equal to zero. While (17) makes definite the price and output paths, the exchange rate and interest rate paths are much more difficult to characterize because they also depend on b_{21}/b_{22} . As shown below, there are two possible interest rate paths and four possible exchange rate paths (see Appendix).

Case 1 *Initial overshooting followed by appreciation*: $(b_{21}/b_{22})(R_2/R_1) < 1$ and thus $b_{21}/b_{22} < 1$ and $(b_{21}/b_{22})(R_1/R_2) < 1$, $\phi\delta - (1-\tau) < 0$ but $|\alpha(\phi\delta - (1-\tau))| > R_f$, $b_{2f} < 0$, $K_1 > 0$, and $e_0 - e^* = K_1 + K_2 > 0$. Since in this case t_2 , t_3 , and t_4 do not exist, the exchange rate and interest rate paths are both unidirectional and the transition is characterized by appreciation and rising interest rate.¹⁸

Case 2 *Initial overshooting followed by delayed undershooting, i.e., by appreciation and then depreciation*: $(b_{21}/b_{22})(R_1/R_2) > 1$ and thus $(b_{21}/b_{22})(R_2/R_1) > (b_{21}/b_{22}) > 1$, $\phi\delta - (1-\tau) < 0$ but $|\alpha(\phi\delta - (1-\tau))| < |R_f|$ or $\phi\delta - (1-\tau) = 0$ or $(1/\alpha)(\delta/(\sigma + \beta\delta)) > \phi\delta - (1-\tau) > 0$, $b_{2f} > 0$, $K_1 < 0$, and $e_0 - e^* = K_1 + K_2 > 0$, implying that $0 < t_2 < t_3 < t_4 < \infty$. The exchange rate overshoots initially and over the interval $(0, t_2)$ there is appreciation and the exchange rate attains e^* at t_2 ; over the interval (t_2, t_3) there is further appreciation and the exchange rate reaches minimum at t_3 ; and, over the interval (t_3, ∞) , there is depreciation. In the case of the interest rate, after the initial decline, it rises and attains i^* at t_3 , it rises further until maximum is reached at t_4 , and then it declines towards i^* .¹⁹

Case 3 *Initial response of neither overshooting nor undershooting followed by delayed undershooting*: $(b_{21}/b_{22}) = (R_2/R_1)$ and thus $(b_{21}/b_{22})(R_2/R_1) > b_{21}/b_{22} > (b_{21}/b_{22})(R_1/R_2) = 1$, $\phi\delta - (1-\tau) = (1/\alpha)(\delta/(\sigma + \beta\delta)) > 0$, $b_{2f} > 0$, $K_1 < 0$, and $e_0 - e^* = K_1 + K_2 = 0$. This case has the same interest rate path as in Case 2. The exchange rate path is also similar in the sense that there is subsequent undershooting; however, since the initial exchange response of neither overshooting nor undershooting is followed by undershooting, the exchange rate will not equal e^* at a finite time and therefore t_2 is non-existent. This case is a special case because $0 < t_1 = t_3 < t_4 < \infty$, i.e., maximum output and minimum exchange rate (and therefore $i_c =$

i^*) occurs at the same time. Since output and the exchange rate has the same turning point, $t_1 = t_3$, it follows that the interval $(0, t_1=t_3)$ is characterized by appreciation and rising output while the interval $(0, t_1=t_3)$ is characterized by depreciation and declining output.²⁰

Case 4 Initial undershooting followed by further undershooting:

$(b_{21}/b_{22})(R_2/R_1) > b_{21}/b_{22} > 1$ but $(b_{21}/b_{22})(R_1/R_2) < 1$, $\phi\delta - (1-\tau) > (1/\alpha)(\delta/(\sigma + \beta\delta)) > 0$, $b_{2J} > 0$, $K_1 < 0$, and $e_0 - e^* = K_1 + K_2 < 0$. This is similar to Case 3 except that here the exchange rate undershoots initially.²¹

As stated earlier, because of uncovered interest rate parity the initial interest rate decline must be associated with an expectation of a subsequent expectation. With perfect foresight, the expectation of a subsequent appreciation means that indeed there will be a subsequent actual appreciation. In all the four cases discussed above, the initial exchange response is immediately followed by appreciation. Clearly an expectation of a subsequent expectation and therefore a subsequent actual appreciation is possible even in cases where the exchange rate does not overshoot initially because in such cases the initial response is followed by undershooting (Cases 3 and 4).²²

4. Conclusions

This paper has examined a model of exchange rate dynamics which incorporates sluggish output adjustment into the Dornbusch variable output model. In this model with sluggish output adjustment, short-run aggregate demand or output, like the price level, is also sticky and cannot jump.

In (both versions of) the Dornbusch variable output model, a decline in the interest rate resulting from monetary expansion is associated with initial overshooting; when output is variable and can adjust instantaneously, such overshooting is dampened or may even be reversed. On the other hand, this paper has shown that when both the price level and output are sticky, the

interest rate must decline in response to monetary expansion but such a decline in the interest rate can be associated with overshooting, neither overshooting nor undershooting, or undershooting. However, because there is an additional constraint, sluggish output adjustment, overshooting (undershooting) is dampened (reinforced). This result is simply the Le Chatelier's principle applied to a dynamic general equilibrium system, that is, when the adjustment of a (another) variable is (is further) constrained, the variables which can freely adjust will have to change by a greater amount.

This paper has also demonstrated that exchange rate overshooting is not necessarily associated with an interest rate decline and thus an expectation of a subsequent appreciation whereas in the Dornbusch model overshooting can occur only if the interest rate declines initially. In particular, because both the price level and output are sticky, the interest rate declines on impact in response to a monetary expansion so as to maintain money market equilibrium. As in the Dornbusch model, with the foreign interest rate unaffected, the uncovered interest rate parity condition requires that the decline in the interest rate be compensated by an expectation of a subsequent appreciation. However, unlike in the Dornbusch model, in this model the exchange rate need not overshoot initially, and yet an expectation of subsequent appreciation is created because expectations depend not only on exchange rate deviation but also on price deviation, and the two are no longer the same sources of information for rational speculators.

Furthermore, given a decline in the interest rate and thus an expectation of a subsequent appreciation arising from a monetary expansion, this paper has also shown that the possible exchange rate paths are overshooting followed by unidirectional adjustment and overshooting (neither overshooting nor undershooting; undershooting) followed by delayed undershooting (delayed

undershooting; further undershooting). In all these four possibilities, the initial exchange response is immediately followed by an appreciation, even in cases where the exchange rate does not overshoot initially because in such cases the initial response is followed by undershooting. Thus, with perfect foresight, an expectation of a subsequent expectation is indeed a subsequent actual appreciation.

This paper has assumed that the negative roots are real and distinct.²³ If on the other hand the roots are imaginary, then the paths would be cyclical but dampened because the model is convergent. However, for the objective of this paper, the case of imaginary roots, while more interesting empirically, would be trivial.²⁴ Finally, this paper could have assumed positive short run aggregate demand elasticities; nevertheless, the main results of this paper would have remained the same.²⁵

Notes

¹ Other factors that can dampen/reverse overshooting are imperfect capital mobility and substitutability (e.g., Frenkel and Rodriguez (1982); Bhandari, Driskill, and Frenkel (1984)) and of course a policy reaction that limits the movement of the exchange rate.

² Bhandari (1982) derives the time paths for fiscal expansion but not for monetary expansion while Levin (1994) does not derive the time paths of the variables, and both assume a perfect-foresight consistent expectation scheme.

³ Equations (3.1.i) and (3.1.ii) are derived from (1.1) and (1.3); de/dt , from (1.1) and (1.3) to (1.5); and, dp/dt , from (1.2).

⁴ The solution to the characteristic equation associated with (3.2), $r^2 + (\pi(\sigma + \beta\delta) - \phi\delta)/V r - \pi\delta/V = 0$, is $r_1, r_2 = \{tr(a) \pm [(-tr(a))^2 - 4det(a)]^{1/2}\}/2$, where $tr(a) = r_1 + r_2 = -(\pi(\sigma + \beta\delta) - \phi\delta)/V$ and $det(a) = r_1 r_2 = -\pi\delta/V < 0$.

⁵ This condition requires $k_2 = 0 = [\phi\delta/V + (-r_1)/(r_2 - r_1)(e_0 - e^*)] + [-\phi\delta - (1-\tau)/V]/(r_2 - r_1)(p_0 - p^*)$, where k_2 is the coefficient associated with the unstable root r_2 .

⁶ At the initial exchange rate, the change in the interest rate that would re-equilibrate the money market is $(\phi\delta - (1-\tau))/V$.

⁷ Equation (8.1.ii) is derived from $y^d e_t = u + \int_{-\infty}^t \tau_0 \exp^{-\alpha(\tau-T)} y_t d\tau + \tau_s y_t - \int_{-\infty}^t \sigma_0 \exp^{-\alpha(\tau-T)} i_t d\tau - \sigma_s i_t + \int_{-\infty}^t \delta_0 \exp^{-\alpha(\tau-T)} (e_t - p_t + p_t) d\tau + \delta_s (e_t - p_t + p_t)$, which implies that $y^d e_t = u + (\tau_0/\alpha + \tau_s) y_t - (\sigma_0/\alpha + \sigma_s) i_t + (\delta_0/\alpha + \delta_s)(e_t - p_t +$

p_r) where, given (8.1.i), $\tau = \tau_0/\alpha + \tau_S$, $\sigma = \sigma_0/\alpha + \sigma_S$, and $\delta = \delta_0/\alpha + \sigma_S$. Differentiating the y^{ds}_t equation with respect to time and using the (8.1.i) yields $dy/dt = \tau_S(dy/dt) - \sigma_S(di/dt) + \delta_S(de/dt - dp/dt) + \alpha(y^{dL}_t - y^{ds}_t)$. Assuming that $\tau_S = \sigma_S = \delta_S = 0$, i.e., short-run aggregate demand or output cannot jump, then the y^{ds}_t equation and the dy/dt equation are given by (8.1.ii) and (8.2.ii). Equation (8.1.ii) has the same form as when output adjusts sluggishly because of production lag (Levin (1894)).

⁸ Equation (9.1) is derived from (1.3); de/dt , from (1.3) to (1.5); dp/dt , from (1.2); and, dy/dt , from (8.2.ii), (8.1.i), and (1.3).

⁹ The other two roots are either real and negative or a pair of complex conjugates with negative real parts.

¹⁰ An alternative and equivalent procedure can be cast in terms of right eigenvectors, as in Buiter and Miller (1980, Appendix); this procedure would be simpler for a model with only one predetermined variable and hence one stable root.

¹¹ Since $c_{32}/c_{31} = [1 - (R_1/R_2)(b_{21}/b_{22})]/[b_{21}((R_1/R_2) - 1)]$, if $(R_1/R_2) < 1$ and $b_{21} < 0$ (> 0 ; > 0 ; > 0) and thus $b_{21}((R_1/R_2) - 1) > 0$ (< 0 ; < 0 ; < 0) and if $(R_1/R_2)(b_{21}/b_{22}) < 1$ (> 1 ; $= 1$; < 1), then $c_{32}/c_{31} > 0$ (> 0 ; $= 0$; < 0) but $|c_{32}/c_{31}| < 1$. See (A6), (A7.1), and (A8.1) in the Appendix.

¹² There are two other expectation schemes equivalent to (12.1): one depends on $p_t - p^*$ and $y_t - y^*$ and the other depends on $e_t - e^*$ and $y_t - y^*$.

¹³ Since $c_{32}/c_{31} = [(1/\beta) - \theta_2]/\theta_1 = -1 + [(1/\beta) - \theta_2 + \theta_1]/\theta_1$ and if $c_{32}/c_{31} < 0$, then $\theta_2 > 1/\beta$; but since $|c_{32}/c_{31}| < 1$, it follows that $(1/\beta) - \theta_2 + \theta_1 > 0$.

¹⁴ The Bhandari (1982, 1983) model can be reduced to the Dornbusch variable-output model if it is assumed that $\alpha = \infty$, so that there is no longer a distinction between short-run and long-run aggregate demand elasticities.

¹⁵ In Bhandari's model, because short-run aggregate demand elasticities are nonzero and thus $y^{ds} (= y)$ can jump, the interest rate will not necessarily decline, overshooting (undershooting) will be reinforced (dampened), and both the exchange rate and output may respond perversely.

¹⁶ Since $dp^* > 0$, the initial price deviation, $p_0 - p^* = -dp^* < 0$, signals that the price level will subsequently rise; when $\theta_2 > 0$ (< 0), this tends to create an expectation of a subsequent appreciation (depreciation) so that the exchange rate need not overshoot (has to overshoot to create an expectation of a subsequent appreciation).

¹⁷ A t is said to exist if it is positive and finite.

¹⁸ When $\phi\delta - (1-\tau) < 0$ but $|\alpha(\phi\delta - (1-\tau))| > R_f$, $\pi/\phi > |R_f|$. Note that coefficient A_2 in (10) can be rewritten as $\alpha(\sigma + \beta\delta)(\phi/\beta)[(\pi/\phi) - (\delta/(\sigma + \beta\delta))]$, implying that if $\pi/\phi > (<) \delta/(\sigma + \beta\delta)$ then $A_2 > 0$ (< 0).

¹⁹ When $\phi\delta - (1-\tau) < 0$ but $|\alpha(\phi\delta - (1-\tau))| < |R_f|$ or $\phi\delta - (1-\tau) \geq 0$, $\pi/\phi > R_f$. In this case $t_1 < t_3 < t_4$ so that the exchange rate reaches minimum at a later (an earlier) time than output (the interest rate) reaches maximum.

²⁰ In this case $\pi/\phi > R_f$ since $\phi\delta - (1-\tau) > 0$ and, since $t_1 = t_3 < t_4$, output reaches maximum and the exchange rate reaches minimum at an earlier time than the interest rate reaches maximum.

²¹ As in Case 3, $\pi/\phi > R_f$ since $\phi\delta - (1-\tau) > 0$, but in this case $t_3 < t_1 < t_4$ and output reaches maximum at a later (an earlier) time than the exchange rate (the interest rate) reaches minimum (maximum).

²² This appreciation is temporary as it is followed by depreciation.

²³ Since the paths of the interest rate and the exchange rate as well as

the real exchange rate is not necessarily unidirectional, the implication is that investment and net exports can be volatile.

²⁴ When the paths are oscillatory, the transition would be characterized by alternating depreciations and appreciations and what this paper would like to show is that the initial exchange response will be followed by appreciation.

²⁵ In terms of Bhandari's (1982, 1983) model, the result that sluggish aggregate demand or output adjustment reinforces (dampens) overshooting (undershooting) will be left intact but the interest rate will no longer necessarily decline. See note 15 above.

Appendix

This appendix derives the paths of the price level, output, interest rate, and the exchange rate following a monetary expansion. The analysis assumes that the two negative roots R_1 and R_2 are real and distinct.

The solution to the dynamic system (3.2) has the following form:

$$e_t - e^* = b_{11}K_1\exp(R_1t) + b_{12}K_2\exp(R_2t) + b_{13}K_3\exp(R_2t), \quad (A1.1)$$

$$p_t - p^* = b_{21}K_1\exp(R_1t) + b_{22}K_2\exp(R_2t) + b_{23}K_3\exp(R_2t), \quad (A1.2)$$

$$y_t - y^* = b_{31}K_1\exp(R_1t) + b_{32}K_2\exp(R_2t) + b_{33}K_3\exp(R_2t), \quad (A1.3)$$

where

$$K_j = c_{j1}(e_0 - e^*) + c_{j2}(p_0 - p^*) + c_{j3}(y_0 - y^*), \quad (A2.4)$$

$$b_{2j} = [a_{13}a_{31} + a_{33}R_j - R_j^2]/[(a_{12}a_{33} - a_{13}a_{32}) - a_{12}R_j], \quad (A2.2)$$

$$b_{3j} = [-a_{12}a_{31} - a_{32}R_j]/[(a_{12}a_{33} - a_{13}a_{32}) - a_{12}R_j], \quad (A2.3)$$

$$= (R_j/\pi)b_{2j}, \quad (A2.3')$$

and $b_{1j} = 1$, c_{ji} is the element in the j^{th} row and the i^{th} column of B^{-1} , and B is the matrix containing b_{ij} . Note that (A2.3') holds because of (1.2).

The terminal condition, which requires

$$K_3 = 0 = c_{31}(e_0 - e^*) + c_{32}(p_0 - p^*) + c_{33}(y_0 - y^*), \quad (A3.1)$$

implies that the stabilizing exchange rate jump is

$$e_0 - e^* = -(c_{32}/c_{31})(p_0 - p^*) - (c_{33}/c_{31})(y_0 - y^*). \quad (A3.2)$$

Given that $K_3 = 0$, (11.1) and (A1.2) imply that $p_0 - p^* = b_{21}K_1 + b_{22}K_2 = -dp^*$ while (11.2), (A1.3), and (A2.3') imply that $y_0 - y^* = b_{21}(\pi/R_1)K_1 + b_{22}(\pi/R_2)K_2 = -dy^* = 0$. Thus, respectively,

$$b_{22}K_2 = -b_{21}K_1 - dp^*, \quad (A4.1)$$

$$b_{22}K_2 = -(R_1/R_2)b_{21}K_1, \quad (A4.2)$$

Solving (A3.2) and (A3.3) simultaneously for the constants K_1 and K_2 yields

$$K_1 = (1/b_{21})[R_2/(R_1 - R_2)]dp^*, \quad (A5.1)$$

$$K_2 = -(1/b_{22})[R_1/(R_1 - R_2)]dp^*. \quad (A5.2)$$

Substitutions of (A4.2), given (A3.1), (A2.1) and (A2.3'), into (A1.1) to (A1.3) yield (15.1) to (15.3) in the text.

The solution for i_t can be derived using (3.1), (A1.2), and (A1.3); alternatively, since $i_t - i^* = E(de/dt) = de/dt$, it can also be derived from (A1.1); given (A2.3') and (A4.2), these yield (15.4) in the text.

The relationship between K_1 and K_2 and coefficients b_{21} and b_{22} , given (A1.1), (11.3), (A4.2), (A2.3), (A2.3'), and (A5.2), can be inferred from

$$e_0 - e^* = (c_{32}/c_{31})dP^* = K_1 + K_2 = \{1 - [(b_{21}/b_{22})(R_1/R_2)]\}K_1, \quad (A6)$$

where

$$b_{21} = [\pi\alpha(\sigma + \beta\delta)/(-R_J)][\delta/(\sigma + \beta\delta) - R_J]/[\alpha(\phi\delta - (1-\tau)) - R_J], \quad (A7.1)$$

$$= [\pi(-R_J)/(\phi/\beta)]/(-\pi/\phi - R_J), \quad (A7.2)$$

$$b_{21}/b_{22} = (R_2/R_1)[((\delta/(\sigma + \beta\delta)) - R_1)/(\alpha(\phi\delta - (1-\tau)) - R_1)]/ \\ [((\delta/(\sigma + \beta\delta)) - R_2)/(\alpha(\phi\delta - (1-\tau)) - R_2))], \quad (A8.1)$$

$$= (R_1/R_2)((\pi/\phi + R_2)/(\pi/\phi + R_1)). \quad (A8.2)$$

It follows from (A8.1) and (A8.2) that $(b_{21}/b_{22})(R_1/R_2) = [.] = (R_1^2/R_2^2)((\pi/\phi + R_2)/(\pi/\phi + R_1))$ and that $(b_{21}/b_{22})(R_2/R_1) = (R_2^2/R_1^2)[.] = (\pi/\phi + R_2)/(\pi/\phi + R_1)$. It also follows from (A7.1) and (A7.2) that $\alpha(\phi/\beta)(\sigma + \beta\delta)[\delta/(\sigma + \beta\delta) - R_J] = [\alpha(\phi\delta - (1-\tau)) - R_J]/(-\pi/\phi - R_J) > 0$.

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