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Macromodels and Walras' Law

by

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## Abstract

Using a short-period macromodel with familiar components, this paper reformulates the aggregate demand (AD) function so that in conjunction with an aggregate supply (AS) function, one has an AS/AD framework with the following features: unlike the textbook treatment based on IS-LM where the  $I = S$  equilibrium condition is satisfied at every point of the AD curve,  $I = S$  only where  $AS = AD$ ; involuntary unemployment is the typical case; and the money wage is not exogenously given. Walras' Law does not hold in the model. It may hold in other short-period models but only because of special assumptions about AD or AS.

Using a short-period macromodel with familiar components, this paper reformulates the aggregate demand (AD) function so that in conjunction with an aggregate supply (AS) function, one has an AS/AD framework with the following features: unlike the textbook treatment based on IS-LM where the  $I = S$  equilibrium condition is satisfied at every point of the AD curve (see e.g. Dornbusch and Fischer, 1990),  $I = S$  only where  $AS = AD$ ; involuntary unemployment is the typical case; and the money wage is not exogenously given. Walras' Law does not hold in the model. It may hold in other short-period models but only because of special assumptions about AD or AS. In particular, it is assumed that AD in the short-period general equilibrium model is identically equal to the output that can be produced by the supply of labor.

### I. The Model

The model pertains to the short period and presupposes a given state of expectations regarding the future. Denoting the price level by  $p$  and the money wage by  $w$ , write  $v = p/w$  for the inverse of the real wage. With the money stock  $M$  given, let

$$(1) \quad Y = f(N), \quad f'(N) > 0$$

$$(2) \quad N = n(v), \quad n'(v) > 0$$

$$(3) \quad Y^d = g(Y, r, M/p), \quad g_Y > 0, \quad g_r < 0, \quad g_{M/p} > 0$$

$$(4) \quad M/p = L(Y, r), \quad L_Y > 0, L_r < 0$$

$$(5) \quad k(Y, r, M/p) = 0, \quad k_Y > 0, k_{M/p} > 0$$

$$(6) \quad Y = Y^d$$

where  $Y$  is real output,  $N$  employment,  $Y^d$  demand from output,  $r$  the interest rate. Eq. (1) is the production function, omitting the given capital stock; (2) derives from the profit-maximization condition that calls for firms to equate the marginal product of labor to the real wage; (3) is the usual consumption plus investment demand function with real balances included as an argument; (4) and (5) are the money and bond markets equilibrium conditions respectively (the left-hand side of (5) is an excess demand function); (6) says the goods market clears. Calling (1)-(6) Model A, the model just suffices to determine the (equilibrium) values of the six variables  $Y, N, v, Y^d, r$  and  $p$ , so  $w$  is also endogenously determined.

Model A can be condensed into a simple AS/AD schema. From (1) and (2),

$$(AS) \quad Y = f^*(v), \quad f^{*'}(v) > 0$$

giving AS as a function of  $v$ . To construct the AD function, use (4) to eliminate  $M/p$  in (3) so that

$$(3a) \quad Y^d = g^*(Y, r)$$

where  $g_Y^* = g_Y + g_{M/p} L_Y > 0$  and  $g_r^* = g_r + g_{M/p} L_r < 0$ . Eliminating  $M/p$  in (5) also,

$$(5a) \quad k^*(Y, r) = 0$$

where  $k_T^a = k_T + k_{M/P}L_T > 0$  and  $k_T^s = k_T + k_{M/P}L_T$  whose sign, like that of  $k_T$ , is unclear. It seems plausible that  $k_T > 0$  but  $k_T^s$  could take either sign. From (3a) and (5a) one can write

$$(3b) \quad Y^d = g^{**}(Y)$$

by eliminating  $r$ . (Graphically, in a diagram with  $Y$  on the horizontal axis and  $r$  on the vertical, plot (3a) for different values of  $Y^d$ . The curve corresponding to a particular value of  $Y^d$  will be upward sloping and curves corresponding to higher values will be farther to the right. Plotting (5a) also, the intersection of (5a) and a particular  $Y^d$  curve then gives a point  $(Y, Y^d)$  for (3b). It is reasonable that  $g^{**'}(Y) > 0$ , so we assume that the slope of (5a) is less than that of any  $Y^d$  curve so that a higher value of  $Y^d$  does get associated with a higher value of  $Y$ . This condition is of course satisfied if (5a) is downward sloping, which follows from  $k_T^s > 0$ ,<sup>11</sup> but the latter is not necessary.)

Eq. (3b) makes  $Y^d$  a function only of  $Y$  with cleared money and bond markets.<sup>12</sup> Using (1a), (3b) can be written as

$$(AD) \quad Y^d = g^{***}(v)$$

giving AD as a function of  $v$ . (AS) and (AD) are plotted in Fig. 1 showing  $e$  as the equilibrium point where  $AS = AD$ . As seems called for by the logic of it, at no other point on the AD curve is the equilibrium condition  $I = S$  satisfied. The standard Keynesian 45° diagram obtains from Fig. 1 by changing the variable on the horizontal axis to  $Y$ , which replaces (AD) with (3b) and, using (1a), converts the AS curve to a 45° line since  $Y =$

$$f^*(v) = f^*(f^{*-1}(Y)) = Y.$$

## II. Involuntary Unemployment and Walras' Law

The usual labor supply function can be written as  $N^s = h(v)$ ,  $h'(v) < 0$ . Model A only requires that  $N \leq N^s$ . In order to depict labor supply in Fig. 1, let  $Y^d = f(N^s) = h^*(v)$  denote the output that can be produced by  $N^s$  workers. Since the functions  $f^*$ ,  $g^{***}$  and  $h^*$  are different in general from one another,

$$(7) \quad Y^d = h^*(v), \quad h^{*'}(v) < 0$$

plots as shown in Fig. 1. The labor market clears if  $N = N^s$  or, equivalently,

$$(8) \quad Y = Y^d.$$

Notice that the inclusion of (7) and (8) would add two new equations but only one new variable,  $Y^d$ , to Model A. Calling (1)-(8) Model B, the latter is overdeterminate. The  $N = N^s$  case would thus be fortuitous, and the general case is  $N < N^s$ .

Involuntary unemployment is measured by  $h(v_e) - n(v_e)$  or, in terms of the corresponding outputs, by  $h^*(v_e) - f^*(v_e)$ . (We indicate equilibrium values of the variables with  $e$  subscripts.) As Keynes (1936, p. 15) put it, "in the event of a small rise in  $[v]$  both the aggregate supply of labor willing to work for the current money wage and the aggregate demand for it at that wage would be greater than the existing volume of employment." In output terms, both  $h^*(v)$  and  $f^*(v)$  are greater than  $f^*(v_e)$  for  $v$  slightly higher than  $v_e$ .

Walras' Law implies that any single market clears if all the other markets do so. (This implication will be the focus of our attention.) Since the money, bond and goods markets clear at  $e$  but not the labor market, Walras' Law fails to hold.<sup>3/</sup> The reason is not that  $w$  is exogenously fixed, as in the textbook Keynesian model, but because  $v$ , which equates  $Y = f^*(v)$  and  $Y^d = g^{***}(v)$ , cannot be expected to equate  $Y$  and  $Y^d = h^*(v)$  also.

It will be recognized that Model B is essentially the full-employment model of Patinkin (1965, ch. 9) which he claimed to be just determinate assuming Walras' Law (Patinkin, 1965, p. 229). As we have seen, Model B is actually overdetermined. In order for Walras' Law to hold, the overdeterminacy has to be removed, which can be done by revisions or assumptions that make two of the functions  $f^*$ ,  $g^{***}$  and  $h^*$  (or their modifications) always have the same values. There are three ways of accomplishing this, and all three have figured in the literature.

### III. Three Other Models

#### The classical model

In Model B, suppose  $AD = AS$  always (so the function  $g^{**}$  in (3b) is the identity relation), which might be called Say's Law in the sense of Keynes. Then there would be only two distinct curves in Fig. 1,  $N = N^s$  at the intersection of the  $Y$  and  $Y^d$  curves, and all markets clear. As Keynes (1936, pp. 21-22) had observed, the following three propositions are equivalent: (i) "the real wage is equal to the marginal disutility of the existing employment," (ii) "there is no such thing as involuntary unemployment," and (iii) "the aggregate demand price is equal to the aggregate supply price for all levels of employment and output."<sup>4/</sup>

Perhaps no one today would accept the assumption that  $AD = AS$  identically. The next model is more subtle.

### Short-period general equilibrium

The original interpretation of Arrow and Debreu (1954) of their formal model of general equilibrium is that its finite time-horizon spans the entire lifetimes of consumers and firms in an economy "where consumers own the resources and control the producers" (Debreu, 1959, p. 74), but it has since been considered to be applicable to the short period as well. In the Arrow-Debreu model a consumer's potential income is basic: his potential income, which depends on his supply of labor and share of firms' profits, is what appears in his budget constraint. Accordingly a revision of Model B is needed.

First, in (3)-(5) replace  $Y$  with  $Y^d$  so that

$$(3') \quad Y^d = G(Y^d, r, M/P)$$

$$(4') \quad M/P = L^*(Y^d, r)$$

$$(5') \quad K(Y^d, r, M/P) = 0$$

and repeating the earlier discussion,

$$(3a') \quad Y^d = G^*(Y^d, r)$$

$$(5a') \quad K^*(Y^d, r) = 0$$

$$(3b') \quad Y^d = G^{**}(Y^d).$$

Second, to allow for  $r$  as a possible determinant of labor supply-- $r$  is

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taken into account in the new classical macroeconomics (see e.g. Peel, 1989)--

let  $N^d = H(v, r)$  so

$$(7') \quad Y^d = H^*(v, r)$$

which can be reduced to

$$(7'') \quad Y^d = H^{**}(v)$$

by putting in (7') the values of  $r$  eliminated in getting (3b') from (3a') and (5a'). Then, from (3b') and (7''),

$$(AD') \quad Y^d = G^{***}(v)$$

Call the above revision Model C.

The key to Model C is the fact that in the Arrow-Debreu model "each individual spends his entire potential income because of the absence of satiation (and since the model covers his entire economic life)" (Arrow and Debreu, 1954, p. 272). In the short-period application this means that  $AD' = Y^d$  identically and therefore, as in the classical case, there would be only two distinct curves in Fig. 1 and all markets clear.

McKenzie (1987, p. 503) has noted that the short-period interpretation of general equilibrium "raises two problems. One is to distinguish between resources devoted to this period's consumption and resources reserved for the support of consumption in future periods. The other is to explain how the decision to reserve a certain quantity of resources for future use is made." It might be supposed that each consumer's utility function includes investment goods among its arguments, evaluating then in terms of their implications for

future consumption, and transferring them to the firms where they own shares. The difficulty is that the uncoordinated decisions of the individual owners of a firm cannot substitute for the investment decision of the firm itself. Another set of problems has to do with the inclusion of money and bonds which are absent in the Arrow-Debreu model. In order to maintain  $Y^d = Y^s$  identically when the money and bond markets clear, one has to require that any net change in each individual's money holdings be accounted for solely by a corresponding opposite change in his bond holdings (so that  $I = S$  identically). This is evidently an artificial restriction.

#### Worker-owned single-worker firms

Returning to Model B, suppose each firm is owned by its single worker, an assumption in effect made e.g. in Lucas (1972). This changes the aggregate supply function to

$$AS') \quad Y = f(N^s) = h^*(v)$$

since each firm's owner provides its only source of labor. Thus  $AS' = Y^d$  identically, and all markets clear.

#### IV. Stability of the Equilibrium Money Wage

Returning to Model A, one would want to know why the money wage does not fall in the presence of excess labor supply, for we have become accustomed to the idea that the price in a market falls when there is excess supply. In a goods market a lower price reduces supply and also promotes sales, so a price movement serves an equilibrating function. The idea does not extend, however, to the labor market. Fig. 1 is drawn under the assumption that Model A is

stable, i.e.  $g^{**}(Y) < 1$  in (3b). Looking at Fig. 1 a fall in  $w$ , which raises  $v$ , increases  $Y^d$  less than it does  $Y$  resulting in excess supply of goods. A fall in  $w$  cannot therefore help to equilibrate the labor market.

In more detail, suppose the equilibrium is perturbed and there is a new  $w < w_e$ . With  $p = p_e$ , there would be pressure from employed workers to raise  $w$  because of higher living costs, and since profits are higher, firms can accommodate a higher  $w$  to prevent productivity losses from demoralization. If labor does not react immediately to the fall in the real wage but firms do, a more complicated course of events is possible. Start again with  $w < w_e$  and other variables at their equilibrium values. From (AS) firms produce a higher  $Y$ , so  $Y^d < Y$  and  $p$  falls. To simplify a little, let  $v$  fall back to  $v_e$  and  $Y$  to  $Y_e$ . Now with  $p < p_e$ ,  $M/p$  is higher than called for so  $r$  is lower in (4) which makes  $Y^d > Y$  and  $p$  rises this time. There would be pressure from employed workers to raise  $w$ , etc., and the fall in  $w$  gets reversed.

Consider the opposite case of  $w > w_e$ . Firms produce a lower  $Y$ , workers are laid off, profits are lower, and there is pressure on  $w$  to fall. In short, the equilibrium money wage is stable, which is consistent with money wage rigidity or stickiness.

#### V. Concluding Remarks

This paper has described a simple macromodel which can be further simplified into an AS/AD schema. AD, like AS, is a function of (the inverse of) the real wage. The money and bond markets clear at every point of AD, and only at its intersection with AS is there a Keynesian equilibrium

where  $I = S$  and the goods market also clears. The standard view is that an exogenously fixed money wage is needed to account for involuntary unemployment, but in the present model the price level and the money wage are both endogenously determined. Adding a labor supply function and a labor market clearing requirement results in overdeterminacy, so Walras' Law does not hold. In an economy where firms hire workers for money wages in order to sell the output they produce, the goods market clearing condition determines employment independently of any excess supply that might exist in the labor market.

Walras' Law does hold in three models whose assumptions remove the overdeterminacy by making two generally different functions have the same values. In the classical model all markets clear under the assumption that  $AD = AS$  always, as Keynes had pointed out. In short-period general equilibrium, aggregate demand in the model is identically equal to the output  $Y^d$  that can be produced by the supply of labor. In a model with worker-owned single-worker firms, aggregate supply is identically equal to  $Y^s$ . Because of their special assumptions, these three models do not seem to be especially convincing.

Notes

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1. Differentiating (5a),  $k_Y^* dY + k_r^* dr = 0$  so  $dr/dY = -k_Y^*/k_r^* < 0$  if  $k_r^* > 0$ .

2. In other words,  $r$  and  $M/p$  drop out of (3) by using (4) and (5). Notice that if  $r$  and  $M/p$  were deleted from (3) in the subsystem (1)-(3) and (6), this subsystem would just suffice to determine  $Y, N, v$  and  $Y^d$ . Restoring  $r$  and  $M/p$  in (3) and adding (4) and (5) just suffices to determine  $p$  and  $r$  also.

3. As Clower (1965) has argued, Walras' Law cannot be considered as a fundamental principle.

4. In the perspective of this paper, Keynes' aggregate demand price can be thought of as  $g^{**}(v)$  multiplied by  $p = vw$  or, expressed in wage-units (deflated by  $w$ ),  $g^{**}(v)v$ .

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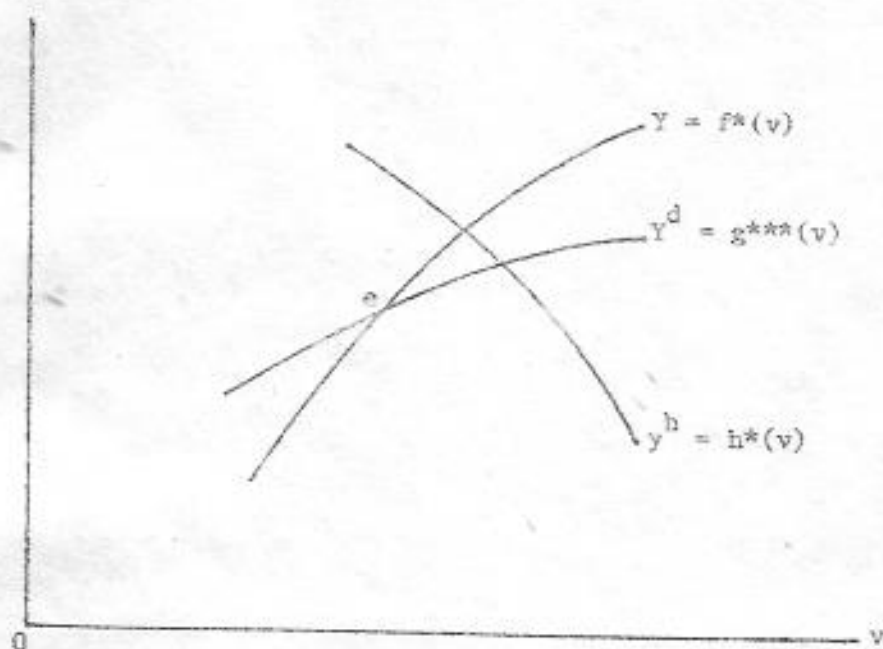


Fig. 1