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Optimum Quantity of Money Instability

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Abstract: It is shown that optimum quantity of money equilibria are unstable in the simplest models. Getting back to the desired path requires a temporary suspension of the money growth rule.

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1. Introduction

One of the most interesting topics in monetary theory is the idea of an optimum quantity of money (OQM for short) which was made prominent by Friedman (1969). In a recent survey of the current state of knowledge regarding OQM Woodford (1990) does not bring out the fact that OQM equilibrium is unstable. This is done in the present paper, which also suggests a simple response to the instability.

2. The basic model

Following largely the formulation of Brock (1975), assume that the representative individual, infinitely long-lived, has a constant endowment of real income y every period and maximizes

$$W = \sum_{t=0}^{\infty} \beta^t (u(c_t) + v(m_t)) \tag{2.1}$$

where β is the time discount factor $(0 < \beta < 1)$, c_t is real consumption and m_t real balances in period t, and $u(\cdot)$, $v(\cdot)$ are their corresponding one-period utilities. The representative individual is to be thought of as the average person who normally does not consume his own endowment and needs cash to buy goods. (Cf. Wilson (1979). We do not assume identical individuals because in such a case there would be no trade and hence no need for money for transactions purposes.) It is assumed that there exists m^0 (const) such that $v'(m) > 0$ and $v''(m) < 0$ for $m < m^0$ and $v'(m) = 0$ for $m \geq m^0$. We also assume that $v'(0) < \infty$ so that the possibility of a barter equilibrium is not necessarily ruled out.

W is maximized subject to the budget constraints

$$p_t\, c_t + M_t - M_{t-1} = p_t\, y + H_t \qquad (t = 0, 1, 2, \ldots) \tag{2.2}$$

where M_{-1} is given, p and M denote the price level and nominal balances respectively so $m_t = M_t/p_t$, and H is a transfer payment (if H is negative, a tax is paid). It is assumed that

$$H_t = (\alpha^{t+1} - \alpha^t)M_{-1} \qquad (\alpha > 0) \tag{2.3}$$

so that, since $M_t = M_{t-1} + H_t$,

$$M_t = \alpha^t\, M_0. \tag{2.4}$$

It is also assumed that the representative person's price expectations $\{p_t\}_0^\infty = (p_0, p_1, p_2, \ldots)$, which turn out to be correct in the absence of unforeseen disturbances, are consistent with equilibrium in every period. It is necessary—see Brock (1975) for details—that

$$u'(c_t)/p_t = v'(m_t)/p_t + \beta\, u'(c_{t+1})/p_{t+1} \qquad (2.5)$$

i.e. the marginal utility from spending a dollar on current consumption must equal the marginal utility from not spending it, which is the R.H.S. of (2.5). Because of (2.4), (2.5) can be written

$$u'(c_t)m_t = v'(m_t)m_t + \beta\, u'(c_{t+1})m_{t+1}/\alpha$$

or, since equilibrium at t requires $c_t = y$,

$$m_{t+1}/m_t = (1 - v'(m_t)/u'(y))\alpha/\beta. \qquad (2.6)$$

It is well known that there is no steady state equilibrium if the exogenous money growth factor α is less than β (because no value of m could then make the R.H.S. of (2.6) equal to 1). A steady state requires $\alpha \geq \beta$. Given a particular value of $\alpha > \beta$, there would be a constant value of m--denote it by m^*--such that the R.H.S. of (2.6) is equal to 1 thus yielding a steady state.

Consider another steady state corresponding to a lower α ($\geq \beta$). This means a higher m^* hence a higher W. A steady state equilibrium generated by any $\alpha > \beta$ is accordingly inefficient. (Note that inefficiency is shown not by "assigning" lower prices--prices are endogenously determined by equilibrium conditions--but by having a lower α.) Putting $\alpha = \beta$ one has a steady state where $v'(m^*) = 0$ and W is maximal, so $m^* \geq m^0$, the (minimum) OQM value. Accordingly, the OQM policy rule is to implement (2.3) with $\alpha = \beta$.

Let $\alpha = \beta$ and $m^* = m^0$ but a disturbance at $t = t_0$ makes $m_t < m^0$ so $v'(m_t) > 0$. Then $m_{t+1} < m_t$ from (2.6) and therefore

$v'(m_{t+1}) > v'(m_t)$ which implies $m_{t+2} < m_{t+1}$, and so on. In other words, there is what might be called a one-sided (specifically, to the left or downward side) instability of the $m^* = m^0$ equilibrium. (If the disturbance makes $m_t = m^0 + k$, $k > 0$, there would only be a new steady state $m^* = m^0 + k$.)

3. Another version

Instability is less apparent in the Benhabib and Bull (1983) formulation, but it is also to be expected in their model. The representative individual maximizes

$$V = \int_0^\infty U(c(t))e^{-\delta t}dt \tag{3.1}$$

subject to

$$c(t) + T(m(t)) + \pi(t)\, m(t) + \dot{m}(t) = y + h(t) \tag{3.2}$$

where y, c and m have similar meanings as in Section 2, and $T(m)$, π and h denote real transactions costs, the inflation rate and a transfer payment respectively. (Benhabib and Bull write $T(y, m)$ but since y is a constant our simplification can be permitted.) Specifically,

$$h(t) = \sigma M(0)e^{\sigma t}/p(t) \tag{3.3}$$

so that nominal balances are given by

$$M(t) = M(0)e^{\sigma t} \tag{3.4}$$

and σ is the growth rate of M. (Benhabib and Bull (1983, p. 102) say that σ is the growth rate of the supply of real balances but they mean nominal.) Equilibrium requires

$$c(t) = y - T(m(t)). \tag{3.5}$$

It is assumed that $T'(m) < 0$ and $T''(m) > 0$ for $m < m^0$, $T'(m) = 0$ for $m \geq m^0$, and $T(0) < y$. From (3.2) and (3.5),

$$\dot{m}(t) = h(t) - \pi(t)\, m(t) \tag{3.6}$$

which tells the sources of change in real balances, viz. transfer payments (taxes are paid if h is negative) and inflation (deflation if π is negative).

Using Euler's equation it is straightforward to derive the following necessary condition (suppressing t):

$$\frac{\dot{m}}{m} = \frac{\delta + \sigma + T'(m)}{1 - T'(m)\, m\, U''(c)/U'(c)}. \tag{3.7}$$

If there exists m^* such that $\delta + \sigma + T'(m^*) = 0$, one has a steady state where $m(t) = m^*$ all t. A steady state does not exist if $\sigma < -\delta$. A lower value of $\sigma > -\delta$ means a higher m^* hence a higher V in (3.1), so the equilibrium corresponding to any $\sigma > -\delta$ is inefficient. Putting $\sigma = -\delta$, one has $m^* = m^0$ and maximal V.

Suppose $\sigma = -\delta$ and $m^* = m^0$. As observed by Benhabib and Bull (1983) and earlier by Calvo (1979), stability requires the denominator in the R.H.S. of (3.7) to be negative when $m < m^*$ (so that then, $\dot{m} > 0$). That is, stability of the $m^* = m^0$ equilibrium requires

$$T'(m)\, m\, U''(c)/U'(c) > 1. \tag{3.8}$$

Can (3.8) be expected to hold?

Note first that if $-T'(m) \geqq 1$, then from (3.5) a unit increase in real balances will raise c by at least one unit (without having to pay for it), in which case (since one unit of c trades for one unit of m) the representative person would not buy the marginal c. Clearly, $-T'(m) < 1$. Second, to get an idea of the other magnitudes involved in (3.8), consider the special case $U(c) = \log c$ which gives $-m\, U''(c)/U'(c) = m/c$. Since the amount m can obviously buy all of c if $m = c$, there is no reason to have $m > c$. Thus in this special case, $m/c \leqq 1$ and therefore the L.H.S. of (3.8) is less than 1. Now in the general case we can expect $-U''(c)/U'(c)$ to be in the order of magnitude of $1/c$, and since $-T'(m)$ would of course be close to 0 in the neighborhood of m^0, the direction of the inequality in (3.8) should be just the opposite. In other words, the OQM equilibrium $m^* = m^0$ is unstable downward.

4. Correcting for the instability

In the basic model, suppose the disturbance occurs at t_0: $m_{t_0} < m^0$. In view of (2.4) the instability implies that the new $\{p_t\}_{t_0}^{\infty}$ is associated with higher inflation (or more precisely, less deflation) than the original sequence of price expectations. What happens is that at t_0 the price level is higher than originally expected, and this induces a revision of expectations in the direction of higher inflation. This in turn, because of the resulting increase in the opportunity cost of holding real balances, leads to lower equilibrium levels of real balances. In order to get back on track, what is called for is suspension of H payments until a time t_1 is reached when $M_{t_1}/P_{t_1} = m^0$. H payments could then be resumed beginning at $t_1 + 1$ in accordance with (2.3) again but with a new calendar where the new time 0 is the old $t_1 + 1$.

In the continuous-time version the higher inflation concomitant with instability can be seen directly because (3.3), (3.4) and (3.6) give $\dot{m}(t)/m(t) = \sigma - \pi(t)$ and therefore $\pi(t) > \sigma$. What is similarly needed is a suspension of h payments until t_1 when $m(t_1) = m^0$, at which time (3.3) could be resumed with a new calendar where the new time 0 is the old t_1. There is no problem with "overshooting," either here or in the discrete-time case, since any $m^* \geq m^0$ is an OQM equilibrium.

5. Conclusion

We have seen that the OQM rule, which decreases the money supply at a rate equal to the rate of time preference, gives an unstable equilibrium. However, the instability is correctible by a contingent rule, viz. whenever real balances fall below the OQM value, suspend the contraction of the money supply until real balances are restored to the required level. Being nondiscretionary, this supplement preserves the spirit of Friedman's original proposal, and we conclude that the OQM concept remains useful, at least in simple models.

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