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Rent Intensity and Indirectly Endogenous Rent

by

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Abstract

Consider rent sourced from the value-added generated by productive investments. The amount dedicated to the pursuit of rent affects the aggregate value added and the size of rent itself, i.e. endogenous rent. We introduce rent intensity as the analytic tool and characterize it by relating it to the Tullock dissipation rate under specific assumptions on rent structure, risk attitude and membership size in the symmetric Cournot-Nash equilibrium. Among others, we show that with proportionally endogeneous rent, with risk aversion of non-Inada variety, complete indifference and dissipation are not attainable.

Introduction

In the social cost of rent-seeking literature, it is natural that the resources spent in the pursuit of a given size of rent X is of paramount importance. It is no wonder that the Tullock dissipation rate, d, defined as the ratio of the overall resource outlays to the given rent, hugs the center stage of a good number of the area's analytic endeavors (Tullock, 1980; Hillman and Katz, 1984; Hillman, 1990; Hillman and Riley, 1990; Fabella, 1989; Fabella, 1992). For risk neutral agents in a symmetric competitive ($n \to \infty$) rent contest, the Cournot-Nash equilibrium dissipation rate d* equals unity which means the resources dissipated in the pursuit of X equals X itself. Risk-aversion reduces the dissipation rate (Hillman and Katz, op.cit.). Uncertainty over the rent X reduces it further (Fabella, 1989). For two risk neutral agents, d* = 0.5. (Tullock, 1980). For two agents with risk aversion implied by the quadratic utility function, the contest results also in d* = 0.5 (Fabella, 1992). With absolute risk aversion, d* rises with X. In general, the dissipation rate rises with the entry of opposition to transfers (Fabella, 1991a,b; Fabella 1993). By knowing the factors influencing behavior of d*, we get a feel of the ebb and flow of the social cost of rent-seeking.

Up to now, the dissipation rate sufficed to quench curiosity about the ravages of rent-seeking on the economy. Its main drawback is its decidedly partial equilibrium character. The impact of rent-seeking on other (presumably productive) sectors of the rent-seeking on other (presumably productive) sectors of the economy can only at best be indirectly inferred and the validity of the inference depends on the nature of the rent. Rent, however, is seldom autonomously available.

In an economywide setting, rent is a claim on a share of the total value produced by the economy. Without the latter, rent may cease to exist. Without the harvest from the soil, the feudal society, which sits on the foundation of a system of rents, will collapse. The latter, in its turn, is the prize for endless feudal warfare which requires a lot of resources to prosecute. The financing of the harvest and the financing of contest for claims on the harvest are competing claims on feudal resources. The more resources are arrayed in the service of war, the smaller is the total harvest and consequently the rent. In this setting, rent is indirecly endogenous and rent-seeking maybe self-limiting. We contrast this with "directly endogeneous rent or transfer" where the rent size is directly related to the level of rent spending. By "indirect" we mean that R affects rent via the budget constraint. In most modern day LDCs, the biggest source of rent is the state's sovereign claim on real resources. Agents choose between value-adding activities which are subject to tax or value redistributing activities hovering over the resources in the control of the state. But the latter depends also on how agents behave. The more agents dedicate to unproductive redistributive pursuits, the smaller the total pie and government's share itself. This may slow down or intensify unproductive activities depending on the circumstances. The system settles where the returns are equalized. This view is very natural in growth models with rent seeking.

We maintain with Murphy, Vishny and Schleifer (1990) and Baumol (1991) that on a systemwide basis, the productive and the unproductive (redistributive) sectors are intimately related and agents' decisions on the deployment of individual resources are so guided. In this context, the dissipation rate as the organizing analytic principle is not very informative. The bottom line is that unless the denominator is fixed, knowing d is not a handle on the amount of resources wasted.

The alternative analytic concept that this paper proposes for games with endogenous rent is the "intensity of rent seeking, ρ," defined as "the ratio of total resources expended to total initial endowment, i.e., $\rho = (\sum^{n} R_i / \sum^{n} A_i)$". So natural is this measure that one wonders why the Tullock dissipation rate took chronological precedence. But there is a good reason. In the most elementary (and most resorted to) model of rent-seeking with risk neutral agents and fixed rent X, the agents' initial endowments wash out of the first partial so that the intensity does not readily arise. In contrast, d* is readily and explicitly available for this model. Since a good deal is known about the latter, we first establish the formal relation between the Tullock dissipation rate and the rent intensity which will then give us insights into the behavior of the rent intensity based on what we know about the dissipation rate.

II. General Model

Every agent i has initial endowment A_i, $i = 1, 2, \ldots n$. I_i of A_i he uses for productive investment and the rest R_i for unproductive (redistributive) investment: $I_i = A_i - R_i$. His net revenue in productive investment is $I_iV = (A_i - R_i)V$, where V is the net value-added constant and embodies value added returns after tax or other deductions if any. In general $V \geq 1$. We assume every agent to be facing the same net value-added constant. The difference between the gross value-added and the net value-added I_iV is the source of rent. Since $\sum I_i$ is the source of all value added, in the economy and the aggregate rent X is just a fraction of total value added X is some function of $\sum^n I_i$, i.e., $X = f(\sum I_i)$, $f' > 0$. Clearly, for accounting balance, f(.) and V are not independent. If, on the other hand he chooses to play the rent game, his probability of winning is $P_i = [R_i^{\alpha}/\sum^n R_j^{\alpha}]$, $0 < \alpha \leq 1$. Agents exhibit a von-Neumann- Morgenstern utility function over wealth W. If i wins the rent game, his wealth is $((A_i-R_i)V + X) = W_w$ and his utility is $U_i(W_w)$. If he loses, his wealth is $(A_i-R_i)V = W_l$ and his utility is $U_i(W_l)$. He maximizes expected utility, i.e.,

$$\max_{R_i}\{P_iU_i(W_w) + [1-P_i]U_i(W_l)\}, \qquad i = 1,2, \ldots, n. \qquad (1)$$

The 1st condition for a maximum is:

$$P_i[U_i'(W_1) - U_i'(W_w)]V + (\partial P_i/\partial P_i)[U_i(W_w) - U_i(W_1)] + P_iU_i'(W_w)/\partial X/\partial R_i) = U_i'(W_1)V, \quad i = 1, 2, \ldots, n \tag{2}$$

where $U_i'(W) = (\partial U_i/\partial W)$. This can be solved for n unknowns $\{R_i^*\}$, $i = 1, 2, \ldots, n$, which constitute the Cournot-Nash equilibrium rent-seeking spending. Assuming instead global symmetry, we have $A_i = A_j = A$, $\forall_{i,j}$ and $R_i = R_j = R$, $\forall_{i,j} = 1, 2, \ldots, n$, and suppressing subscripts (2) simplifies to

$$(1/n)[U'(W_1) - U'(W_w)]V + [\alpha(n-1)/nR][U(W_1)] - U(W_w)] + (1/n)U'(W_w)(\partial X/\partial R) = U'(W_1)V \tag{3}$$

which can be solved for the lone symmetric Cournot-Nash equilibrium (SCNE) rent-seeking spending, R^*. Rearranging, we have:

$$[X/nR][[\alpha(n-1)/n]C = TV - D \tag{4}$$

where $C = [U(W_w) - U(W_1)]/X$, is the chord slope (the slope of the line connecting $U(W_1)$ and $U(W_w)$), $T = ((n-1)/n)U'(W_1) - (1/n)U'(W_w)$ is the tangent slope (the average of the slopes tangent at W_1 and W_w) and $D = (1/n)U(W_w)(\partial X/\partial R)$ is the valuation slope (the marginal utility to an agent of the increment to rent). Since $[nR^*/X] = d^*$, we have

<u>Claim 1</u> : $d^* = \alpha[(n-1)/n][C/TV-D)]$. (5)

This is the SCNE Tullock dissipation rate. Multiplying and dividing $[nR^*/X]$ by nA gives the SCNE rent intensity:

Claim 2 : $\rho^* = \alpha[(n-1)/n][C/(TV-D)][X/nA]$. (6)

III. Exogenous Rent

The purpose of this section is to characterize ρ^* under the more familiar assumption of exogenous rent. Although ρ^* and d^* are intimately related ($\rho^* = d^*[X/nA]$), the impression given by d^* may differ from that by ρ^*. In the following, we will exploit this relation between ρ^* and d^* and Fabella's (1992) results on d^*. With exogenous rent, $D = 0$, so we have from (6):

$$\rho^* = \alpha[(n-1)/n][C/TV][X/nA]. \quad (7)$$

Note that if no value-added is generated, (or (A_i-R_i) is not invested) $V = 1$ and d^* is exactly as in Fabella (1992). For comparison with current literature, we let $V = 1$.

Claim 3: In exogenous rent contests with $V = 1$,

(i) $\rho^* = \alpha((n-1)/n)(X/nA)$ for risk neutral agents in particular, for $n = 2$ and $\alpha = 1$, $\rho^* = (0.25(X/A)$.

(ii) $\rho^* \to 0$ for risk-neutral or risk averse agents if $n \to \infty$.

(iii) $\rho^* \to 0$ as $X \to 0$ for risk-neutral or risk averse agents.

(iv) $\rho^* \to 1$ as $X \to \infty$ for risk-neutral or risk averse agents.

(v) If the utility function is $U(W) = a - e^{-bw}$, $a,b > 0$, with constant absolute risk aversion variety equal to b,

$$\rho^* = \alpha(n-1)b[1-e^{bx}][(n-1) + e^{-bx}]^{-1}[X/nA]. \quad (8)$$

Since no R appears on the left-hand-side, we have a complete solution for ρ^* and $(d\rho^*/dX) > 0$.

(vi) If the utility function is quadratic and risk averse, i.e., $U(W) = aW - bW^2$, $a,b > 0$, we have

$$\rho^* = \alpha[(n-1)/n](1+\lambda)^{-1}[X/nA]$$

where $\lambda = [(n-2)/n][bX/[U'(A-R) - bX]]$. For $n = 2$, $\rho^* = (0.25)[X/A]$, the risk neutral rent intensity.

Remark: $\rho^* = 0$ we call "complete indifference"; $\rho^* = 1$ we call "complete obsession."

Proof: (i) For risk neutral agents, $C = T$, and the claim is obvious. (ii) For risk neutral agents, this is obvious since $(n-1)/n \rightarrow 1$ and $(X/nA) \rightarrow 0$ as $n \rightarrow \infty$. For risk averse agents, we know that $T \rightarrow U'(A-R) > C$ as $n \rightarrow \infty$ and $d^* \rightarrow \alpha(C/T) < \alpha$. But $(X/nA) \rightarrow 0$ as $n \rightarrow \infty$. (iii) Obvious for risk neutral agents. As $X \rightarrow 0$, C and T approach a common value with risk averse agents. But $[X/nA] \rightarrow 0$. (iv). Let the denominator of C and the numerator of $[X/nA]$ cancel, and let $X \rightarrow \infty$. If agents are risk neutral, $U(A-R+X) \rightarrow \infty$, but

$U'(A-R+X)$ is constant. If risk averse and unbounded, $U(A-R+X) \to \infty$ while $U'(A-R+X)$ becomes progressively smaller. If risk averse and bounded, $U(A-R+X)$ approaches a fixed number, but $U'(A-R+X) \to 0$. In both cases, $\rho^* \to 1$ (its upper limit). (v) d^* for this case is derived in Fabella (op.cit.). Also shown there is that d^* rises with X. Obviously, ρ^* rises with X. (vi) Again, d^* is derived in Fabella (op.cit.). The claim for $n = 2$ is obvious. Q.E.D.

We have characterized rent intensity for exogenous rent and no productive sector value-added by formally relating it to the dissipation rate of which a good deal is known. We have generated the rent intensity counterpart {(0.25)[X/A]} of the famous Tullock dissipation rate (0.5) for two risk - neutral players and $\alpha = 1$ and shown this to be robust against risk aversion of the quadratic type. ρ^* and d^* behave in similar fashion towards X when the utility function is constant absolute risk aversion.

The principal interest in Claim 2 revolves around three results ((ii), (iii) and (iv)). These demonstrate how intuitively more appealing is ρ^* than d^*. As $X \to 0$, d^* approaches a finite value while ρ^* approaches zero. As far as the individual and society is concerned, rent seeking disappears as a problem. The dissipation rate does not quite reflect this. Again as $X \to \infty$, with bounded utility (Fabella, op.cit.), $d^* \to 0$ which makes sense because X is the denominator of d^*. $d^* \to 0$ does not mean resources are not being wasted, they are just small relative to X. The rent intensity reflects the intuition since $\rho^* \to 1$ when $X \to \infty$. Finally while d^* approaches a finite value as n becomes very large, ρ^* approaches zero. Again resources are being wasted (d^* positive) but relative to the total resources of the economy, the damage is vanishingly small. The latter is clearly our concern when economic performance is in question. Thus, even in the exogenous rent case, rent intensity as an analytic tool naturally recommends itself. It is, however, in the endogenous rent cases that rent intensity is most useful.

IV. Proportionally Endogenous Rent

When rent is indirectly endogenous, $D \neq 0$. Strictly positive value-added means $V > 1$. For economic sense $(TV-D) > 0$ which is the case if $D < 0$ ($(\partial X/\partial R) < 0$). We will determine the limiting values of d^* and ρ^* for special sets of circumstances rather than in general. These sets of circumstances will involve the structure of X, the risk attitude and number of participants.

Definition 1 : Rent Structure: Rent is proportionally endogenous if $X = f(I) = BI$, $B > 0$.

Thus, rent is directly proportional to aggregate productive investment. This is the simplest possible configuration.

Remark : Thus, we have $X = B \sum^{n}(A_j - R_j)$ and $(dX/\partial R_j) = B < 0$.

By symmetry, we have $X = nB(A-R)$. As observed, V and B may be related.

Definition 2: Let $d^{**} = \lim_{n \to \infty} d^*$. The d^{**} is the symmetric competitive Cournot-Nash equilibrium (SCCNE) dissipation rate. Let $\rho^{**} = \lim_{n \to \infty} \rho^*$. This is the SCCNE rent intensity.

We have:

Claim 4: Let X be proportionally endogenous and agents be risk neutral. Then $d^{**} = (\alpha/V)$.

Proof: Under risk neutrality, C = T and U'(.) is a constant. Furthermore, $(\partial X/\partial R_j)/n = -B/n \to 0$ as $n \to \infty$. Thus, $D \to 0$ as $n \to \infty$. The $\lim_{n \to \infty} d^* = \alpha/V = d^{**}$

Q.E.D.

If $\alpha = 1$ and $V = 1$ as in most studies, complete dissipation is robust at SCCNE under proportional rent endogenization. If $V > 1$, however, complete dissipation fails to obtain. How about ρ^*? Note that under exogenous rent and risk neutrality, $\rho^{**} = 0$. In contrast:

Claim 5: Let X be proportionally endogenous and agents be risk-neutral. Then $\rho^{**} = \alpha B/(V+\alpha B) > 0$.

Proof: By the structure of X, $[X/nA] = B - B(R/A)$ so that

$$(R^*/A) = [(n-1)/n]\alpha[C/(TV-D)][B-B(R^*/A)]. \qquad (9)$$

Solving for (R^*/A) we have:

$[R^*/A][1+d^*B] = d^*B$

$[R^*/A] = d^*B/[1+d^*B]$.

The limit of d^* as $n \to \infty$ is (α/V) (Claim 4)

$$\lim_{n \to \infty} (R^*/A) = \alpha B/[V+\alpha B] > 0. \qquad \text{Q.E.D.}$$

For the simplest endogenous rent structure, the proportional rent structure, the rent intensity at SCCNE does not go to zero. Note that $B = (\partial X/\partial I_j)$ and the higher is B the higher is the SCCNE rent intensity. The reason for the result is that as $n \to \infty$, the rent size also approaches infinity and continuous to attract interest. Every new agent k adds to the total rent via I_k even as he then becomes a contender for X.

Definition 3: A utility function U(w) is "non-Inada" if

$U'(w) > 0$, $U''(w) < 0$ and $U'(0) = z < \infty$.

Ex: The constant absolute risk aversion utility function

$U(w) = a - e^{-bw}$, $b > 0$ is non-Inada.

Claim 6: Let X be proportionally endogenous and agents exhibit non-Negishi utility functions. Then $d^{**} = \alpha[C/TV] < \alpha$.

Proof: As $n \to \infty$, $[(\partial X/\partial R)/n] = (-B/n) \to 0$. Because agents have non-Negishi utilities, the highest value $U'(w)$ attains is $z < \infty$. Thus, $D \to 0$ as $n \to \infty$. As $n \to \infty$, $d^* \to \alpha(C/TV)$ and $T \to U'(A-R) > C$. thus, $d^{**} = (C/TV) < \alpha$. Q.E.D. (11)

The SCCNE dissipation rate has the same structure under exogenous and endogenous rent with the addition of the non-Negishi condition. Underdissipation still holds even with endogenous rents due to risk aversion.

<u>Claim 7</u>: Let X be proportionally endogenous, and let agents exhibit non-Inada utility functions. Then $\rho^{**} = \alpha B/[(TV/C) + \alpha B]$.

<u>Proof</u>: Again under the conditions, $D \to 0$ as $n \to \infty$. Likewise $d^* \to \alpha(C/TV)$. Following the procedure in Claim 5, we have from (9)

$$\rho^{**} = \alpha B/[(TV/C) + \alpha B] > 0. \qquad \text{Q.E.D.} \qquad (12)$$

This is also obvious from $d^* = \alpha(C/TV)$ since $\rho^{**} = d^{**}(X/nA) = d^{**}(B\,D(R/A))$. Since $T > C$, $TV/C > V$ and ρ^{**} for non-Inada risk averse agents is less than the counterpart for risk-neutral agent.

The small numbers contest, of course, leaves $D \neq 0$. this complicates the result but is still manageable for special cases:

<u>Claim 8</u>: Let X be proportionally endogenous and let agents be risk neutral. Then

$$d^* = \alpha(n-1)/(Vn+B) \qquad (13)$$

$$\rho^* = \alpha B(n-1)/(Vn+B+\alpha B(n-1). \qquad (14)$$

Proof: For risk-neutral agents $T = C$ and also $U'(A-R+X) = C$. Thus, from (5) we get $d* = \alpha((n-1)/n)[V+B/n]^{-1}$ where $(-B) = (\partial X/\partial R_j)$, and n cancels out. Now $X = B - B(R/A)$ so $(R*/A) = [\alpha(n-1)/(Vn+B)][B-B(R*/A)]$ from (6). Solving for $(R*/A)$ gives $(R*/A) = [\alpha(n-1)B/(Vn+B)] + \alpha(n-1)B]$ which gives (14).

For a check, let $n \rightarrow \infty$ and $d* \rightarrow (\alpha/V)$ is in (Claim 4) and $\rho* \rightarrow \alpha B/(V+\alpha B)$ as in (Claim 5).

V. An Economy with Endogeneous Rent

The question addressed in this section is whether greater productivity in the value-adding sector always dampens rent-seeking as one would normally expect in a fixed rent case. There are n agents in this society, each having an endowment A_i. Each agent confronts two investment options: one value-adding (productive) into which he sinks I_i and the other redistributive (unproductive) into which he sinks R_i and $A_i = I_i + R_i$, $i = 1,2, \ldots n$. For I_i, he realizes rI_i, r the common and fixed interest rate representing value-adding sector productivity. He is charged the amount rtI_i, where $0<t<1$, is the tax rate on value added rI_i. The total $\sum^{n}$

$P_i = (R_i/\sum^n R_j)$ of winning X, which is not subject to tax. Agent i is risk-neutral and maximizes

$$EY_i = (A_i-R_i)(1+r(1-t)) + htr\sum^n(A_j-R_j)(R_i/\sum^n R_j) \qquad (15)$$

with respect to R_i. Here $V = [1+r(1-t)] > 1$. If i is a Cournot-Nash agent, the 1° condition is

$$-(1+r(1-t)) - htr[R_i/\sum^n R_j] + [htr\sum^n(A_j-R_j)]\sum^{n-1}R_j/[\sum^n R_j] = 0 \qquad (16)$$

$$i = 1, 2, \ldots, n.$$

These n equations can be solved for n unknowns $\{R_i^*\}$, $i = 1, 2, \ldots, n$, the Cournot-Nash equilibrium rent seeking expenditures. If we assume symmetry, we have $A_i = A_j = A$ and $R_i^* = R_j^* = R^*$ and (16) collapses into

$$htrn(A-R)(n-1)/n^2R = [1 + r(1-t)] + htr/n. \qquad (17)$$

Note that $U'(w) = 1$, $(\partial X/\partial R_j) = (-htr)$ and so $(htr/n) = D$ in our previous discussion. Note that, as required, $(htr/n) \rightarrow 0$ as $n \rightarrow \infty$. Since $X = htrn(A-R)$ and $X/nR = (1/d)$, we have from (17)

$$d^* = [[1+r(1-t)][n/(n-1)] + thr/(n-1)]^{-1} \qquad (18)$$

and

$$d^{**} = [1+r(1-t)]^{-1} \qquad (19)$$

This latter is the symmetric competitive Cournot-Nash equilibrium dissipation rate. On the other hand solving for (R/A) from (17) gives

$$\rho^{**} = [thr][1+r(1-t) + thr]^{-1}. \tag{20}$$

This is the SCCNE rent intensity. Now thr = B so $\rho^{**} = B/[[d^{**}]^{-1} + B] = d^{**}B/[1+d^{**}B]$ as before. We now have some interesting results.

Claim 7: (i) $(\partial d^{**}/\partial r) < 0$. (ii) $(\partial \rho^{**}/\partial r) > 0$.

Proof: (i) is obvious from (19). From (20), we have

$$(\partial \rho^{**}/\partial r) = [[1+r(1-t)]th - thr(1-t)\}/[.]^2 = th/[.]^2 > 0.$$

Thus, a rise in the value-adding productivity (r rises) reduces the SCCNE dissipation rate but raises the rent intensity - an instance where the two behave in opposite fashion. The reason for this divergence is that since X is endogenous, a rise in r also raises X and in this case raises X more than it raises nr. The dissipation rate is falling but the resources wasted is actually rising because the rent intensity is rising. In an economy where rent grows proportionally with society's output, increased productivity in the value-added sector may intensify rather than dampen rent seeking. This clearly demonstrates the inadequacy of the dissipation rate. However, in the exogenous rent case, this divergence will never happen.

Conclusion

In this paper we introduce the concept of rent intensity as a compliment to the Tullock dissipation rate which is the organizing analytic principle in a good part of rent-seeking literature with exogenous rent. We characterize rent intensity by relating it formally to the Tullock dissipation rate at the symmetric Cournot-Nash equilibrium. With exogenous rent and risk-neutral agents, rent intensity approaches zero (complete indifference) as n becomes very large or as the rent approaches zero. It approaches 1 (complete obsession) as the rent approaches infinity. We also show that the rent intensity for agents with risk aversion of the quadratic utility type equals that or risk-neutral agents for n = 2. (Claim 3)

When rent is indirectly endogenous, i.e., it is sourced from aggregate value-added whose size depends on agents' productive investments, the Tullock dissipation is no longer informative. In this paper, we dwell only on the simplest rent structure: proportionally endogenous. Productive investments now generate value-added. For risk-neutral agents, the symmetric competitive Cournot-Nash equilibrium dissipation rate will never show complete dissipation if value-added generated is non-zero. The corresponding rent-intensity is strictly positive so complete indifference is not attained (Claim 5). For risk averse agents with "non-Inada" utility function (e.g., the constant absolute risk aversion utility), the dissipation rate and the rent intensity are smaller as expected and a priori underdissipation and incomplete obsession obtain (Claim 7). We also give the dissipation rate and rent intensity when agents are risk neutral and the number of agents is finite. Finally, we give an example of an economy with endogenous rent satisfying the strictures we proposed in the previous section. We show that as the interest rate increases the dissipation rate decreases but the rent intensity rises (Claim 8). This emphasizes the inadequacy of the dissipation rate under endogenous rent which never arises under exogenous rent.