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Generalized Sharing Scheme in Teams

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Abstract

We show that the Generalized Sharing Scheme which is exhaustive, allows a team of identical members voluntarily supplying observable effort to attain Pareto efficient production under increasing returns provided team size is allowed to vary.

I. Introduction

The sharing scheme and the optimizing objective constitute the "constitution" of the production unit. Who gets what part of the pie spells out the all important incentive structure of the working unit and determines where the collective objective is attained. Team members take the allocation provisions as parameters when they make voluntary effort supply decisions. These decisions determine the size and welfare features of the team output. The sharing scheme also determines whether a "residual" exists and a "residual claimant" has a role to play. The existence of a residual transforms the outfit into a principal-agent unit with the principal or capitalist as the residual claimant. In this paper, we are concerned only with "pure" teams, i.e., where the sharing scheme is exhaustive, and no principal exists.

While team theory can take on many layers of complication, we will confine ourselves to the most basic features where known results are most transparent and comparisons readily made. First of all, we will deal only with welfare maximizing teams and ignore other objectives such as output maximization or the maximization of the minimum share (the Rawlsian objective). We will confine our work to static team theory. Thus, we will keep clear of supergame versions of the team game (Macleod, 1984; Guttman and Schnytzer, 1987). We will also avoid effort nonobservability (Holmstrom, 1982) and the possibility of moral hazard. We will assume effort to be observable but we assume asymmetric information with respect to each members true capacity. Thus, the emphasis shifts from moral hazard to adverse selection. Effort supply is wholly voluntary. We will also confine ourselves to the case of selfish but identical team members.

The classic result here clearly is Sen's (1966). Combining two conflicting principles of distribution, viz., "according to work" and "according to need," (the first is usually formalized by proportional sharing, i.e., a member's share in the total pie equals his share in total effort; the second is egalitarian which divides the pie by the number of members) Sen spelled out the condition for Pareto optimal production level in symmetric teams: the proportion of income to be distributed according to work must equal the elasticity of output with respect to total effort (where labor is the only production input). Clearly, decreasing returns in production is required. Sen's result is of the popular Cournot solution genre. This is important because Cournot solutions are "subgame perfect" and require no additional mechanism to enforce. This feature also characterizes the result of Fabella (1988) which shows that in teams with voluntary, not necessarily identical and observable effort, proportional sharing (i.e. according to effort) supports Pareto optimal output if and only if the production function unique is up to a constant of proportionality and exhibits constant returns to scale. Still along the general direction of Sen, Nitzan and Schnytzer (1987) showed that when knowledge of the marginal rate of substitution between income and leisure is available, manipulation of the degree of egalitarianism leads to optimal output whenever the production function is of decreasing returns variety.

Non-Cournot behavior in team theory takes the form of effort interdependence associated with effort matching. The strength and weakness of this approach is discussed by Putterman (1985) who points to enforceability as the main problem. Cameron (1973) observes that Pareto efficiency results if every members acts on the assumption that its every move will be matched by all the other members. Perfect cohesion or emulation (Chinn, 1979) has similar result when social forces are strong enough sanctions. One way to dodge the self-enforcing dilemma is to put the game in the repeated game context (Macleod, op.cit.) Guttman and Schnytzer, op.cit.; Putterman, op.cit.). To avoid the enforceability problem in the static setting, we stay wholly within the Cournot fold.

This enquiry on the condition(s) for the existence of Pareto efficient production will pay special attention to the role scale economies in production plays. As already noted, known conditions for Pareto efficiency seem to exclude increasing returns to scale. One reason why this hasn't bothered more people is because, as is well-known, increasing returns also block the existence of equilibrium (and thus of Pareto optimal solutions) in neoclassical markets. Increasing returns is especially important in teams with exhaustive and proportional sharing because it leads to effort undersupply in comparison to the Pareto efficient solution. The reason is simple: the more industrious are unable to appropriate all the benefits from additional effort. Thus, undersupply of effort naturally accompanies increasing returns. Most real world problems in this organization (e.g., in partnerships and cooperatives) are of the undersupply variety. In contrast, decreasing returns tends to be accompanied by effort oversupply relative to the Pareto levels! Thus, the team problem appears most interesting only with increasing returns. Unfortunately, this is precisely where the literature is largely silent!

In II, we propose the Generalized Sharing Scheme and give conditions for Pareto efficiency both for decreasing and increasing returns technology.

II. The Model

The team consists of n identical members. Member i contributes effort l_i to the total effort $L = \sum_{i=1}^{n} l_i$. The team production function F, defined over L, is twice differentiable, strictly increasing and homogeneous of degree r. Total output is shared according to the following sharing scheme :

<u>Definition</u>: The Generalized Sharing Scheme (GSS) applies if the ith share in total output, S_i, is

$$S_i = g_i + (1 - \sum_{j=1}^{n} g_j)n^{-1}, \quad i = 1, 2, 3, \ldots, n, \qquad (1)$$

where g_j, $j = 1, 2, \ldots, n$ defined over (l_j/L), is continuous, twice differentiable and obeys the following additional properties: (i) $g_j' \geq 0$, (ii) $g_j'' \leq 0$, $g_j(0) = 0$, (iii) if

$g_j' > 0$, then $g_j'(0) \longrightarrow \infty$, (iv) $0 \leq g_j \leq 1$, $\forall j = 1, 2, \ldots, n$.

Remarks: (i) GSS is exhaustive i.e., $\sum^{n} S_j = 1$.

(ii) If $g_j(l_j/L) = (l_j/L)$ $\forall j = 1, 2, \ldots, n$, then $S_j = (l_i/L)$, the proportional sharing scheme.

(iii) If $g_j(l_j/L) = b > 0$, $\forall j = 1, 2, \ldots, n$, then $S_i = (1/n)$, the egalitarian sharing scheme.

(iv) If $g_j(l_j/L) = (1-a)(l_j/L)$, $0 \leq a \leq 1$, then $S_j = (1-a)(l_j/L) + (a/n)$, the Sen sharing scheme.

Example: Let $g_i(l_i/L) = (l_i/L)^{1/2}$: $g_i(0) = 0$, $g_i' = (1/2)(l_i/L)^{-1/2} > 0$, $g_i'' = -(1/4)(l_i/L)^{-3/2} < 0$ and $g_i' \longrightarrow \infty$ as $(l_i/L) \longrightarrow 0$. Furthermore, $\sum^{n} S_i = \sum^{n}(l_i/L) + \sum^{n}(1/n) - \sum^{n}[\sum^{n}(l_j/L)^{1/2}/n] = 1$.

Every member i has a utility function $U_i = S_iF - V_i(l_i)$, where $V_i(.)$ is twice differentiable, convex and increasing in l_i. Pareto efficient production (cooperative output) is attained when every member i supplies the cooperative effort level (to be characterized below). The cooperative welfare is defined as $W = \sum^{n} U_i = F - \sum^{n} V_i(l_i)$ and the cooperative effort levels which together maximizes W, are generated by

$$F' - V_i' = 0, \qquad i = 1, 2, \ldots, n. \qquad (2)$$

The solution to (2), $\{l_i^{**}\}$, $i = 1, 2, \ldots, n$, are the cooperative effort levels and are Pareto efficient. Implicitly, this also means that the marginal product of labor equals the marginal rate of substitution between income and leisure which is the Pareto efficient condition in a neoclassical market. Thus, (2) is an attractive reference point regardless of one's opinion about social welfare functions. Since individual effort supply is voluntary, member i will supply l_i^* which maximizes U_i and is characterized by

$$S_i F' + F(\partial S_i/\partial l_i) - V_i' = 0, \quad i = 1, 2, \ldots, n. \tag{3}$$

Each of the n conditions in (3) depends on the effort levels of every other member the team. Let the solution to (3) be $\{l_i^*\}$. $\{l_i^*\}$ is the Cournot effort levels and would generally be Pareto inefficient, i.e., $\{l_i^*\}$ would differ from $\{l_i^{**}\}$. We rewrite (3) as:

$$F'[S_i + (F/F')(\partial S_i/\partial l_i)] - V_i' = 0, \quad i = 1, 2, \ldots, n \tag{4}$$

Clearly, how different $\{l_i^*\}$ from $\{l_i^{**}\}$ depends on how different from one is the expression $[S_i + (F/F')(\partial S_i/\partial l_i^*)]$. In this analysis, we confine ourselves to the symmetry case so that $S_i = S_j = S$, $l_i^* = l_j^* = l^*$, $l_i^{**} = l_j^{**} = l^{**}$, $V_i = V_j = V$ and $g_i = g_j$, $\forall i, j$. Thus, we can drop subscripts from (4) and solve

$$F'[S + (F/F')(\partial S/\partial l^*)] = V' \tag{5}$$

for l^*. The following is obvious:

<u>Lemma 1</u> : The Cournot effort level equals the Pareto efficient effort level in the symmetric team if and only if

$$[S + (F/F')(\partial S/\partial l^*)] = 1. \tag{6}$$

From (1), we take $(\partial s_i/\partial l_i)$ and applying symmetry we get:

$$(\partial s_i/\partial l_i) = g'((n-1)/n)^2 L^{-1}) + (1/n) \sum_{i=1}^{n-1} g' n^{-1} L^{-1}). \quad (7)$$

By symmetry, S = (1/n), so applying (7) to (6) and simplifying gives

$$(F/F'L)g'((n-1)/n)[(n-1)[((n-1)/n) + (1/n)] = (n-1)/n. \quad (8)$$

The following is now obvious from (8):

Proposition 1: A team under Generalized Sharing Scheme with n identical members whose efforts are observable and voluntary attains Pareto efficient production iff

$$(F'L/F) = g'. \quad (9)$$

Proof : (if) Suppose (9) is true, $(F/F'L) = (1/g')$ follows and by appropriate manipulation so does (8). Subtracting (1/n) from both sides of (8) gives (6) and Pareto efficient production is attained. (only if) From (6), one easily gets (9). Q.E.D.

Remark (v): $(F'L/F) = r$, the degree of homogeneity of F is also the labor elasticity of output. Thus, it reflects the returns to scale property of F.

The following summarize the known relations between returns to scale and Pareto.

Proposition 2: (i) Under egalitarian sharing, the Pareto efficient production is impossible. (ii) Under proportional sharing, Pareto efficient production occurs if and only if $r = 1$. (iii) Under Sen sharing, if and only if $r = 1-a$.

Proof: (i) From Remark (iii), $g' = 0$ and from (9), the Pareto production level is attained iff $(F'L/F) = g' = 0$. But this is impossible if F is strictly increasing in L and $F < \infty$. (ii) For proportional sharing, $g(l_i/L) = (l_i/L)$ and $g' = 1 = r$ (iii) For Sen sharing, $g(l_i/L) = (1-a)(l_i/L)$ and $g' = (1-a)$.

Q.E.D.

We now give the main result:

Proposition 3: For any r such that $\infty > r \geq h$ where $h = g'(0.5)$, suppose $g' > 0$ and $g'' < 0$ and membership is continuous rather than discrete, then there exists a membership size $n = n^*$, so that the symmetric team under Generalized Sharing Scheme attains Pareto efficient production.

Proof: Let $r \geq h$. For n members, we have $g'(1/n)$ of which by the property of g in Definition 1, the smallest value is $g'(0.5)$, i.e., when there are but two members. If $\infty > r = g'(0.5)$ then let $n = 2$. If $r > g'(0.5)$, then raise n progressively. As $n \longrightarrow \infty$, $g' \longrightarrow \infty$, again by the property of g, so at some $n^* < \infty$, $r = g'(1/n^*)$.

Q.E.D.

Remark (vi): Clearly, this allows r to be strictly greater than one and increasing returns to scale can coexist with Pareto efficiency.

Example: Let $g = (1/L)^{1/2}$. Suppose $r = 1.5$. Then $1.5 = (1/2)(1/n)^{-1/2} = (1/2)n^{1/2}$ and $n = 9$. Suppose $r = 3$, then $3 = (1/2)n^{1/2}$ and $n = 36$. For both these cases, $h = (0.5)2^{0.5} = 0.707$. Thus, Pareto efficiency is assured if $r \geq 0.707$ as long as n is allowed to rise without limit.

It is interesting to note that the Pareto efficient or cooperative production is attained under increasing returns by manipulating the size of the team membership. There is here operative, however, an outside environment, a world outside the team with which the team interacts. This represents a break from the traditional concepts of a team as operating in isolation. The fact that the team may have to expand membership to attain efficiency may not be so alien but that it may have to "fire" excess members for the purpose seem to militate against the core concept of a team especially in this case where members are identical. Note that the advantage of the ordinary firm in a principal-agent mold is that it can "free" for cause, a fact which motivates efficiency. The existence of the outside environment in the case of the ordinary firm which taken as natural in the principal-agent context constitutes an aberration in the idea of the team. And yet in most real world context in which we encounter teams, the outside environment is central, e.g., in athletic teams or in the amorphous notion of, say, Japan, Inc. One way to motivate this is to view the team as part of a bigger organization, say, for example, as a new computer design team within a larger computer company. There may be a number of such teams in competition in one company.

That the role of outside environment may, however, prove crucial for efficiency in teams is suggested, though remotely, by developments in literature on cooperation in Prisoner's Dilemma games for truly rational players. As Aumann and Sorin (1989) observe "rationality in games depends critically on irrationality". Players must ascribe some irrationality to one another with some probability. Only then do Pareto efficient solutions get attained. Absent irrationality, one needs an outside environment to forge cooperative solutions. When $n \neq n$ and cannot be varied, manipulating some other parameter may deliver Pareto efficiency.

<u>Proposition 4</u>: Let $g_i = (l_i/L)^{\alpha}$, $0 \leq \alpha \leq 1$, in the Generalized Sharing Scheme of a symmetric team. Suppose $r \in (0,1]$. Then there is a particular $\alpha = \alpha^*$ such that Pareto efficient production is sustained.

<u>Proof</u>: For the symmetric team $g' = \alpha(1/n)^{\alpha-1}$. Let $h(\alpha) = \alpha(1/n)^{\alpha-1}$. For $\alpha = 1$, $h(\alpha) = 1$ and for $\alpha = 0$, $h(\alpha) = 0$. Because $h(\alpha)$ is continuous in α between 0 and 1, and $r \in (0,1]$ there is a particular $\alpha = \alpha^*$ so that $h(\alpha^*) = r$. Q.E.D.

By manipulating α, one can always guarantee Pareto efficient production in a symmetric team provided the production function is either constant or decreasing returns to scale. "α" here plays the role the egalitarian bias parameter "a" plays in Sen's where the degree of egalitarianism can be manipulated to force Pareto efficiency if decreasing returns obtains.

Conclusion

Attaining Pareto efficient production in teams with increasing returns in production is an open challenge in team theory. Here we concentrate on teams with observable and voluntary effort and identical membership. We introduce the Generalized Sharing Scheme which, in addition to being exhaustive, also imbeds the other known sharing schemes. We then show that, provided team size is allowed to vary continuously and without limit, Pareto efficient production is guaranteed for production function of any degree of homogeneity strictly greater than one (Proposition 3). Parameter manipulation can also force Pareto efficiency for a given team membership when decreasing returns obtains for given specifications of the Generalized Sharing Scheme (Proposition 4).

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