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A NOTE ON AN ANTINOMY IN COMPARATIVE ADVANTAGE

by

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Abstract

This note points out that the pattern of trade predicted by comparative advantage may or may not hold, depending on the initial distribution of endowments. A necessary and sufficient condition for the "perverse" case to occur in a pure-exchange economy is that the education in the value of consumption on the part of those who lose from trade exceed the increase in the value of consumption on the part of those who gain, where the evaluation is carried out at autarky prices. Examples are then presented which illustrate that the "antinomy" may occur even when there are only two goods and both are normal. The question then raised is whether the principle of comparative advantage can really be regarded as a purely positive proposition.

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Since the observation by Drabicki and Takayama [1979], it has been recognized that where more than two goods exist, pairwise comparisons of autarkic and free trade prices do not in general suffice to predict the pattern of trade. Current thinking, however (e.g. Dixit and Norman [1980:94] and Ethier [1984:139]), seeks to extract a weaker principle of comparative advantage in the form of a "negative correlation" between net exports and the difference between autarky and free trade prices, apparently regardless of dimension. This note argues that the usual demonstration is based on a confounding of positive and normative categories and shows how, in a familiar class of trade models, even the weak negative correlation itself may be violated and a country may export goods whose autarky prices are higher, even when there are only two goods.

In the following, p, c, x, and z are the price, consumption, output, and net-export vectors, respectively, e and g are the expenditure and national-product functions, u is (aggregable) utility and v is the endowment vector for the economy. Autarky

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values are denoted by a "°" superscript while free-trade values are primed. The usual argument for n goods then proceeds from the following chain, namely

(1) $\quad p^{\circ}c' \geq e(p^{\circ},u^{\circ}) = g(p^{\circ},v) \geq p^{\circ}x'$

which yields $p^{\circ}(x' - c') = p^{\circ}z' \leq 0$. If, furthermore trade is balanced, then $p'z' = 0$. These two observations lead to the desired result that

(2) $\quad (p^{\circ} - p')z' \leq 0$

or that net-exports are "negatively correlated" with the differences between autarky and free-trade prices.

The crucial step in the preceding argument is the first inequality in (1), which says that the value of free-trade consumption is greater than that under autarky when both bundles are evaluated at autarky prices. The implicit assumption being made is that either there is only one consumer (as in the above) or that "domestic lump-sum transfers or commodity taxes are deployed...to make free trade Pareto superior to autarky" [Dixit and Norman 1980:94].

In this, however, there seems to be a confusion of categories, since the principle of comparative advantage is meant as a positive proposition which seeks to predict the pattern of trade independently of the normative Pareto-superiority of free trade over autarky. Furthermore, if anything, the recent debates on

lump-sum versus commodity taxation (e.g. Kemp and Wan [1986] and Dixit and Norman [1986]) have in effect merely shown the great difficulties attending attempts to put such compensation schemes into practice. One ought not to expect, however, that a theory of comparative advantage should be dependent on whether in fact a compensation scheme is devised and implemented.

To illustrate the consequences of relaxing the assumption of a single consumer or of the implementation of compensation, consider an internationally small, two person pure-exchange economy. Endowments, consumption, and utility level of person i are denoted by x_i, c_i, and u_i respectively, $i = A,B$, with $x_A + x_B = x$, where x is fixed. As in the preceding, "noughts" and "primes" distinguish the autarky and utility levels of the variables, respectively.

Suppose equilibrium prices p° prevail in autarky, resulting in chosen feasible consumptions c_A° and c_B° and corresponding utility levels u_A° and u_B°. If now a free-trade regime is introduced in the form of (exogenously given) world prices p', we suppose that new feasible consumption bundles c_A' and c_B' are chosen and that the resulting new utility levels are related to the autarky levels by

(3a) $\quad u_A' < u_A^\circ$

(3b) $\quad u_B' > u_B^\circ$.

That is to say, person A is a net loser from trade while B is a net gainer. Now the budget restraint for person i is given by $H_i(p) = \{c_i / pc_i \leq px_i\}$.

Revealed preference and (3b) imply that c_B' does not belong to $H_B(p^0)$. But (3a) is consistent with either (a) $c_A' \in H_A(p^0)$ or (b) $c_A' \notin H_A(p^0)$. We impose the following restriction:

Condition 1. $c_A' \in H_A(p^0)$

Lemma. Suppose (3a)-(3b) and Condition 1 are fulfilled. Then

(4) $p^0c_A' \leq p^0c_A^0$ and $p^0c_B' \geq p^0c_B^0$.

That is, valued at autarky prices, the value of consumption is greater (resp. less) in autarky than under free trade for A (resp. B).

Proof. One simply notes that from Condition 1 $p^0c_A' \leq p^0x_A = p^0c_A^0$. Likewise, $c_B' \notin H_B(p^0)$ implies $p^0c_B' \geq p^0x_B = p^0c_B$. If preferences are strictly convex, then the strict inequalities in (4) would hold. We shall make that further assumption.

Now (4) is equivalent to:

(5) $p^0(c_A^0 - c_A') > 0$ and $p^0(c_B' - c_B^0) > 0$.

Consider now the following:

Condition 2. $p^0(c_A^0 - c_A') > p^0(c_B' - c_B^0)$, namely that the autarky-value loss in A's consumption is greater than the autarky-value gain in B's.

Proposition. Condition 2 is necessary and sufficient for net exports to be positively correlated with the difference between autarky and free-trade prices.

Proof. Condition 2 may be rewritten as

$$p^0(c_A' + c_B') < p^0(c_A^0 + c_B^0)$$

or $$p^0c' < p^0c^0$$

where c^0 and c' are aggregate consumption under autarky and free trade, respectively. Subtracting both sides of the last inequality from the value of the fixed goods-endowment x yields

$$p^0(x - c') < p^0(x - c^0)$$

or $$p^0z' < 0$$

since under autarky $x - c^0 = 0$. Subtracting the balanced trade condition $p'z' = 0$ from both sides yields:

$$(p^0 - p')z' > 0$$

which was to be demonstrated. This shows sufficiency. Necessity may be proved simply by working backwards.

The following two examples are meant to show that the above argument also holds for the familiar pure-exchange model with two

goods. In that case, the fulfillment of Condition 2 says that barring compensating transfers, the economy in question would paradoxically end up exporting that good which was relatively more expensive in autarky, and it is not necessary to go beyond two goods to derive an "antinomy" in comparative advantage. Furthermore, the the examples show that the occurrence of Conditions 1 and 2 is independent of the presence of goods-inferiority. In what follows c_{ij} and x_{ij} are person i's consumption and endowment of good j.

Example 1. Suppose persons A and B have Leontiev-type utility functions and as a result consume in fixed proportions, namely, $c_{A1} = c_{A2}$, and $c_{B1} = (7/3)c_{B2}$. Total endowments are given by $x_1 = 7$ and $x_2 = 11$, distributed so that $x_{A1} = 2$; $x_{B1} = 9$; $x_{A2} = 6$; and $x_{B2} = 1$. It is evident the only equilibrium[1] is one in which consumption is given by $c_{A1}^0 = c_{A2}^0 = 4$, $c_{B1}^0 = 7$, and $c_{B2}^0 = 3$. The price ratio (p_1/p_2) being denoted simply by p, autarkic equilibrium requires that p fulfill A's budget restraint:

$$pc_{A1}^0 + c_{A2}^0 = px_{A1} + x_{A2}$$

Substituting the given values yields $p^0 = 1$ as the price which supports the unique equilibrium, given the distribution of endowments (which may also be arrived at by using B's budget restraint). The situation is depicted graphically in Figure 1.

[1]As it turns out, this equilibrium is not stable in the usual sense that the excess-demand curve for Good 1 increases with p. It is, however, the only equilibrium.

Now suppose Good 1 is more expensive abroad than at home so that $p' = 3$, and this price prevails upon the opening of trade. Then direct computation using the budget restraints and demand functions shows the new consumption levels to be: $c_{A1}' = c_{A2}' = 3$; $c_{B1}' = 49/6$; $c_{B2}' = 7/2$. Comparing these with the fixed supplies shows that $c_1' = 67/6 > 11 = x_1$, and $c_2' = 6.5 < 8 = x_2$. That is, Good 1, though relatively cheaper in autarky, would be exported, and Good 2 exported. That Condition 2 is satisfied may be verified directly.

Example 2. Suppose we maintain the assumption of homotheticity--which is the same as saying all goods are normal[2] -- but now allow for some substitutability between goods. Let the ratios of Good 1 to Good 2 consumption by A and by B depend on p in the following manner:

$$c_{A2}/c_{A1} = 5p^{-.02} \qquad c_{B2}/c_{B1} = (25/16)p^{-.02}$$

and endowments be given by $x_{A1} = 2$; $x_{B1} = 32$; $x_{A2} = 106$; and $x_{B2} = 9$. In autarkic equilibrium, the fixed supply of Good 1 must equal consumption, hence,

$$c_{A1} + c_{B1} - 34 = 0$$

Using the consumption and substituting the values of the endowments yields the following excess-demand function in p:

$$E(p) = \{(2p + 106)/(p + 5p^{-.02})\}$$

[2]It should be noted, however, that the normality of a good, in the sense of having a positive derivative with respect to income, is quite compatible with its exhibiting the Giffen-characteristic with respect to price.

$$+ ((32p + 9)/(p + (25/16)p^{.02})) - 34$$

A rough graph of the curve is shown as Figure Two. It is evident that there are two solutions to $E(p) = 0$, which are (approximately) $p_s = .9 \times 10^{-5}$ and $p_u = 1$. Of these, however, only p_s is stable, so we may take $p^o = p_s$. Suppose now the exogenous free-trade price is greater than unity, say $p' = 7$. This is a case where Good 1 is relatively more expensive abroad and relatively cheaper in autarky. But an examination of the diagram will show that nonetheless Good 1 is imported and the more expensive good is exported. Again the fulfillment of Condition Two may be directly verified. Examining the diagram, it is evident why this has occurred. The world price is to the left of $p_u = 1$. Such a position would have been unstable in autarky, but that is irrelevant, since the trading equilibrium itself is stable at p'. (In this case because of the small-country assumption, but this could be changed, at the cost of some transparency.) What has been shown in these examples is that one need not even leave the world of "two-ness" in order to demonstrate an "antinomy" in comparative advantage.

In both the examples cited, person A was a net seller of the commodity which is cheaper on the world market and therefore loses from trade as a result. The income loss is so great, however, that as a consequence the domestic consumption of the other commodity is curtailed enough to allow it to be exported. These examples merely point to the implicit importance of the distributional issue in the positive theory of comparative

advantage. Even in a simple model, trade may cause redistributive effects so large that the trade-patterns suggested by comparative advantage are reversed. If so, then the larger question this suggests is to what extent it is at all possible to develop a theory of trade patterns independently of normative propositions regarding distribution.

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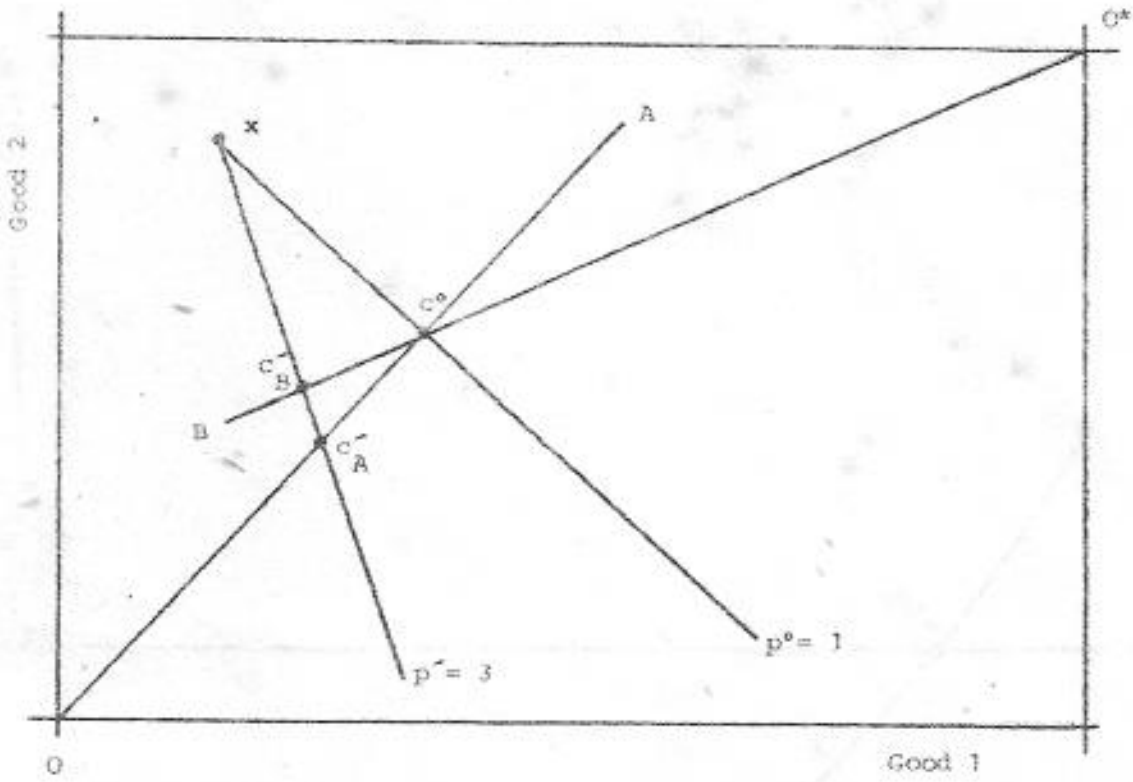

Figure 1

Here OA and O*B represent the price-consumption lines of persons A and B, respectively; x is the endowment point, and c° is the autarkic consumption point, attained through prices $p^\circ = 1$.

With a higher relative price for Good 1, as shown by p', consumption points c'_A and c'_B are attained. It is readily seen that at that price, there is domestic excess supply of (resp. demand for) Good 1 (resp. Good 2) which is made good through exports (resp. imports). The fulfillment of Condition 2 may be seen by noting that a line parallel to p° drawn through c'_B (not shown) would lie above a similar one drawn through c'_A.

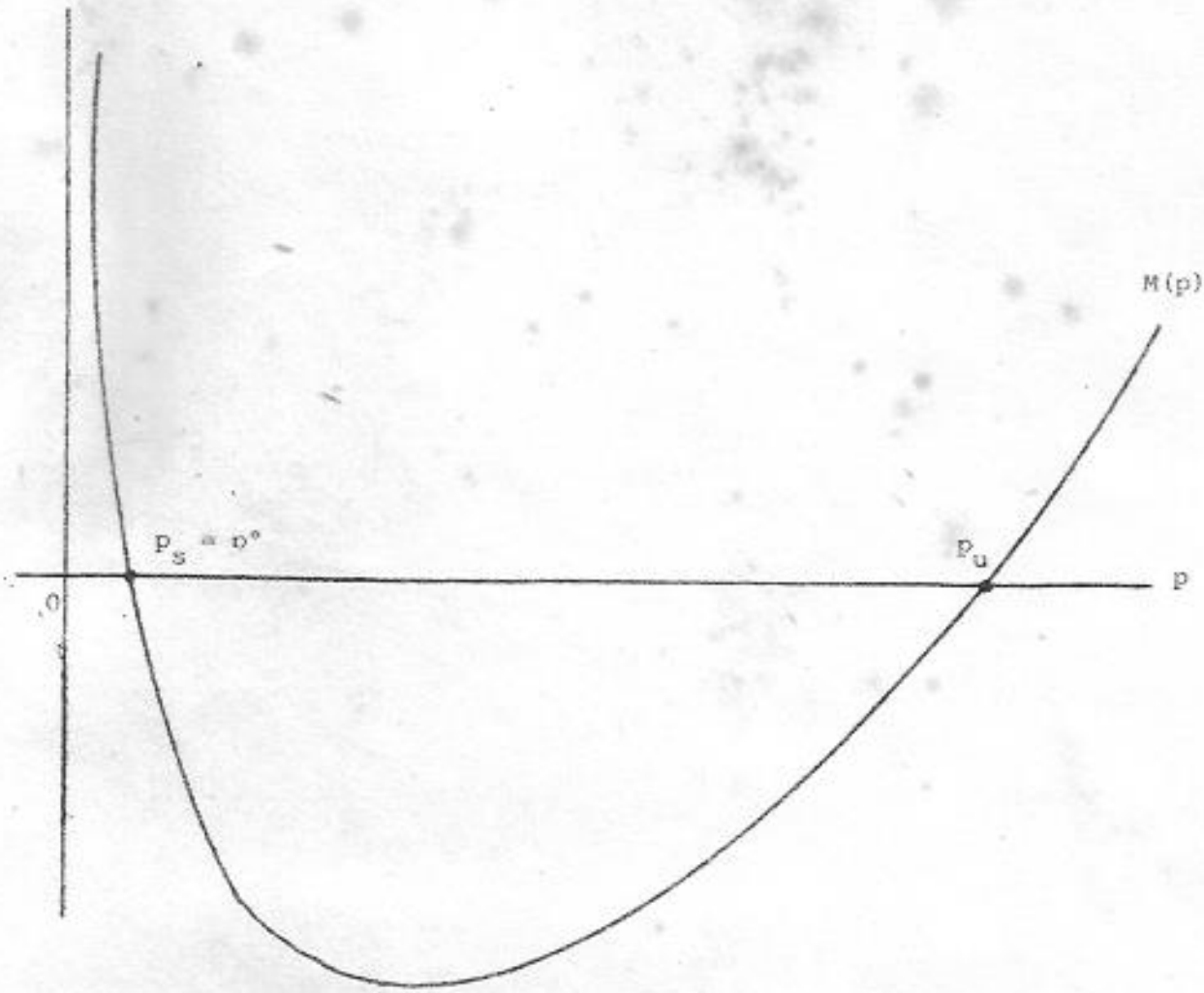

Figure 2