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Monetary Policy, Fiscal Policy, and Sterilized Intervention in a Model of Exchange-Rate and Aggregate-Demand Dynamics

by

Fidelina B. Natividad

Abstract

This paper incorporates a fully-sterilized intervention policy and imperfect capital substitutability into a model characterized by sluggish agrágate demand, sticky prices, and rational expectations. It shows that intervention may eliminate overshooting resulting from monetary expansion but may only dampen overshooting resulting from fiscal expansion.

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1. Introduction

In his pioneering work on the issue of exchange rate overshooting, Dornbusch (1976) has shown that the combination of sticky prices, fixed income, continuous asset market equilibrium, and rational expectations causes the exchange rate to unambiguously overshoot its new long-run equilibrium value in response to monetary expansion. With short-run variability of output, however, overshooting may be dampened or may even be reversed (Dornbusch, appendix). Other factors that have been shown to contribute to undershooting are imperfect capital mobility and substitutability (Frenkel and Rodriguez (1982) and Bhandari, et al (1984)) and policy reaction (Pappell (1985) and Mussa (1985)).[1]

These papers, however, assume that exports and imports adjust instantaneously to changes in prices and exchange rates despite the empirical evidence suggesting that this adjustment is distributed over a number of years (Goldstein (1980) and references therein). Exceptions are Bhandari (1983) and Levin (1986) who distinguish between short-run and long-run elasticities of the trade balance by introducing sluggish aggregate-demand adjustment and trade-flow lags, respectively, into the Dornbusch variable-income model. They show that, unlike the Dornbusch result, the exchange rate will unambiguously exhibit overshooting in response to a fiscal disturbance but may or may not exhibit overshooting in response to a monetary disturbance. The implication, therefore, is that fiscal policy may be the more important source of exchange-rate volatility.

This paper also deals with monetary policy, fiscal policy, and the issue of exchange-rate overshooting. However, the focus of the paper is on the role of intervention and sluggish aggregate-demand adjustment and, therefore, the Dornbusch variable-income model is extended by introducing not only sluggish aggregate-demand adjustment to allow for dictinction between short-run and long-run elasticities of the trade balance but also imperfect capital substitutability to allow for fully-sterilized intervention policy.[2] This paper shows that intervention may eliminate overshooting resulting from monetary expansion but may only dampen overshooting resulting from fiscal expansion.

2. The Model

The model is described by the following set of relationships:

$$(1.1)\quad y_t = y^{dS}_t \equiv y_0 + \int_{-\infty}^{t} \tau^o \exp^{-\alpha(t-T)} y_T d_T + \tau^S y_t - \int_{-\infty}^{t} \sigma^o \exp^{-\alpha(t-T)} i_T d_T$$

$$- \sigma^S i_t + \int_{-\infty}^{t} \delta^o \exp^{-\alpha(t-T)} (e_T - p_T + p_f) d_T + \delta^S (e_t - p_t + p_f)$$

$$(1.2)\quad dp/dt = \pi(y_t - y^*)$$

$$(1.3)\quad m - p = \emptyset y - \beta i$$

$$(1.4)\quad i = i_f + E(de/dt) - (1/\Phi)(f_0 + f_1 nfa - f_2 r)$$

$$(1.5)\quad r = r_0 - u((e-p+p_f)-(e^*-p^*+p_f))$$

where e = log of nominal exchange rate measured in units of domestic currency per unit of foreign currency; $e-p+p_f$ = log of real exchange rate; i, i_f = domestic and foreign interest rates; nfa = lof of net foreign assets; p, p_f = logs of domestic and foreign price levels; r = log of reserves; y = log of short-run income or output; y^{dS} = log of short-run aggregate demand; and, "*" denotes a long-run equilibrium value.

Eq. 1.1, drawn from Bhandari, states that the short-run level of income is demand determined and that the short-run level of aggregate demand is a function of the past and present values of income, domestic interest rate and real exchange rate. On the other hand, the equation for long-run aggregate demand, y^{dL}, or that level of aggregate demand that would obtain when all adjustments have been completed, is:

(1.1.i) $y^{dL} = y_0 + \tau^L y - \sigma^L i + \delta^L(e-p+p_f)$.

Since y^{dL} is also the level of aggregate demand that would obtain with all endogenous variables held constant at time t and since $\int_{-\infty}^{t} \exp{-\alpha(t-T)}\, d_T = 1/\alpha$, then Eq. 1.1 implies that:

(1.1.ii) $y^{dL} = y_0 + (\tau^o/\alpha + \tau^S)y_t - (\sigma^o/\alpha + \sigma^S)i_t + (\delta^o/\alpha + \delta^S)(e_t - p_t + p_f)$

where consistency between Eqs. 1.1.i and 1.1.ii requires that $\tau^L = \tau^o/\alpha + \tau^S$, $\sigma L = \sigma^o/\alpha + \sigma^S$; and $\delta^S = \delta^o/\alpha + \delta^S$. Notice that as α approaches ∞, the short-run elasticities approach the long-run elasticities. Notice also that y^{dL} is fixed at time t; in contrast, y, which is equal to y^{dS} at each instant, can jump at a point in time whenever the exchange rate jumps; this characteristic of y as a dependent jump variable is important in the specification of boundary conditions.

The price adjustment equation, Eq. 1.2, shows that the price level adjusts over time in response to the deviation of short-run income from its natural level y^*. Eqs. 1.1 and 1.2, therefore, describe the goods market.

The asset market, on the other hand, is described by Eqs. 1.3 to 1.5. Eq. 1.3 is the money market equilibrium condition where money demand is a function of income and the domestic interest rate while Eq. 1.4 is the foreign exchange market equilibrium condition where, under perfect foresight, E(de/dt) = de/dt.[3] Eq. 1.4 embodies the assumption of perfect capital mobility so that the return on domestic assets, i, always equals the net return on foreign assets, $i_f + E(de/dt) - (1/\Phi)(f_0 + f_1 nfa - f_2 r)$; it also embodies the assumption of imperfect capital substitutabilty, as reflected by the risk-premium term, $-(1/\Phi)(f_0 + f_1 nfa - f_2 r)$. The risk premium is affected by intervention. Intervention, on the other hand, takes the form of increasing (reducing) reserves whenever the real exchange rate is below (above) its long-run equilibrium value, as shown in Eq. 1.5. It is assumed that such intervention is fully sterilized and, hence, cannot affect the money supply.[4]

Thus, this model reduces to the Donbusch model when $\alpha = \infty$, $\Phi = \infty$ and $u = 0$ and is also similar to the Bhandari and Levin models when $\Phi = \infty$ and $u = 0$. The innovation of this paper, therefore, is the inclusion of fully-sterilized intervention policy.

2.1. The Steady-State. The steady-state of the model is attained when $de/dt = E(de/dt) = 0$, $dp/dt = 0$, and $dy/dt = 0$, and is described by:

(2.1) $e^* = p^* - p_f - (1/\delta^L)y_0 + (\sigma^L/\delta^L)i^* + ((1-\tau^L)/\delta^L)y^*$

(2.2) $p^* = m - \emptyset y^* + \beta i^*$

(2.3) $i^* = i_f - (1/\Phi)(f_0 + f_1 nfa - f_2 r^*)$

(2.4) $r^* = r_0$

where y^* is assumed to be exogenously fixed at its natural level.

2.2. Dynamic Properties. The state-space representation of the model is given by:

(3.1) $(i-i^*) = (1/\beta)(p-p^*) + (\emptyset/\beta)(y-y^*)$

(3.2) $(r-r^*) = -u(e-e^*) + u(p-p^*)$

$$(3.3)\quad \begin{vmatrix} de/dt \\ dp/dt \\ dy/dt \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & 0 & \pi \end{vmatrix}\begin{vmatrix} (e-e^*) \\ (p-p^*) \\ (y-y^*) \end{vmatrix}$$

where $a_{11} = Fu$ $\qquad V^S = (1-\tau^S)\beta + \emptyset\sigma^S$

$a_{12} = (1 - \beta Fu)/\beta$ $\qquad V^L = (1-\tau^L)\beta + \emptyset\sigma^L$

$a_{13} = \emptyset/\beta$ $\qquad F = (1/\Phi)f_2$

$a_{31} = (1/V^S)[\beta\delta^S Fu + \alpha\beta\delta^L]$

$$a_{32} = (1/V^S)[\delta^S(1 - \beta Fu) - \alpha(\sigma^L + \beta\delta^L)]$$

$$a_{33} = (1/V^S)[\O\delta^S - \pi(\sigma^S + \beta\delta^S) - \alpha V^L]$$

and $0 < (1-\tau^L) < (1-\tau^S) < 1$.[5] The short-run static equations, Eqs. 3.1 and 3.2, and the system of differential equations, Eq. 3.3, fully describe the motion of the system over time.

The characteristic equation associated with Eq. 3.3 is:

$$(4)\quad R^3 + A_1R^2 + A_2R + A_3 = 0$$

where
$$A_1 = -(R_1 + R_2 + R_3) = -(a_{11} + a_{33})$$
$$= -(1/V^S)[(\O\delta^S + V^S Fu) - \pi(\sigma^S + \beta\delta^S) - \alpha V^L]$$
$$A_2 = R_1R_2 + R_1R_3 + R_2R_3 = -(a_{13}a_{31} + \pi a_{32} - a_{11}a_{33})$$
$$= -(1/V^S)[\alpha(\O\delta^L + V^L Fu) - \alpha\pi(\sigma^L + \beta\delta^L) + \pi(\sigma^S Fu + \delta^S)]$$
$$A_3 = -R_1R_2R_3 = -\pi(a_{12}a_{31} - a_{11}a_{32}) = -\alpha(\pi/V^S)(\sigma^L Fu + \delta^L) < 0$$

and R (R_j, $j = 1,2,3$) is a root. The discriminant of Eq. 4, A_3, is unambiguously negative, implying that the system has either three positive roots or one positive root. When the roots are all positive, the system is totally unstable. To show that there exists only one positive root, notice that if $[(\O\delta^S + V^S Fu) - \pi(\sigma^S + \beta\delta^S)]$ and $[(\O\delta^L + V^L Fu) - \pi(\sigma^L + \beta\delta^L)]$ are both positive (negative), then A_2 is negative (ambiguous) while A_1 is ambiguous (positive). In either case there is only one variation in the sign of coefficients since the coefficient of R^3 is positive and A_3 is negative; by Descartes' rule of signs, there exists only one positive root and, therefore, the system possesses saddlepoint property.

Given that Eq. 4 has the solutions $R_1 < 0$, $R_2 < 0$, and $R_3 > 0$, the solution to Eq. 3.3 is of the following form:

$$(5.1)\quad e(t) = e^* + b_{11}K_1\exp^{R_1t} + b_{12}K_2\exp^{R_2t} + b_{13}K_3\exp^{R_3t}$$

(5.2) $p(t) = p^* + b_{21}K_1\exp^{R_1t} + b_{22}K_2\exp^{R_2t} + b_{23}K_3\exp^{R_3t}$

(5.3) $y(t) = y^* + b_{31}K_1\exp^{R_1t} + b_{32}K_2\exp^{R_2t} + b_{33}K_3\exp^{R_3t}$

where $b_{1j} = 1$

$$b_{2j} = [(a_{13}a_{31} - a_{11}a_{33}) + (a_{11} + a_{33})R_j - R_j^2]/$$

$$[(a_{12}a_{33} - a_{13}a_{32}) - a_{12}R_j]$$

$$b_{3j} = [-(a_{12}a_{31} - a_{11}a_{32}) - a_{32}R_j]/[(a_{12}a_{33} - a_{13}a_{32}) - a_{12}R_j]$$

$$= (R_j/\pi)b_{2j}$$

$$K_j = c_{j1}(e(0)-e^*) + c_{j2}(p(0)-p^*) + c_{j3}(y(0)-y^*)$$

and c_{ji} (j = 1, 2, 3) is the element in the j^{th} row and the i^{th} column of the inverse of the matrix containing b_{ij}'s while $e(0)$, $p(0)$, and $y(0)$ are the values of e, p, and y following some disturbance.

Starting from an initial long-run equilibrium where $e = e^*(0)$, $p = p^*(0)$, and $y = y^*$, any exogenous shock to the system that affects the equilibrium price level and/or short-run income will yield a new long-run equilibrium characterized by $e = e^*$, $p = p^*$ or $p^*(0)$, and $y = y^*$. If the initial conditions are such that both K_1 and K_2 are zero (nonzero) but K_3 is nonzero (zero), then the model is explosive (convergent); thus, Eqs. 5.1 to 5.3 show the saddlepoint behavior of the system.

Consider now the bounded solution to the dynamic system, Eq. 3.3. Since the system has three state variables, three linearly independent boundary conditions must be specified to obtain a unique solution.

For the predetermined, backward-looking variable, the price level (which is sticky and cannot jump), the boundary condition is that this variable is equal to its initial value. This means that the price level following some disturbance, $p(0)$, is the same as the initial equilibrium price level, $p^*(0)$, and, therefore:

(6.1) $p(0) - p^* = - dp^*$.

That is, the initial disequilibrium in the price level equals the negative of the change in the equilibrium price level.

The boundary condition for the nonpredetermined, forward-looking variable, the exchange rate, is standard in linear rational expectations models. This transversatility condition ensures that the system is dynamically stable. For a previously unanticipated and immediately implemented shock, Dixit (1980) has shown that dynamic stability can be achieved by ensuring that K_3, the product of the row eigenvector associated with the positive root and the column vector of the initial values of the state variables measured as deviations from the new long-run equilibrium, equals zero. This requires that:

(6.2) $e(0) - e^* = - (c_{32}/c_{31})(p(0)-p^*) - (c_{33}/c_{31})(y(0)-y^*)$

$$\text{where } c_{32}/c_{31} = (b_{22}R_2 - b_{21}R_1)/(R_1 - R_2)b_{21}b_{22} = C_1/C_3$$

$$c_{33}/c_{31} = \pi(b_{21} - b_{22})/(R_1 - R_2)b_{21}b_{22} = C_2/C_3$$

$$C_1 = R_1R_2[a_{12}(a_{12}a_{31} - a_{11}a_{32}) + a_{32}(a_{12}a_{33} - a_{13}a_{32})]$$

$$C_2 = \pi[(a_{12}a_{31} - a_{11}a_{32})(a_{12}a_{33} - a_{13}a_{32}) - a_{12}((a_{12}a_{31} - a_{11}a_{32})(R_1 + R_2) + a_{32}R_1R_2)]$$

$$C_3 = \pi[(a_{12}a_{31} - a_{11}a_{32})^2 + a_{32}((a_{12}a_{31} - a_{11}a_{32})(R_1 + R_2) + a_{32}R_1R_2)]$$

and both c_{32}/c_{31} and c_{33}/c_{31} are ambiguous in signs.[6] Since the price level is sticky, the exchange rate must jump to place the system onto the path converging toward the new long-run equilibrium. Such exchange-rate jump, indicated by Eq. 6.2, ensures that the system is dynamically stable.

Thus, for a given initial value of the price level, the stabilizing exchange-rate jump determined by Eq. 6.2 ensures that the system will converge to the new steady-state. However, Eq. 6.2 is not unique unless an additional linear boundary condition, relating to income, is specified.

A boundary-value problem arises in this model due to the nature of the state variables, in particular income. Specifically: the price level is a predetermined, backward-looking variable; the exchange rate is a non-predetermined, forward-looking variable; and, income, because it can jump at a point in time whenever the exchange rate jumps and because it also has a component that is characterized by an adaptive process (see Eq. A2 in the Appendix and Eq. 1.1), is a nonpredetermined but backward-looking variable. The presence of a nonpredetermined but backward-looking variable implies that the number of negative roots exceeds the number of predetermined variables. In this case, the additional boundary condition must take the form of a linear restriction on the state variables at the initial date. Following Buiter (1984), this restriction yields the initial disequilibrium in income:

$$(6.3)\quad y(0) - y^* = dy = (1/W)[(\beta\delta^s/V^s)de^* + (c_{32}/c_{31})(\beta\delta^s/V^s)dp^* + b_k dz_k/V^s]$$

$$\text{where } b_k z_k = \beta y_o + \sigma^s dm + \beta\delta^s p_f + \beta X_t$$

$$W = 1 + (c_{33}/c_{31})(\beta\delta^s/V^s)$$

and dz_k is the change in the k^{th} exogenous variable or shift parameter while X_t is the collection of integral terms in Eq. 1.1 which is fixed at time t (see Appendix for derivation).

The boundary conditions, Eqs. 6.1 to 6.3, imply that:

$$(7)\quad e(0) - e^* = (1/W)[(c_{32}/c_{31})dp^* - (c_{33}/c_{31})[(\beta\delta^s/V^s)de^* + b_k dz_k/V^s].$$

Eq. 7 is the key equation of the model. It determines the unique and stabilizing exchange-rate jump that places the system onto the convergent path and, therefore, whether at $t = 0$ the short-run exchange rate will overshoot, undershoot, neither overshoot nor overshoot, or respond perversely in the sense that it will move in a direction opposite to that of the long-run equilibrium exchange rate.

After the jump, the system moves along the path converging toward the new steady-state. Using Eq. 6.2 for $(p-p^*)$ and Eq. 3.3 for de/dt, it can be shown that along this stable path:

(8) $E(de/dt) = de/dt = -\theta_1(e-e^*) - \theta_2(y-y^*)$

where $\theta_1 = a_{12}/(c_{32}/c_{31}) - a_{11}$

$\theta_2 = a_{12}(c_{33}/c_{31})/(c_{32}/c_{31}) - a_{13}$

and, by assumption, $0 < \theta_1, \theta_2 < \infty$.[7] Using Eq. 6.2 again for $(e-e^*)$ in Eq. 8, it can be shown that the dynamics of the exchange rate and other variables during transition are governed by income and the price level.[8]

In summary, any exogenous shock to the system yields a new steady-state characterized by a saddlepoint. The adjustment involves two stages. First, at $t = 0$, the exchange rate and income jump to place the system on the stable arm of the saddlepoint and trigger the adjustment of other variables. Second, at $0 \leq t < \infty$, the system moves along the stable path toward the new steady-state and during this transition the slowly adjusting variables, the price level and income, govern the dynamics of the system.

3. Analysis of Disturbances

This section examines the effects of previously unanticipated discretionary policies. These policies are implemented at $t = 0$ and complemented by a fully-sterilized intervention policy at $0 \leq t < \infty$.

3.1. Fiscal Policy. Consider first an increase in government spending. This can be analyzed by writing y_o in Eq. 1.1 as follows: $y_o = n_o + n_1 g$. Using Eqs. 2.1 to 2.4, the steady-state effects are: $de^*/dg = -(1/\delta^L)$ and $dp^*/dg = di^*/dg = dr^*/dg = 0$. The long-run effect of fiscal expansion is to cause e^* as well as $(e^*-p^*+p_f)$ to fall. The fall in e^* is such that the resulting decrease in net exports fully offsets the increase in government spending, thereby leaving y^* unchanged.

The impact effects, using Eqs. 2.1 to 2.4, 3.1, 3.2, 6.3, and 7, are:

$$(9.1)\ d(e(0)-e^*)/dg = -(c_{33}/c_{31})(1/W)(\beta/V^S)(1-\delta^S/\delta^L)n_1 < 0$$

$$(9.2)\ d(y(0)-y^*)/dg = dy(0)/dg = (1/W)(\beta/V^S)(1-\delta^S/\delta^L)n_1 > 0$$

$$(9.3)\ d(i(0)-i^*)/dg = di(0)/dg = (\emptyset/\beta)(1/W)(\beta/V^S)(1-\delta^S/\delta^L)n_1 > 0$$

$$(9.4)\ d(r(0)-r^*)/dg = dr(0)/dg = -u[d(e(0)-e^*)/dg] < 0$$

where $0 < \delta^S/\delta^L < 1$. Since it is assumed that $0 < \theta_1, \theta_2 < \infty$, it follows that $0 < (c_{33}/c_{31}) = (a_{13}+\theta_2)/(a_{11}+\theta_1) < \infty$ and $0 < (c_{33}/c_{31})(1/W) = [(a_{13} + \theta_2)/((\emptyset\delta^S + V^S Fu)/V^S + \theta_1 + \theta_2(\beta\delta^S)/V^S)] < \infty$.

It is clear from Eqs. 9.1 to 9.4 that fiscal expansion causes income to increase and that to maintain money market equilibrium, interest rate must increase; in the foreign exchange market, de/dt must be positive to compensate the holders of foreign assets, implying that the exchange rate must overshoot its new long-run equilibrium value to maintain equilibrium in both markets (see Eqs. 3.1, 1.4, 1.5, and 8).

Is it possible for c_{33}/c_{31} to be zero and, hence, W to equal 1, in which case there is neither overshooting nor overshooting? If $c_{33}/c_{31} = 0$, then either $\theta_2 = -a_{13}$ or $\theta_1 = \infty$ and, therefore the assumption that $0 < \theta_1, \theta_2 < \infty$ is violated. On the other hand, if $c_{33}/c_{31} < 0$, then $\theta_2 < 0$ and all responses are perverse. Thus, c_{33}/c_{31} must be positive.

There is a role for endogenous intervention in dampening exchange rate overshooting caused by fiscal expansion (see Eqs. 1.4, 1.5, and 8). However, the intervention parameter, u, may only be set such that θ_1 (θ_2) becomes infinitely large (small) so that $(1/W)(c_{33}/c_{31})$ becomes infinitely small but not zero. Imperfect asset substitution and fully-sterilized intervention can, therefore, only dampen but cannot eliminate overshooting.

3.2. Monetary Policy. Consider now an increase in money supply. The steady-state effects, using Eqs. 2.1 to 2.4, are: $de^*/dm = dp^*/dm = 1$ and $di^*/dm = dr^*/dm = 0$. Thus, in the long-run, the system is homogeneous with respect to changes in the money supply.

The impact effects, using Eqs. 2.1 to 2.4, 3.1, 3.2, 6.3, and 7, are:

$$(10.1)\quad d(e(0)-e^*)/dm = (1/W)[(c_{32}/c_{31}) - (c_{33}/c_{31})(\sigma^S + \beta\delta^S)/V^S]$$

$$(10.2)\quad d(y(0)-y^*)/dm = dy(0)/dm = (1/W)[\sigma^S/V^S + (\beta\delta^S/V^S)(1 + (c_{32}/c_{31}))]$$

$$(10.3)\quad d(i(0)-i^*)/dm = di(0)/dm =$$

$$(1/W)[-((1-\tau^S) - \emptyset\delta^S)/V^S + (c_{32}/c_{31})\emptyset\delta^S/V^S - (c_{33}/c_{31})\delta^S/V^S]$$

$$(10.4)\quad d(r(0)-r^*)/dm = dr(0)/dm = -\,u[1 + d(e(0)-e^*)/dm]$$

where $(1/W)[(c_{32}/c_{31}) - (c_{33}/c_{31})(\sigma^S + \beta\delta^S)/V^S] = [((1-\tau^S) - \emptyset\delta^S - V^S Fu)/V^S - \theta_2(\beta\delta^S + \sigma^S)/V^S]/[(\emptyset\delta^S + V^S Fu)/V^S + \theta_1 + \theta_2(\beta\delta^S)/V^S)]$ and since by assumption $a_{12} > 0$ and $\theta_1 > 0$, it follows that $(c_{32}/c_{31}) = a_{12}/(a_{11} + \theta_1) < \infty$.

The exchange rate will neither overshoot nor undershoot, i.e., Eq. 10.1 will be zero, if:

$$(10.5)\quad \emptyset\delta^L + V^L Fu - (1-\tau^L) = (1/\sigma)(\Omega_1/\Omega_2)$$

$$\text{where } \Omega_1 = a_{12}\{[\pi(\sigma^S + \beta\delta^S)][\pi(\sigma^S + \beta\delta^S)/V^S + (R_1 + R_2)] + R_1R_2\}$$

$$\Omega_2 = \pi(\sigma^S + \beta\delta^S)/V^S - a_{32}V^S R_1R_2/(\sigma^L Fu + \delta^L) > 0.$$

and, by assumption, $\Omega_2 > 0$. With instantaneous aggregate-demand adjustment ($\alpha = \infty$), perfect capital substitutability ($F = 0$), and non-intervention ($u = 0$), then Eq. 10.5 reduces to the Dornbusch's condition. With $\alpha < \infty$, it can be shown that the exchange rate overshoots (undershoots) if $(\emptyset\delta^L + V^L Fu - (1-\tau^L))$ is negative (positive) and α is greater (less) than $\Omega_1/(\emptyset\delta^L + V^L Fu - (1-\tau^L))\Omega_2$. It is also possible for the exchange rate to exhibit perverse response and this happens if the right-hand side of Eq. 10.1 is negative and greater than one in absolute value.

If money supply increases and the exchange rate initially exhibits overshooting (neither overshooting nor undershooting, undershooting, perverse response), then $d(r(0)-r^*)/dm$ will be negative (negative, negative, positive). Income will unambiguously increase since by assumption $c_{32}/c_{31} > 0$ while the domestic interest rate may fall, remain the same, or rise.

Again, there is a role for intervention. However, now one may choose:

$$(10.6)\quad u = [(1-\tau^L) - \emptyset\delta^L - (1/\alpha)(\Omega_1/\Omega_2)]/V^L F$$

so that there is neither overshooting nor undershooting (see Eq. 10.5). This shows that intervention may not only dampen but may also eliminate overshooting resulting from monetary expansion; in contrast, intervention may only dampen overshooting resulting from fiscal expansion.

4. Concluding Remarks

Most papers on exchange rate dynamics neglects the distinction between short-run and long-run elasticities of the trade balance by assuming that exports and imports adjust instantaneously to changes in prices and exchange rates. Exceptions are Bhandari and Levin who introduce sluggish aggregate-demand and trade-flow lags, respectively, into the Dornbusch variable income model. They show that, unlike the Dornbusch result, the exchange rate will unambiguously overshoot its new long-run equilibrium value in response to a fiscal disturbance but may or may not exhibit overshooting in response to a monetary disturbance.

This paper has also dealt with monetary policy, fiscal policy, and the issue of exchange-rate overshooting in a model with sluggish aggregate demand. However, the focus of this paper has been to examine how fully-sterilized intervention can influence exchange rate movements resulting from either fiscal or monetary expansion in a model characterized by sticky prices, imperfect capital substitutability, sluggish aggregate demand, and rational expectations. Thus, the Dornbusch variable-income model has been extended by introducing not only sluggish aggregate-demand adjustment to allow for distinction between short-run and long-run elasticities of the trade balance but also imperfect capital substitutability to allow for fully-sterilized intervention policy.

It is shown that, unlike monetary policy, fiscal policy will always result in exchange rate overshooting even if accompanied by fully-sterilized intervention policy. This means that intervention may only dampen (may not only dampen but may also eliminate or even reverse) exchange rate overshooting resulting from fiscal (monetary) expansion. However, in this model where there are two slowly adjusting variables, dynamic responses may no longer be monotonic and overshooting may not be simply an instantaneous response but may also be a consequence of the dynamic path itself, i.e., monetary or fiscal expansion may lead to prolonged overshooting or even cyclical fluctuations of the exchange rate.[9] Intervention, therefore, may play an even more important role.

Appendix

The dy/dt equation is derived by first differentiating Eq. 1.1 with respect to time. Since $y^{ds} = y$ and $dp_f/dt = dy_o/dt = 0$, this gives:

$$\text{(A1)}\quad dy/dt = \tau^o y_t + \tau^s dy/dt - \sigma^o i_t - \sigma^s di/dt + \delta^o(e_t - p_t + p_f)$$
$$+ \delta^s(de/dt - dp/dt) - \alpha X_t$$

$$\text{where } X_t = \int_{-\infty}^{t} \tau^o \exp^{-\alpha(t-T)} y_T d_T - \int_{-\infty}^{t} \sigma^o \exp^{-\alpha(t-T)} i_T d_T$$
$$+ \int_{-\infty}^{t} \delta^o \exp^{-\alpha(t-T)} (e_T - p_T + p_f) d_T.$$

Using Eq. 1.1 for X_t and Eq. 1.1.ii for y^{dL}, (A1) can be rewritten as:

$$\text{(A2)}\quad dy/dt = \tau^s(dy/dt) - \sigma^s(di/dt) + \delta^s(de/dt - dp/dt) + \alpha(y^{dL} - y^{ds}).$$

(A2) can be reduced to its final form by using Eq. 3.1 for i and to derive di/dt, Eq. 1.1.i for y^{dL}, Eq. 1.2 for dp/dt, and Eq. 3.3 for de/dt. Since $y = y^{ds}$, this yields the dynamic equation for y (see Eq. 3.3).

The derivation of the initial disequilibrium in income, $(y(0)-y^*)$, is based on Buiter (1984). First, Eq. 1.1 can be rewritten as:

$$\text{(B1)}\quad [y - y_o - \tau^s y + \sigma^s i - \delta^s(e-p+p_f)] = X_t$$

indicating that while y is nonpredetermined, [.] can be treated as predetermined since the collection of integral terms, X_t, is fixed at time t. Next, solving (B1) for y, using Eq. 3.1 for i, and taking the total differential of the resulting equation yields:

$$\text{(B2)}\quad dy = (1/V^s)(\beta\delta^s de + d(\beta y_o + \sigma^s m + \beta\delta^s p_f + \beta X_t)) = y(0) - y^*.$$

where, since y^* is exogenously fixed, the change in income (dy) following some disturbance also equals the initial disequilibrium in income $(y(0)-y^*)$. Finally, using Eq. 6.2 to get de, and substituting it into (B2) yields the equation for $y(0) - y^*$, Eq. 6.3 in the text.

Notes

[1] For a survey, see Krugman and Obstfeld (1985).

[2] Both Bhandari and Levin assume regressive-expectations schemes (which may be perfect-foresight consistent as Levin has shown). However, if perfect foresight is imposed directly, as in this paper, one has to specify the boundary conditions. Thus, this paper is also an application of Buiter's (1980) method for solving a saddlepoint problem in a perfect foresight model where one state variable is nonpredetermined but backward-looking which implies that the number of negative roots exceeds the number of predetermined variables.

[3] Eq. 1.4 is an inverted net private foreign asset demand function (see Frankel (1983)). It is assumed that $f^d = -\Phi(i - i_f - E(de/dt))$, where $f^d = f$, f^d is the log of net private foreign asset demand and f, equalling $f_0 + f_1 nfa - f_2 r$, is the log of net private foreign asset supply.

[4] It is assumed that $m = m_1 c + m_2 r$ and $c = c_0 - c_1 r$ where c is the log of domestic credit. Thus, $m = m_1 c_0 + (m_2 - m_1 c_1) r$, and the case where $(m_2 - m_1 c_1) = 0$ corresponds to full sterilization (see Makin (1981)).

[5] Eqs. 3.1 and 3.2 are derived using Eqs. 1.3 and 1.5 (in deviation forms). The equation for de/dt is derived using Eq. 1.4 (in deviation form), Eq. 3.1, Eq. 3.2, and the assumption of perfect foresight. The equation for dp/dt, Eq. 1.2, is an assumed feature of the model and is given by Eq. 1.2 while the equation for dy/dt, like the equation for de/dt, is derived from the entire model (see Appendix for derivation).

[6] If $a_{12} = 0$ (< 0), then $a_{32} < 0$, $a_{12}a_{33} - a_{13}a_{32} > 0$, $c_{32}/c_{31} < 0$, and c_{33}/c_{31} is positive (ambiguous). If $a_{12} > 0$, then a_{32}, $a_{12}a_{33} - a_{13}a_{32}$, c_{32}/c_{31}, and c_{33}/c_{31} are all ambiguous. However, it is assumed here that $a_{12} > 0$, $a_{32} < 0$, $a_{12}a_{33} - a_{13}a_{32} > 0$ and, therefore, $c_{33}/c_{31} > 0$; in addition, it

is assumed that $a_{12}(a_{12}a_{31} - a_{11}a_{32}) > |a_{32}(a_{12}a_{33} - a_{13}a_{32})|$ and, thus, $c_{32}/c_{31} > 0$.

[7] The assumption that $\theta_1 > 0$ implies that a_{12} and c_{32}/c_{31} have the same signs. It is assumed here that $a_{12} > 0$ and $\theta_1 > 0$, which imply that $c_{32}/c_{31} > 0$. On the other hand, the assumptions that $\theta_1 > 0$ and $\theta_2 > 0$ imply that $c_{33}/c_{31} > 0$.

[8] Two equations that are equivalent to Eq. 8 are: $E(de/dt) = de/dt = -\theta_3(y-y^*) + \theta_4(p-p^*)$ and $E(de/dt) = de/dt = -\theta_5(e-e^*) + \theta_6(p-p^*)$ where $\theta_3 = a_{11}(c_{33}/c_{31}) - a_{13}$, $\theta_4 = a_{12} - a_{11}(c_{32}/c_{31})$, $\theta_5 = a_{13}/(c_{33}/c_{31}) - a_{11}$, and $\theta_6 = -a_{13}(c_{32}/c_{31})/(c_{33}/c_{31}) + a_{12}$ (see Eq. 3.3 for de/dt and Eq. 6.2). However, Eq. 8 is chosen for its intuitive appeal.

[9] After the jump, the time paths of the state variables are described by Eqs. 5.1 to 5.3 and 6.1 to 6.3. Using these equations, it can be shown that the responses would not necessarily be monotonic nor unidirectional. Specifically, in the case where R_1 and R_2 are real: (1) the exchange rate's response to a fiscal expansion would be initial overshooting followed by further overshooting or delayed undershooting or unidirectional adjustment and (2) the exchange rate's possible responses to a monetary expansion would be initial overshooting (undershooting, perverse response) followed by (a) further overshooting (undershooting, perverse response) or (b) delayed undershooting (overshooting, overshooting or undershooting) or (c) monotonic adjustment. In the case where R_1 and R_2 are complex, the exchange rate would exhibit cyclical behavior in response to either fiscal or monetary expansion.

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