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EGALITARIAN BIAS AND TEAM PRODUCTIVITY: AN IMPOSSIBILITY

by

Raul V. Fabella

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Egalitarian Bias and Team Productivity: An Impossibility

Raul V. Fabella
University of the Philippines
Quezon City, Philippines

Abstract

We show that an output sharing scheme with an egalitarian bias is inconsistent with first best production level in a team with self-interested nonidentical members.

Running Head: Egalitarian Bias and Team Productivity

Dr. Raul V. Fabella
University of the Philippines
School of Economics
Diliman, Quezon City
Philippines, 3004

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The exhaustive allocation of output in a cooperative production arrangement that sustains first best production level remains a subject of abiding interest to economists. Sen's (1966) classic paper confronting the two conflicting principles of distribution according to "work" and according to "need", gave the condition under which voluntary allocation of labour attains the Pareto optimal production level pursued by the central authority: the proportion of income to be distributed according to work should equal the ratio of the elasticity of output with respect to labor to the relative share of income to value of total output. Nitzan and Schnytzer (1987) showed that the knowledge of the magnitude of the marginal rate of substitution between income and leisure allows optimal choice of the institutional setting depending on the degree of egalitarianism. The "carrot and stick option" results in the oversupply, while the "constant threat of unemployment option" results in an undersupply of labor. Fabella (1988) showed that "natural team sharing" (or what Sen (op.cit.) calls the allocation according to "work") sustains first best production level under a unique (up to a constant of proportionality) production function which generates equality of average product and marginal product of labor. When the effort levels are nonobservable, Holmstrom (1982) showed that no sharing scheme that exhausts output will sustain first best production.

In this paper, we investigate the capacity of a sharing scheme displaying an egalitarian bias as defined in Sen (op.cit.) and Nitzan and Schnytzer (op.cit.) to sustain first best production level. In Section II, we review the "cooperative program" that generates the cooperative first best effort levels and thus production level. We then present the noncooperative (Cournot-Nash) counterpart as the "individual program," where we assume nonidentical self-interested members. In Section III, assuming effort levels observable, we show that the sharing scheme with an egalitarian bias is incapable of sustaining first best production level. We start with the framework of Holmstrom (op.cit.).

II. The Cooperative and the Individual Program

Consider a team of $n \geq 2$ members. Following Holmstrom (op.cit.), we endow every member i with a utility function $U_i = X_i - V_i(l_i)$ where X_i is ith share in total output and $V_i(l_i)$ is increasing and strictly convex on l_i, the ith effort level. The production function F is nondecreasing and concave over $L = \sum_{j=1}^{n} l_j$ following Sen (op.cit.) and Nitzan and Schnytzer (op.cit.). All functions are differentiable. The cooperative program is

$$\max_{\{l_i\}}[F(L) - \sum_{i=1}^{n} V_i(l_i)] \qquad i = 1, 2, \ldots, n. \quad (1)$$

The 1° condition gives

$$F' = V_i' \qquad i = 1, 2, \ldots, n. \qquad (2)$$

where $F' = \delta F/\delta l_i$ and $V_i' = dV_i/dl_i$. (2) determines the "cooperative first best effort levels," $\{l_i^*\}$, and thus the first best production level $F(\{\sum_{i=1}^{n} l_i^*\})$. (2) is Sen's (op.cit.) condition (11) if his U is an identity over income.

Assuming members to be individually rational, i.e., self-interested, each maximizes U_i across effort levels with the knowledge that total product is shared exhaustively and that his/her share is s_iF, where $0 < s_i < 1$ is ith proportional share, and $\sum_{i=1}^{n} s_i = 1$. Member i then faces the problem

$$\max_{l_i}[s_iF - V_i(l_i)] \qquad i = 1, 2, \ldots, n. \qquad (3)$$

If we assume for the moment with Holmstrom that effort levels are unobservable or that the sharing scheme for any reason is divorced from effort levels (say, an effort-independent social welfare function governs allocation as in Macleod (1984) or if every member gets the average output), then team members take s_i as given. The 1° condition in this situation is

$$F' = V_i'/s_i \qquad i = 1, 2, \ldots, n. \qquad (4)$$

This determines the "noncooperative (Cournot-Nash) effort levels," $\{l_i^{nc}\}$. Since for every i, $s_i < 1$, it can be expected that $\{l_i^{nc}\} < \{l_i^*\}$ and $F(\{l_i^{nc}\}) < F(\{l_i^*\})$. A general proof is provided in Holmstrom (op.cit.).

III. Team Sharing with Egalitarian Bias

As in Sen (op.cit.), the output sharing scheme may be based on either or a combination of two principles: (a) "to each according to his (her) needs" and (b) "to each according to his (her) work." The latter is interpreted here to mean as done in accordance with the worker's proportionate contribution to total effort level or what was called the "natural team sharing" in Fabella (op.cit.). Suppose the team decides to partly serve principle (a) by injecting an egalitarian bias into the sharing scheme. Following Sen (op.cit.) and Nitzan and Schnytzer (op.cit.), we have

Definition 1 : A sharing scheme $\{s_i\}$ possesses an egalitarian bias if, $\forall_i = 1, 2, \ldots, n$, $s_i = \{(1-a)(l_i/L) + a/n\}$, $0 < a \leq 1$. Whenever this is the case, we write $\{s_i^e\}$.

Remark 1: Note that the worker's share in total work effort is l_i/L, $L = \sum_{j=1}^{n} l_j$. If $a = 0$, we are back to the "natural team sharing." If $a = 1$, $s_i = 1/n$, $\forall_i = 1,2,\ldots n$, implying complete equality.

Remark 2: Note that a is within the half-closed interval, $(0, 1]$, so that an egalitarian bias is always assured. Thus the i^{th} share in total output is nudged downwards (upwards) in the direction of the average share if $l_i/L > (<)(1/n)$.

Remark 3: The sharing scheme is exhaustive since $\sum_{i=1}^{n} s_i =$

$1-a+a=1$

Remark 4: The egalitarian bias parameter a is the same across members.

The individual program is now

$$\max_{l_i}[\{(1-a)(l_i/L) + a/n\}F - V_i(l_i)] \quad i = 1, 2, \ldots, n. \quad (5)$$

The 1° condition for maximum is:

$$(1-a)[((L-l_i)/L)(F/L) + (l_i/L)F'] + (a/n)F' = V_i' \quad (6)$$

which simplies to:

$$\{(1-a)[((L-l_i)/L)(F/F'L) + (l_i/L)] + a/n\}F' = V_i' \quad (7)$$

Thus, for first best production level $F(L^*)$ to be sustained, it is necessary and sufficient that the expression

$$\{(1-a)[((L-l_i)/L)(F/F'L) + (l_i/L)] + a/n\} = 1 \quad (8)$$
$$\forall_i = 1, 2, \ldots, n$$

We first define "nonidentical members."

Definition 2: Two members h and k are nonidentical if, for effort level $\bar{l} > 0$, $V_h'(\bar{l}) \neq V_k'(\bar{l})$.

The following is immediate from (7).

Lemma 1: Let members h and k be nonidentical. Let the optimum effort levels be l^*_h and l^*_k for h and k, respectively. Then $l^*_h \neq l^*_k$.

Proof: Suppose $l^*_h = l^*_k$. Then the left hand side expression of (7) for h equals the right hand side expression of (7) for k, a contradiction. Q.E.D.

Clearly, if all members of the team are identical, we have a symmetric game case and all effort levels are equal. In a symmetric game case, the egalitarian bias is redundant since $(l^*_i/L) = (l^*_j/L) = 1/n$, $\forall_i \neq j = 1, 2, ..., n$. It is in the case of nonidentical membership where an egalitarian bias matters.

The following is obvious, repeats Sen (op.cit.) and is given only for contrast:

Proposition 1: It is not possible to sustain first best production level with $\{s_i^e\}$ if $a = 1$.

Proof: If $a = 1$, we have from (7) $F'/n = V_i'$ and workers are undersupplying effort levels. Q.E.D.

The question then boils down to whether for some $0 < a < 1$, first best production level is sustained.

We have the following:

Lemma 2: If the first best production level is sustained under $\{s_i^e\}$, then $(F/L) > F'$ and vice-versa.

Proof: The necessary and sufficient condition (8) for first best production level under $\{s_i^e\}$ becomes

$$((L-l_1)/L)(F/F'L) + (l_1/L) = (1-(a/n))/(1-a) > 1$$

for $n \geq 2$. This simplifies to

$$(1-(l_1/L))(F/F'L) > 1 -(l_1/L)$$

giving $(F/L) > F'$.

Suppose $(F/L) > F'$. Let $a = a^*$ be such that

$$((L-l_1)/L)(F/F'L) + (l_1/L) = (1-(a^*/n))/(1-a^*).$$

We show that $0 < a^* \leq 1$. The above equality simplifies into

$$(F/F'L) + (l_1/L)(1-(F/F'L)) = (1-(a^*/n))/(1-a^*).$$

Consider the left hand side (lhs) alone. Let $(F/F'L) = c > 1$. Then the lhs is $\{c-(l_1/L)(c-1)\}$. If $1 < c < \infty$, then $(l_1/L)(c-1) < (c-1)$ if $(l_1/L) < 1$. But $c-(c-1) = 1$ so that $\{c-l_1/L)(c-1)\} > 1$. Now consider the right hand side (rhs) which is continuous and strictly increasing in a^*. It is clear that $1 < \text{rhs} \leq \infty$ for $0 < a^* \leq 1$. Thus, there is an $a = a^*$, $0 < a^* \leq 1$, so that the above equality is satisfied. Q.E.D.

This says simply that the average product should exceed the marginal product of labor. Now since (8) is true for all members, it is true for any two distinct members k and h. Since a and n are constants, it is obvious from (8) that, for members k and h,

$$\{((L-l_k)/L)(F/F'L) + l_k/L\} = \{((L-l_h)/L)(F/F'L) + l_h/L\} \quad (9)$$

This resolves readily into:

$$(l_k - l_h) + (F/F'L)(l_h - l_k) = 0. \quad (10)$$

We now have the following:

Lemma 3: Nonidentical members h and k each render the first best effort level under $\{s_i{}^e\}$ if and only if $(F/L) = F'$.

Proof: (if) Condition (10) is satisfied immediately if $(F/F'L) = 1$. (only if) Suppose (10) holds. Now (10) becomes $l_k(1-(F/F'L)) + l_h((F/F'L) - 1) = 0$ where the parenthesized expressions have equal absolute values but opposite signs. This is true if either $l_k = l_h$, which contradicts the distinctness of members k and h, (Lemma 1) or if $(F/F'L) = 1$. Thus the latter must hold. Q.E.D.

We finally have:

Proposition 2: (Impossibility) The sharing scheme with an egalitarian bias $\{s_i{}^e\}$ is inconsistent with the first best production level in a team of self-interested nonidentical members.

Proof: Lemma 2 shows that for the result to occur $(F/L) > F'$. Lemma 3 shows that for the same to occur $(F/L) = F'$ if there is at least one distinct member. A contradiction.

This result is consistent with Sen's (op.cit., 14b) condition when "social consciousness" is either absent or incomplete, to wit,

$$(1-a) = (F'L/F)/\beta \quad (11)$$

where $\beta = 1$ is our framework since only labor input is considered. Note that if $(F'L/F) = 1$, then the condition is not satisfiable with $a > 0$ (our egalitarian bias case). The contribution of the paper is in showing that for nobody in a team, where at least one member is distinct, to shirk ($F' = V_i'$, $\forall_i = 1, 2, \ldots, n$), $(F'L/F)$ has to be equal to 1 and the contradiction follows. The reason why this problem did not arise in Sen (op.cit.) is that he considered only symmetric voluntary allocation among identical households who then render identical labor. The case of $l_h \neq l_k$ was not allowed.

Conclusion

Having seen how stringent is the condition for "natural team sharing" to sustain first best production level (Fabella, op. cit.), we investigated whether a sharing scheme with egalitarian bias $\{s_i^e\}$ can sustain first best production level among self-interested individuals. We showed that whereas the condition for first best effort under $\{s_i^e\}$ requires $(F/L) > F'$ for every individual member (Lemma 2), it requires $(F/L) = F'$ for it to hold across all members where at least one member is distinct (Lemma 1 and 3). Thus it is impossible for $\{s_i^e\}$ to sustain first best production level in a team with nonidentical members.

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