Discussion Paper No. 8805 June 1988

EQUITY, EFFICIENCY AND ALTRUISM IN COOPERATIVE TEAMS

by

Raul V. Fabella

Equity, Efficiency and Altruism in Cooperative Teams

Raul V. Fabella
University of the Philippines
Quezon City, Philippines

Abstract

We show that in a cooperative team characterized by symmetric nondiscriminating altruism, imperfect altruism and strictly positive equity bias are inconsistent with first best efficiency when members are nonidentical.

The apparent success of cooperative labor arrangements in Japan and Germany (e.g., Ouchi, 1981; Gordon, 1982) has rekindled renewed interest in the theory of production cooperatives. Reinforcing this view is a growing evidence that some cooperative arrangement in the form of profit sharing is superior to a straight wage contract in enhancing productivity (Fitzroy and Kraft, 1987; Jones and Svejnar, 1982). All these appear to go against the grain of the accepted understanding of cooperative team efficiency. Alchian and Demsetz (1971) emphasized the aspect of effort level undersupply in partnerships due to their susceptibility to free riding. The situation suggests the need for a principal (the capitalist) who monitors work effort in return for a claim on the residual. Holmstrom (1982) buttressed the Alchian and Demsetz viewpoint by showing that where effort levels are nonobservable, no exhaustive output sharing scheme can be devised that does not sacrifice output. On the other hand, a bonus and penalty scheme that responds to output does attain first best efficiency. A capitalist is again called for. The presumed superiority of straight wage contract, may, however, involve an overestimate of the monitoring capacity of the principal, or an underestimate of the shirking creativity of workers. Workers can enlarge their surplus share in nonpecuniary form by restricting effort and output (e.g., Frank, 1984). The growing emphasis on human capital makes monitoring more tenuous. The advantage of straight wage contracts erodes further if peer monitoring which is incentivized in cooperative arrangements succeeds in reducing shirking (Fitzroy and Kraft, op.cit). It may also stem from failure to take into account of unselfish behaviour as is common in oligopoly theory (Kurz, 1985). Fitzroy and Kraft's (op.cit.) claim that "no generally accepted microeconomic foundation for group incentives and cooperative behavior has yet emerged..." remains largely valid.

The microeconomic theoretical pool that applies to and is potentially fruitful in this area comes from the early and abiding interest in the two fundamental conflicting output sharing principles, namely, (a) "to each according to his (her) need" and (b) "to each according to his (her) work." Sen's (1966) seminal paper showed that in a production cooperative with observable effort levels (a) with perfect altruism, first best efficiency is compatible with every degree of egalitarianism in output sharing; (b) with non- or incompletely altruistic members, first best efficiency is attained if the surplus is allocated according to work equals the ratio of the elasticity of output with respect to labor and the relative share of labor income in total value of output. Sen (op.cit.) dwelt however only on the symmetric case of his model. For nonaltruistic members, Nitzan and Schnytzer (1988) largely following Sen (op.cit.) but using only output as the index of social welfare pointed out the strategic importance of the marginal rate of substitution between leisure and income in the optimal choice of the institutional setting given the degree of egalitarianism. Fabella (1988) showed that "natural team sharing" (or what Sen (op.cit.) calls allocation according to work) sustains first best efficiency among self-interested members if and only if output is directly proportional to the sum of all effort levels. This tends to confirm the pessimism of Alchian and Demsetz (op.cit.). Thus the static cooperative team theory does not still form a satisfactory microeconomic framework to explain the evidence. In the supergame version of the team game, Macleod (1984) showed that there exists an output-based "trigger strategy equilibrium" which sustains solutions Pareto-superior to the Cournot-Nash position. It naturally inherited the usual strong assumptions of the framework.

In this paper we investigate further the role of altruism and its interaction particularly with equity in the pure cooperative sharing arrangement. We address questions: When do teams with altruistic membership attain first best efficiency? Is this compatible with some egalitarian bias? In Section II, we review the "cooperative program" which generates the definition of of the first best production efficiency. We also present the "individual program" of voluntary allocation. In Section III, we define "symmetric nondiscriminatory altruism," the equity-biased sharing scheme and nonidentical membership. We proceed to prove that with imperfect altruism among nonidentical members, no egalitarian bias can coexist with first best efficiency.

II. The Cooperative and the Individual Programs

Consider a team of $n \geq 2$ members. Following Sen (op.cit.), the welfare function of member i consists two parts. The first is ith private utility function defined as:

$$U_i = X_i - V_i(l_i) \qquad i = 1, 2, \ldots, n. \qquad (1)$$

where X_i is the ith share in total output and $V_i(l_i)$, the ith disutility associated with supplying effort level l_i, is differentiable, increasing and strictly convex in l_i. Note that in Sen (op.cit.), the utility function is also separable in income and effort. Member i also attaches some value to the well-being of other members of the team. Let b_{ij} be the weight member i attaches to the utility of individual j. Then member i's welfare function is:

$$W_i = U_i + \sum_{\substack{j=1 \\ i \neq j}}^{n} b_{ij} U_j \qquad (2)$$

where U_i gets a weight of one ($b_{ii} = 1$) and $b_{ij} \leq 1$. In this paper, we propose to treat only individuals with nondiscriminatory altruism.

<u>Definition 1:</u> Member i displays nondiscriminatory altruism

if $b_{ij} = b_{ik}, \forall j \quad k = 1, 2, \ldots, n;\ j, k \neq i.$

We further simplify the model by imposing symmetry.

Definition 2: A team is characterized by symmetric nondiscriminatory altruism (sna) if $b_{ij} = b_{ik} = b$, $\forall i, j, k = 1, 2, \ldots, n$; $i \neq j, k$.

With sna members are assumed to treat only themselves differently. With symmetric nondiscriminatory altruism we rewrite (2) as

$$W_i = U_i + b\sum_{\substack{j=1 \\ j\neq i}}^{n} U_j. \qquad (3)$$

Remark 1: With sna, individuals are identical with respect to altruism but may not be identical in general.

Definition 3: A team with sna is characterized by perfect altruism if $b = 1$; by imperfect altruism if $0 < b < 1$.

The social welfare function W from the viewpoint of the team as a whole is a nondiscriminatory summation of private individual utilities defined, following Sen (op.cit.), as:

$$W = \sum_{i=1}^{n} U_i. \qquad (4)$$

We assume the production function F to be differentiable, nondecreasing and concave on $L = \sum_{i=1}^{n} l_i$. Since output is shared exhaustively, $F = \sum_{i=1}^{n} X_i$. Using (4), the cooperative program is

$$\max_{\{l_i\}} \{F(L) - \sum_{i=1}^{n} V_i(l_i)\}. \qquad (5)$$

The 1° conditions for maximum are:

$$F' = V_i' \qquad \forall_i = 1, 2, \ldots, n. \qquad (6)$$

where $F' = dF/dL$ and $V_i' = dV_i/dl_i$.

Note that this is identical to Sen's (op.cit.) condition (11) if i's utility function is assumed to be an identity over income. (6) generates the "cooperative first best effort levels," $\{l_i^*\}$ and thus the "first best production level," $F(\sum_{i=1}^{n} l_i^*)$. Whenever this is the case, we say that the team attains "first best efficiency."

The individual program using (3) is now

$$\max_{l_i} \{s_i F + b\sum_{\substack{j=1 \\ j\neq i}}^{n}[s_j F - V_j(l_j)] - V_i(l_i)\}. \qquad (7)$$

where s_i is ith proportional share in output F and $s_i F = X_i$. Assuming for the moment and for expository purpose that s_i is unresponsive to effort level l_i (say, $s_i = 1/n$, the average), and $b = 0$, then we have from the 1° condition, $F'/n = V_i'$, $\forall_i = 1,2,\ldots, n$, and i is undersupplying effort level. This means that the F implied is less than $F(\sum_{i=1}^{n} l_i^*)$. Holmstrom (op.cit.) provides the general proof.

III. The Main Result

Output can be distributed according to either or a combination of the two principles "to each according to his (her) needs" and "to each according to his (her) work."

Following Sen (op.cit.) we have the following:

Definition 4: A sharing scheme displays equity bias if

$\forall_i = 1,2, \ldots, n,\ s_i = \{(1-a)l_i/L + a/n\},\ 0 < a \leq 1.$

We denote this scheme by $\{s_i^*\}$.

Remark 2: If $a = 0$, $s_i = l_i/L$ which is the "natural team sharing" in Fabella (op.cit.) or allocation according to work in Sen (op.cit.). If $a = 1$, we have complete equality at $s_i = 1/n$. Thus the share of i is nudged downwards (upwards) in the direction of the average as $(l_i/L) > (<)(1/n)$.

Remark 3: Note that a lies somewhere in the half-open interval (0,1] and a bias in favor of equality is always assured.

Using Definition (4) the individual program becomes

$$\max_{l_i} \{[(1-a)(l_i/L)+a/n+b\sum_{\substack{j=1\\j\neq i}}^{n}((1-a)(l_j/L)+a/n)]F-b\sum_{\substack{j=1\\j\neq i}}^{n}V_j(l_i)-V_i(l_i)\}. \quad (8)$$

It is easy to show that (8) is a strictly concave program if $(F/F'L) \geq 1$. We assume this to be the case. The 1st conditon for maximum gives:

$$\{(1-a)[((L-l_i)/L)(F/F'L) + l_i/L+b(\sum_{\substack{j=1\\j\neq i}}^{n} l_j/L)(1-F/F'L)] + a[1+b(n-1)/n]\}F' = V_i' \quad (9)$$

$$\forall_i = 1,2,\ldots, n.$$

(9) generates the (non-cooperative) Cournot-Nash effort levels $\{l_i^{cn}\}$ and the Cournot-Nash production level, $F(\sum_{i=1}^{n} l_i^{cn})$. Thus the first best effort level, l_i^*, is supplied voluntarily if and only if

$$(1-a)[((L-l_i)/L)(F/F'L)+l_i/L+b(\sum_{\substack{j=1\\j\neq i}}^{n} l_j/L)(1-F/F'L)] + a[(1+b(n-1))/n]=1 \qquad (10)$$

$$i = 1,2,\ldots, n.$$

It is obvious from the structure of the individual program (7) and from (10) that with complete altruism (b=1), first best production efficiency is attained regardless of the value of a (as in Sen, op.cit.). The distribution aspect is of no consequence. But perfect altruism is more likely to be the exception than the rule.

Suppose that team members are imperfectly altruistic. Can the team as a whole serve the principle of sharing according to needs (i.e., with subsidies) and still be first best efficient? We first have the following:

Definition 5: Two members h and k are nonidentical if for effort level $l > 0$, $V_h'(l) \neq V_k'(l)$. Otherwise, h and k are identical.

Lemma 1 : If h and k are nonidentical, then $l_h^{cn} \neq l_k^{cn}$.

Proof: Suppose $l_h^{cn} = l_k^{cn}$ in (9).

Then the left hand side of (9) in the case of h equals the left hand side of (9) in the case of k. A contradiction.

Q.E.D.

<u>Lemma 2</u> : Let the team be sna. For every member to supply first best effort level under $\{s_i^e\}$ it is necessary and sufficient that $(F/F'L) > 1$.

<u>Proof:</u> (necessity) The necessary and sufficient condition (10) for member i to supply first best effort level under $\{s_i^e\}$ can be rewritten as

$$(F/F'L)[1 - (l_i + b\sum_{\substack{j=1 \\ j\neq i}}^{n} l_j)/L] + (l_i + b\sum_{\substack{j=1 \\ j\neq i}}^{n} l_j)/L = (1-a(1+b(n-1))/n)/(1-a). \quad (11)$$

Under $\{s_i^e\}$, $a > 0$ and the right hand side of (11) is strictly greater than 1. Simplifying, we have:

$$(F/F'L)(1 - (l_i + b\sum_{\substack{j=1 \\ j\neq i}}^{n} l_j)/L) > (1 - (l_i + b\sum_{\substack{j=1 \\ j\neq i}}^{n} l_j)/L)$$

and $(F/F'L) > 1$.

(sufficiency) Suppose $(F/F'L) > 1$. Let $0 < b < 1$. We show that there is $0 < a^* \leq 1$, so that (11) is true. Letting $(F/F'L) = c > 1$, we rewrite the left hand side (lhs) of (11) as

$$c - (c - 1)\ ([l_i + b(L-l_i)]/L).$$

But $[l_i + b(L-l_i)]/L) < 1$ and $c - (c-1) = 1$ so that lhs > 1. Now consider the right hand side (rhs) of

(11). Note that this is continuous and strictly increasing in a, i.e.,

$$\frac{d(rhs)}{da} = \frac{1-(1+b(a-1))/n}{(1-a)^2} > 0$$

since $0 < b < 1$. Furthermore, $1 < rhs \leq \infty$ for $0 < a \leq 1$. Thus for some $a = a^*$, $0 < a^* \leq 1$, (11) is satisfied.

Q.E.D.

Lemma 3: Let the team be sna. Let $0 < b < 1$. Let members h and k be nonidentical. Then h and k both supply first best effort level if and only if $F/F'L = 1$.

Proof: Consider the necessary and sufficient condition (10) for first best effort level. Since this is true for all i, it is true for h and k. Since a, b, n are the same across i, we have the following:

$$((L-l_h)/L)(F/F'L) + (l_h/L) + b(L-l_h)/L)(1-(F/F'L)) =$$
$$((L-l_k)/L)(F/F'L) + (l_k/L) + b(L-l_k)/L)(1-(F/F'L)).$$

Simplifying, we get:

$$(F/F'L)(l_k-l_h) + (l_h-l_k) + b(1-(F/F'L))(l_k-l_h) = 0. \quad (12)$$

(if) Suppose $(F/F'L) = 1$. Then we get $(l_k-l_h) - (l_k-l_h) = 0$. (only if). Suppose (12) to be true.

Simplifying (12) further, we have:

$$(l_k-l_h)(1-(F/F'L))(b-1) = 0$$

If $l_k \neq l_h$ and $0 < b < 1$, then $(F/F'L) = 1$. But by Lemma 1 $l_k \neq l_h$ if h and k are nonidentical. Q.E.D.

Thus, the nonidentical team membership assumption requires an additional stringent condition that, in fact, contradicts the condition in Lemma 2. The following is now obvious:

Proposition: If the team be sna. Let $o < b < 1$.

If the team members are nonidentical, first best efficiency is inconsistent with $\{s_1 e\}$.

Thus with nonidentical membership, imperfect altruism does not allow any degree of egalitarian bias to coexist with first best production efficiency. Egalitarian bias can coexist with first best production efficiency under identical membership but the bias is of no use in this case since everyone gets the average share anyway. Where it matters, egalitarian bias is untenable without sacrificing output. Equity and efficiency always conflict in a team characterized by imperfect sna and nonidentical membership. In addition, one can easily show using a known result (Fabella, 1988, Proposition 3) that a team characterized by sna, imperfect altruism and zero equity bias attains first best efficiency if and only if F = AL.

Conclusion

The paper started by pointing out the apparent discrepancy between the theory of cooperative arrangements and the growing evidence to the contrary. The theory due to Alchian and Demsetz (op.cit.) and Holmstrom (op.cit.) seem to point towards inefficiency. Macleod's (1984) dynamic version of the team game is an exception but the conditions are, as in the case of similar models, also very stringent. The promise in Sen (op.cit.) appears dimmed by the stringency of the production function condition for first best efficiency in Fabella (1988). Perfect altruism does the job but as we saw here, the more general imperfect altruism case falls far short: with some egalitarian bias, first best efficiency is not attained in a cooperative team of nonidentical membership.

References

Alchian, A.A. and H. Demsetz, 1972, Production, Information Cost, and Economic Organization, American Economic Review LXII, 777-795.

Fabella, R.V., 1988, Natural Team Sharing and Team Productivity, Economics Letters, forthcoming.

Fitzroy, F. R. and K. Kraft, 1987, Cooperation, Productivity and Profit Sharing, Quarterly Journal of Economics, 23-35.

Frank, R.H., 1984, Are Workers Paid Their Marginal Product? American Economic Review LXXIV, 549-71.

Gordon, R. J. 1982, Why U.S. Wage and Employment Behavior Differs from that in Brittain and Japan, Economic Journal XCII, 13-44.

Holmstrom, B., 1982, Moral Hazard in Teams, Bell Journal of Economics 13, 324-340.

Jensen, M.C. and W.H. Meckling, 1976, Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure, Journal of Financial Economics, 305-360.

Jones, D.C. and J. Svejnar, 1982, Participatory and Self-Managed Firms (Lexington Books, Lexington).

Macleod, N.B., 1984, A Theory of Cooperative Teams, CORE Discussion Paper No. 8441, CORE, Université Catholique de Louvain.

Moulin, H., 1987, The Pure Compensation Problem: Egalitarianism versus Laizzez Fairism, Quarterly Journal of Economic CII, 769-784.

Nitzan, S. and A. Schnytzer, 1987, Egalitarianism and Equilibrium Output in Producer Cooperatives, Economics Letters 24, 133-137.

Ouchi, W.G., 1981, Theory Z (Addison-Wesley, Reading, M.A.).

Sen, A.K., 1966, Labour Allocation in a Cooperative Enterprise, Review of Economic Studies 33, 361-371.