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Liquidity, Risk, and Efficient Forward
Foreign Exchange Markets

by

Cristino Arroyo III

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ABSTRACT

In this paper I suggest a partial answer to the question: if forward and spot currency markets are not "efficient" in the sense that on average the speculative profits agents can make in this market given information at the time expectations are formed are nonzero, why are they not so? Research that has suggested the inclusion of risk premia has met with mixed results in empirical tests. This paper suggests that treating currency as having differential liquidity services in a dynamic programming framework provides an alternative model that can accommodate some empirical facts that risk premia alone cannot account for.

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Caveat

The purpose of this paper in the form that I have chosen to present it in is, by way of description, expository and suggestive rather than anything I would deem complete or rigorous at this point, for it represents work that was initiated some months before and is still ongoing. More precisely my purpose here is merely to draw your interest to one of the long-standing issues in international finance that has yet to be adequately resolved by academic research. In this paper I suggest a partial answer to the question: if forward and spot currency markets are not "efficient" in the sense that on average the speculative profits agents can make in this market given what they know at the time are not zero, why are they not so? Research that has been based on the alternative hypothesis that risk premia account for nonzero expected profits have not given us adequate explanations for this empirical fact. My suggestion is that if one treats spot currency as money also (as opposed to being a pure financial asset) then there is a

formed. Together with the assumption of risk-neutrality (as well as the absence of liquidity preference, imperfect markets and transactions costs) rational expectations implies the well-known efficient markets hypothesis

$$(1) \quad E(r_{t+k} | I_t) = 0, \quad k = 1, 2, \dots$$

where r_{t+k} is the rate of return to speculative activity at time $t+k$ periods off and I_t is the information set at time t .

Interest has been generated, in recent years, as to how well this relationship holds up in foreign exchange markets, specifically, the market for forward currencies. In this context, the simple efficient markets hypothesis becomes (for the two-currency case)

$$(1') \quad E\left(\frac{S_{t+k} - F_{t,k}}{F_{t,k}} \middle| I_t\right) = 0, \quad k = 1, 2, \dots$$

where S_{t+k} is the spot rate at $t+k$ (the number of domestic currency units one can get for one foreign currency unit) and $F_{t,k}$ is the k -period ahead forward rate that obtains at time t . Letting $s_{t+k} = \ln S_{t+k}$ and $f_{t,k} = \ln F_{t,k}$ we can rewrite (1') approximately

$$(1'') \quad E(s_{t+k} - f_{t,k} | I_t) = 0$$

by a Taylor expansion of $\ln x$ around $x_0 = 1$, where $x = S_{t+k}/F_{t,k}$. Numerous papers have been concerned with whether or not (1') or (1'')

holds up to empirical tests. Several studies (Kaserman 1973; Bilson 1976; Bilson and Levich 1977; Frenkel 1977, 1978; Stockman 1978) test equations similar to

$$(2) \quad S_{t+k}^i = \alpha^i + \beta^i F_{t,k}^i + v_t^i, \quad i = 1, \dots, N$$

against the null hypothesis that $\alpha = 0$ and $\beta^i = 1$. With the

exception of Frenkel (1977) the null was not rejected. Frenkel did indicate the presence of a significant α^i that could be explained by transactions costs. Levich (1977), however, finds that such disparities between S_{t+k} and $F_{t,k}$ are smaller than what most researchers are inclined to think transactions costs are.

While the inability of regressions on (2) to reject $\alpha^i = 0$ and $\beta^i = 1$ suggest that $F_{t,k}$ is an unbiased predictor of S_{t+k} , it is a weak test of the simple efficient markets hypothesis. A more powerful approach is to discover whether $S_{t+k} - F_{t,k}/F_{t,k}$ or $s_{t+k} - f_{t+k}$ is correlated with economic variables that are likely to be components of I_t . Any such observed correlation will be a direct rejection of (1') or (1''). Natural candidates for such elements of the information set are the lagged forecast errors ($s_t - f_{t-k,k}$, $s_{t-1} - f_{t-k-1,k}$, ...) Cornell (1977) finds that the size of these forecast errors is not significantly large either way but there was some correlation with past forecast errors. Levich (1977), however, found no such serial correlation to hold over a larger sample period and for a larger set of forward rates (1, 3, and 6-month as opposed to just 1-month rates in Cornell (1977).) Levich's study, however, suffers from some problems with the econometrics, which we shall examine very short.

It was the valuable, though negative, contribution of Geweke and Peige (1979) and Hansen and Hodrick (1980, 1983) to point out that the reassuring initial results were not all that solid. Hansen and Hodrick (1980) test the relationship

$$(3) \quad s_{t+13}^i - f_t^i = \alpha^i + \beta^i (s_t^i - f_{t-13}^i) + \gamma^i (s_{t-1}^i - f_{t-14}^i) + v_{t,13}^i$$

against the null that $(\alpha, \beta, \gamma) = (0, 0, 0)$ and reject this for the Italian lira, Deutsche mark and Swiss franc, though there was no evidence

of significantly nonzero α^1 's. In the regression (3), estimates of β^1 and γ^1 provide a measure of the first and second-order serial correlation, respectively, provided that $E[(s_t - f_{t-1})v_t] = 0$ and $E[(s_{t-1} - f_{t-1})v_t] = 0$. This can be verified as true in a model where $v_{t,k}$ is defined as

$$(4) \quad v_{t,k} = (s_{t+k} - f_{t,k}) - E(s_{t+k} - f_{t,k} | I_t)$$

with $(s_t - f_{t-1}, s_{t-1} - f_{t-1}) \in I_t$. The problem with regressions like (3) is that $E(u_{t,k} u_{t+h,k}) = 0$ only for $h \geq k$ and not for $h < k$ and only in the case when $k = 1$ do we expect to see no serial correlation between $u_{t,k}$ and $u_{t+h,k}$. Because such serial correlation exists, the estimates of the covariance matrix of (3), $\hat{\Omega} \equiv E(\underline{u} \underline{u}')$, $\underline{u} = (u_{t,k})_{t=1, \dots, T}$ will be generally incorrect and significance tests of hypotheses on $(\alpha^1, \beta^1, \gamma^1)$ will be misleading.

Cornell (1977), Levich (1978), and Geweke and Feige (1979) have used $k = 1$ to sidestep this problem. In doing so they have sacrificed degrees of freedom thereby lessening the strength of their results. At first blush it would appear that GLS on (3) rather than OLS would be sufficient to correct the serial correlation. However, GLS requires

$$E(u_{t,k} | s_{t+k} - f_{t,k}, s_{t+k+1} - f_{t+1,k}, s_{t+k-1} - f_{t-1,k}, \dots) = 0$$

which is not at all true when $v_{t,k}$ is defined by (4). If GLS were to be applied to (3) in a model where $u_{t,k}$ is given by (4) then the GLS estimates of $(\alpha^1, \beta^1, \gamma^1)$ would be inconsistent. In order to get around this problem it is necessary to make appropriate modifications in estimating $\hat{\Omega}$ when applying OLS on (3). The details of the procedure are in Hansen (1979) and Hansen and Hodrick (1980) and will be omitted from this discussion. Suffice it to say that when Hansen

and Hodrick (1980) make the appropriate corrections in the estimation procedure for (3) they found evidence against the coefficients being all zero. They, in fact, conducted an even stronger test of the efficient markets hypothesis by applying corrected OLS on

$$(5) \quad s_{t+13}^i - f_t^i = \alpha^i + \sum_{j=1}^J \beta^{ij} (s_t^j - f_{t-13}^j) + u_t^i$$

for $i = 1, \dots, 5$ currencies (j indexes these currencies on the right-hand side). They reject the null hypothesis that $(\alpha^i, \beta^{ij})_{j=1, \dots, 5} = 0$ for the Canadian dollar, Deutsche mark, and Swiss franc, though again they find no strong evidence against $\alpha^i = 0$. When the set of currencies is expanded to include the lira and yen, rejection of the null is even more strongly indicated.

In an earlier study, Geweke and Feige (1979) look at OLS estimation of

$$(6) \quad \frac{s_{t+1}^i - f_t^i}{s_t^i} = \alpha^i + \beta^i \left(\frac{s_t^i - f_{t-1}^i}{s_{t-1}^i} \right) + v_t^i$$

for $i = 1, \dots, 7$ currencies. As mentioned above the use of observations generated at intervals equal to the forecast interval avoids serial correlation problems in the u_t 's but has less power for the reason that there are less degrees of freedom than if one were to allow for sampling intervals finer than the forecast interval. Over the two sample periods they look at they report evidence of nonzero intercept terms that also seem to change with time. They also find nonzero slope parameters that are significant. A more powerful joint test of the system

$$(7) \quad \frac{s_{t+1}^i - f_t^i}{s_t^i} = \alpha^i + \sum_{j=1}^7 \beta^{ij} \frac{s_t^j - f_{t-1}^j}{s_{t-1}^j} + u_t^i$$

was done for $i = 1, \dots, 7$ currencies using joint GLS (SUR, Zellner-Aitken) estimation. Again the null that $(\alpha^i, \beta^{ij}) = 0$ for all i, j is rejected in both sample periods. They note, however, that the joint test, while powerful enough to reject the simple efficiency hypothesis, does not provide insight as to why we are rejecting it, which brings us now to the problem of interpretation of evidence against the simple efficiency hypothesis.

The problem with all empirical tests of the efficient markets hypothesis is that we are testing a joint hypothesis of the truth of all the assumptions made in order to derive (1') or (1'') including all the "auxiliary hypotheses" of no risk aversion, transactions costs, liquidity preference, as well as the very form of the model that gets to be estimated (i.e., (2) together with (4), for example.) When we reject the efficient markets hypothesis it is often not clear that we are rejecting it because people do not really form rational expectations or because one of the auxiliary assumptions does not approximate reality closely enough. It is therefore a natural extension of this line of research to seek to reject all the plausible explanations suggested by the auxiliary assumptions given above.

Let us summarize the findings we have discussed to this point. The stylized facts that have come to attention are: (a) the presence of some intercept term in regressions like (2) which is usually smaller than transactions costs and appears to be unstable over time, (b) the presence of some serial correlation in the own-currency forecast errors

$s_{t+k}^i - f_{t,k}^i$ along with cross-correlation with lagged forecast errors of other currencies, $s_t^j - f_{t,k,k}^j$, etc.

A considerable amount of (to my mind misdirected) attention has been given to explaining fact (a) by researchers since Hansen and Hodrick and Geweke and Feige. Inasmuch as the early evidence suggested intercept terms which were smaller than transactions costs, researchers have been since inclined to interpret a significant intercept term as a risk premium a la the Capital-Asset Pricing Model (CAPM) of the finance literature:

$$(8) \quad \frac{S_{t+k}^i - F_{t,k}^i}{F_{t,k}^i} = R_t^f + \left\{ [E(R_t^m) - R_t^f] \cdot \frac{\text{cov} \left(\frac{S_{t+k}^i - F_{t,k}^i}{F_{t,k}^i}, R_t^m \right)}{\text{var}(R_t^m)} \right\} + \text{white noise}$$

where R_t^f is the return on a security that is uncorrelated with the market portfolio and R_t^m is the (random) return on the market portfolio. A simple CAPM study will have to allow for the term

$$\beta = \frac{\text{cov} \left(\frac{S_{t+k}^i - F_{t,k}^i}{F_{t,k}^i}, R_t^m \right)}{\text{var}(R_t^m)}$$

to be time-varying. Lately it has become more fashionable to go with new trends in financial modelling that suggest β also be made a function of "underlying" economic "factors," which is the approach suggested by the Arbitrage Pricing Theory (APT) as developed in Ross (1976); Cox, Ingersoll and Ross (1985); etc.

Apart from the usual criticisms that can be levelled on the theoretical foundations of CAPM and also of APT, my main criticism of this approach to explaining stylized fact (a) is that the direct application of CAPM or APT to forward speculative returns recognizes that forward currency is valuable for transferring wealth across states of the world but it fails to properly account for the value of spot currency in financing transactions in the market for goods. Put another way,

this genre of models treats currency as just any other financial asset (like a share in a corporation, or a bond) and not also as money. If the empirical results of such studies could explain fact (a) favorably, then my criticism would not be all that serious. The results, however, have been mixed to poor (see Frankel, 1982; Frankel and Engel, 1984; Engel and Rodrigues 1986, 1987). In this paper, I venture to suggest how the liquidity services of currency might be able to help explain not only fact (a) but also fact (b).

Fact (b) itself has been somewhat neglected by the research, though people are quite conscious of it. I believe this is due in part to the fact that it seems to be a more difficult bit of reality for the finance-based models to accommodate. However there is another class of models, which have their origins in Lucas (1978, 1982) but which have since been adapted by Svenson (1985), Stockman (1980), and Svenson and Stockman (1985) for the purposes of modelling international capital and commodity markets that appears to show promise in explaining the serial correlation in the forecast errors of the spot exchange rate. Our model belongs to this class of models and we shall turn to it briefly.

The economy is described as follows: there are two countries in the world made distinct only by the issuance of national currencies and the endowment of its constituent individuals. All quantities below will be in per capita terms unless otherwise specified.

The home country issues \bar{M}_t "dollars" at time t and its individuals are endowed every period with a stochastically varying amount of the home good, \bar{X}_t . Analogously, the foreign country issues \bar{N}_t "pounds" annually and foreign individuals get a random amount of \bar{Y}_t units of foreign good every period. We will suppose that the money supplies evolve stochastically according to

$$(1) \quad \begin{aligned} \bar{M}_t &= \rho_t \bar{M}_{t-1} \\ \bar{N}_t &= \rho_t^* \bar{N}_{t-1} \end{aligned}$$

where ρ_t and ρ_t^* are stochastic with joint distributions to be specified later.

Apart from the differences in their endowments, all individuals in the world are otherwise alike, so it is meaningful to speak of the representative individual at home or in the foreign country. Individuals have preferences defined over the infinite stream of stochastic consumption from a reference period τ onwards according to

$$(2) \quad E_\tau \sum_{t=\tau}^{\infty} \beta^t U(x_t, y_t)$$

where U is a smooth and strictly concave utility function over $(x_t, y_t) \geq 0$ and $\beta^t \in (0, 1)$. E_τ denotes statistical expectation conditioned on information at τ . The additively time-separable structure of preferences is a concession to the problem of time-consistency of the "optimal" consumption plan.¹

In a model without fiat money or tradeable financial assets individuals simply maximize (2) by choice of x_t^s, y_t^s subject to a sequence of budget constraints $p_t^s x_t^s + p_t^{*s} y_t^s \leq p_t^s \bar{x}_t^s + p_t^{*s} \bar{y}_t^s$, $s = 1, \dots, S$ where s indexes the (possibly infinite) states of the world at t . In this classical Arrow-Debreu setup the above problem is well-defined under regularity conditions on endowments and the consumption space and equilibrium exists with

$$(x_t^s, y_t^s) = \left(\frac{1}{2} \bar{x}_t^s, \frac{1}{2} \bar{y}_t^s \right)$$

since all individuals are risk-averse and alike in risk-averse preferences. It is presumed that in this setup trading is centralized

at an abstract time and place in economic prehistory at which individuals write contracts for delivery of goods contingent on certain states of the world occurring. There is no role for fiat (unbacked) money.

Let us now introduce the market for securities. The f^{th} security, in this model, is a claim to a random dividend d_t^f if held into period t . We shall denominate d_t^f in dollars.

As pointed out in Lucas (1982) there is a certain arbitrariness about which securities get to be traded in equilibrium as long as there are enough securities whose returns allow for a complete set of income transfers across states of the world so that no incomplete markets problem arises.² We assume that there are enough such securities, besides those we select arbitrarily in what follows, to replicate a complete set of Arrow-Debreu securities. Any redundant securities will be held in zero amounts and priced according to no-arbitrage conditions.

So let $z_t \in R^F$ be the quantities of the F securities held by the representative individual at t . Let $d_t \in R^F$ be the vector of dividends returned by the F securities in dollars at t . We will focus on four securities: the first two securities are claims to the endowment processes \bar{x}_t and \bar{y}_t . The next two are claims on the net transfers of money. Thus

$$\begin{aligned}
 d_t^1 &= p_t \bar{x}_t \\
 d_t^2 &= e_t p_t^* \bar{y}_t \\
 d_t^3 &= (\rho_t - 1) \bar{M}_{t-1} \\
 d_t^4 &= e_t (\rho_t^* - 1) \bar{M}_{t-1}
 \end{aligned}
 \tag{3}$$

where e_t is the rate of exchange of pounds for dollars at the time within the interval $(t, t+1)$ when securities markets are open. In this model e_t is not the usual spot exchange rate but is instead the forward exchange rate, as will become evident when we discuss the timing convention used later to support a demand for money. These assets will trade at prices $q_t \in R^F$ at time t . Our assumption on the joint distribution of $W_t \equiv (\rho_t, \rho_t^*, \bar{X}_t, \bar{Y}_t)$ is that W_t is first-order Markovian:

$$(4) \quad W_t \sim P(W_t | W_{t-1}) = \text{prob}\{W_t \leq k' | W_{t-1} = k\}.$$

We now introduce a role for fiat money in this model. This requires a departure from the standard Arrow-Debreu market institution of centralized trading of contracts for the delivery of contingent commodities. This can be done in a number of ways³ but here we shall do so by the imposition of finance constraints (also called cash-in-advance or Clower constraints) on the purchase of goods by the consumer. The motivation for this approach is as follows: first, assume that within the period t the market for goods and the market for assets (financial securities and currencies) are alternately open then closed, with the goods markets open first as a matter of convention only. Next, assume that individuals cannot issue currency (they are legally restricted from doing so, if necessary) so that the supplies of currency are, in fact, \bar{M}_t and \bar{N}_t dollars and pounds, respectively. Lastly, assume that, owing to decentralized trading in goods, currency minimizes transactions costs in the sense that with money transactions costs are zero but without it they are infinite.

These three assumptions are enough to give a role for fiat money in the model. Let us, however, be more explicit about the last

assumption; as the exact form of the liquidity constraint has nontrivial consequences on the equilibrium of the model. Specifically, suppose transactions costs in the purchase of home goods X_t are infinite without domestic money and zero with it. Then if M_t^O is the quantity of dollars held by the individual at t , his domestic finance constraint is:

$$(5.a) \quad P_t X_t \leq M_t^O$$

Were we to make a similar assumption in the purchase of foreign goods we would get a similar finance constraint for Y_t . We, however, shall depart from this to allow for differential liquidity services of various currencies. We assign additional liquidity services to the domestic money as follows: consider the individual who enters the period t with M_{t-1} worth of dollars and N_{t-1} worth of pounds carried over from $t-1$. If M_t^O of the M_{t-1} dollars are used to finance X_t purchases, then the finance constraint for Y_t we specify is

$$(5.b) \quad P_t^* Y_t \leq N_{t-1} + \frac{M_{t-1} - M_t^O}{S_t}$$

where S_t is the rate at which a pound exchanges for dollars at goods markets, i.e., the "spot exchange rate" in this model. This specification in (5.a) and (5.b) allows for differential liquidity services yet avoids the so-called "exchange rate indeterminacy" problem of Kareken and Wallace (1981) which would obtain if we instead specified both currencies as having the same liquidity service in either home goods or foreign goods markets.⁴

The assumption that individuals cannot issue their own money amounts to the constraints

$$\begin{aligned} 0 &\leq M_{t-1} \leq \bar{M}_{t-1} \\ (6) \quad 0 &\leq N_{t-1} \leq \bar{N}_{t-1} \\ M_{t-1} - M_t^0 &\geq 0 \end{aligned}$$

It remains to specify the amount and time of availability of information assumed known by individuals making choices at t . We assume that at the start of t , before goods markets open, W_t becomes known and that individuals do not forget W_{t-1}, W_{t-2}, \dots as well as all variables dated $t-1$ and earlier and the function F . (The Markovian assumption in (4), however, means we will actually only need to know W_{t-1} and state variables dated $t-1$.) After W_t is realized goods markets are open during which individuals trade according to the finance constraints (5.a) and (5.b). When goods trading closes asset markets then open into which individuals bring in $M_t^0 - P_t X_t$ leftover dollars, $N_{t-1} + \frac{M_{t-1} - M_t^0}{S_t} - P_t^* Y_t$ leftover pounds, $d_t Z_{t-1}$ dividends in dollars, $q_t Z_{t-1}$ worth of securities in dollars and $(\rho_t - 1)\bar{M}_{t-1}$ or $(\rho_t^* - 1)\bar{N}_{t-1}$ worth of new money transfers. They choose new moneyholdings M_t and N_t and new security holdings Z_t according to

$$\begin{aligned} (7) \quad M_t + e_t N_t + q_t Z_t &\leq M_t^0 - P_t X_t + e_t \left[N_{t-1} + \frac{(M_{t-1} - M_t^0)}{S_t} - P_t^* Y_t \right] \\ &\quad + (q_t + d_t) Z_{t-1} + (\rho_t - 1) \bar{M}_{t-1} \end{aligned}$$

No new information becomes available until asset markets close. This last assumption is an important one as it allows for $M_t^0 - P_t X_t > 0$

and $N_{t-1} + \frac{M_{t-1} - M_t^0}{S_t} - P_t^* Y_t > 0$ there is now a precautionary, and

not only a transactions demand for currency. This leads to a nontrivial (unit) velocity of money⁵ and is Svenson's (1983) improvement on the standard cash-in-advance model. Our timing convention are summarized in Figure 1.

The complete statement of the domestic individual's maximization program is

$$\max E_{\tau} \sum_{t=\tau}^{\infty} \beta^t U(X_t, Y_t)$$

$$(X_t, Y_t, M_t^0, M_t, N_t, Z_t)$$

$$\text{s.t. } P_t X_t \leq M_t^0$$

$$P_t^* Y_t \leq N_{t-1} + \frac{M_{t-1}^0 - M_t^0}{s_t}$$

$$M_t^0 \leq M_{t-1}$$

$$M_t + e_t N_t + q_t Z_t \leq (M_t^0 - P_t X_t) + e_t \left[N_{t-1} + \frac{(M_{t-1}^0 - M_t^0)}{s_t} - P_t^* Y_t \right]$$

$$+ (q_t + d_t) Z_{t-1} (\rho_t - 1) \bar{M}_{t-1} (M_t^0, X_t, Y_t, M_t, N_t) \geq 0.$$

The analogous problem for the foreign individual is the same except for the last constraint in which $(\rho_t - 1) \bar{M}_{t-1}$ is replaced by $(\rho_t^* - 1) \bar{N}_{t-1}$.

The additive separability of preferences and the Markovian assumption (4) permits us to rewrite the above domestic consumer's problem in the equivalent form of a dynamic program

$$(MAX) \quad \max_{\{M_t^0, x_t, y_t, M_t, N_t, Z_t\}} U(x_t, y_t) + \beta \int v(M_t, N_t, Z_t, \bar{M}_t, \bar{N}_t, w_t) dF(w_t | w_{t-1})$$

subject to

$$(L) \quad x_t \leq \frac{M_t^0}{P_t}$$

$$(L^*) \quad y_t \leq \frac{N_{t-1}}{P_t^*} + \frac{M_{t-1} - M_t^0}{S_t P_t^*}$$

$$(UB) \quad M_t^0 \leq M_{t-1}$$

$$(B) \quad x_t + \frac{e_t P_t^*}{P_t} y_t + \frac{M_t}{P_t} + \frac{e_t N_t}{P_t} + \frac{q_t Z_t}{P_t} \leq \frac{e_t M_{t-1}}{S_t P_t} + \frac{e_t N_{t-1}}{P_t} + \left(1 - \frac{e_t}{S_t}\right) \frac{M_t^0}{P_t} + \frac{(a_t + d_t) Z_{t-1}}{P_t} + \frac{(\rho_t - 1) \bar{M}_{t-1}}{P_t}$$

$$(NN) \quad \{M_t^0, x_t, y_t, M_t, N_t\} \geq 0$$

$$Z_t \in R^F$$

where v is the fixed point of the mapping $T: V \rightarrow V$, where V is the space of continuous bounded functions that map states

$\ell_t \equiv (M_t, N_t, Z_t, \bar{M}_t, \bar{N}_t, w_t)$ into R , and T is given by

$$T(v(\ell_{t-1})) = \max_{\{x_t, y_t\}} U(x_t, y_t)$$

$$+ \beta \int v(\ell_t) dF$$

s.t. (L), (L*), (B), (UB) and (NN). Under the metric

$$\Delta(v, \tilde{v}) = \sup_{\ell} |v(\ell) - \tilde{v}(\ell)|$$

the space V is a Banach space, and so the existence of such a $v \in V$ above for which

$$(FP) \quad v = Tv \quad \text{for all } \lambda$$

follows from the Banach fixed point or contraction mapping theorem once it is shown that T is, in fact, a contraction mapping.⁶

Details of the proof are in Sargent (1997, Appendix A.7). We shall take (MAX), (L), (L*), (UB), (B), (MN), and (FP) as our departure point for the results that follow.

Results

Let $(\theta_t, \theta_t^*, \mu_t, \lambda_t)$ be Lagrange multipliers corresponding to (L), (L*), (UB), and (B) respectively. The first-order conditions on (x_t, y_t) from performing (MAX) are, with $x_t, y_t > 0$:

$$(1) \quad \frac{U_{x_t}}{U_{y_t}} = \frac{\lambda_t + \theta_t}{e_t p_t^* \frac{\lambda_t}{p_t} + \theta_t^*}$$

$$(2) \quad U_{y_t} = e_t p_t^* \frac{\lambda_t}{p_t} + \theta_t^*$$

The first-order conditions for choice of M_t, N_t and Z_t are, with $M_t, N_t > 0$.

$$(3') \quad \beta \int v_{M_t} dF = \frac{\lambda_t}{p_t}$$

$$(4') \quad \beta \int v_{N_t} dF = e_t \frac{\lambda_t}{p_t}$$

$$(5') \quad \beta \int v_{Z_t} dF = e_t \frac{\lambda_t}{p_t}$$

From the fact that $v = Tv$ and our definition of T , we have

$$(3'') \quad v_{M_{t-1}} = \frac{e_t}{s_t} \frac{\lambda_t}{p_t} + \frac{\theta_t^*}{s_t p_t^*} + \mu_t$$

$$(4'') \quad v_{M_{t-1}} = e_t \frac{\lambda_t}{P_t} + \frac{\theta_t^*}{P_t^*}$$

$$(5'') \quad v_{Z_{t-1}} = (q_t + d_t) \frac{\lambda_t}{P_t}$$

so that combining (3') with (3''), (4') with (4''), etc. gives

$$(3) \quad \beta \int \left(\frac{e_{t+1}}{s_{t+1}} \frac{\lambda_{t+1}}{P_{t+1}} + \frac{\theta_{t+1}^*}{s_{t+1} P_{t+1}^*} + u_{t+1} \right) dF = \frac{\lambda_t}{P_t}$$

$$(4) \quad \beta \int \left(e_{t+1} \frac{\lambda_{t+1}}{P_{t+1}} + \frac{\theta_{t+1}^*}{P_{t+1}^*} \right) dF = e_t \frac{\lambda_t}{P_t}$$

$$(5) \quad \beta \int \left(q_{t+1} + d_{t+1} \right) \frac{\lambda_{t+1}}{P_{t+1}} dF = q_t \frac{\lambda_t}{P_t}$$

which are the (usual) pricing equations for currency and securities.

For example, (5) is usually solved recursively to get

$$q_t = \left(\frac{\lambda_t}{P_t} \right)^{-1} \int \sum_{j=1}^{\infty} \left(\beta^j \frac{\lambda_{t+j}}{P_{t+j}} d_{t+j} \right) dF$$

which will be one of a set of simultaneous pricing equations involving λ_t , P_t , and other market and shadow prices that will be solved in the general equilibrium of this model.

The first order condition of (MAX) subject to (L), (L*), (B), (UB), (NN), (FF) with respect to M_t^0 is for $M_t^0 > 0$:

$$(6) \quad \left(1 - \frac{e_t}{s_t} \right) \frac{\lambda_t}{P_t} + \frac{\theta_t}{P_t} - \frac{\theta_t^*}{s_t P_t^*} - u_t = 0$$

The remaining conditions on the multipliers are:

$$(7) \quad (B) \quad \text{and } \lambda_t \geq 0$$

$$(8) \quad x_t - \frac{\mu_t^0}{P_t} \leq 0$$

$$\theta_t \left(x_t - \frac{\mu_t^0}{P_t} \right) = 0$$

$$\theta_t \geq 0$$

$$(9) \quad y_t - \left(\frac{M_{t-1}}{P_t^*} + \frac{M_{t-1} - M_t^0}{S_t P_t^*} \right) \leq 0$$

$$\theta_t^* \left[y_t - \left(\frac{M_{t-1}}{P_t} + \frac{M_{t-1} - M_t^0}{S_t P_t^*} \right) \right] = 0$$

$$\theta_t^* \geq 0$$

$$(10) \quad M_t^0 - M_{t-1} \leq 0$$

$$\mu_t (M_t^0 - M_{t-1}) = 0$$

$$\mu_t \geq 0$$

together with

$$v_{M_{t-1}} = \frac{(\rho_{t-1})}{P_t}$$

There are a number of results that proceed out of the individual's maximization problem alone even before we consider the general equilibrium of the model. In this paper we focus on the liquidity and risk premia on forward currency implied by this model. We shall not examine the general equilibrium results here (to do so in this model will require assuming an explicit form for $U(x_t, y_t)$ and the process F , so one can solve for the unknown functions $(v, \lambda_t, \theta_t, \theta_t^*, \mu_t, P_t, P_t^*, e_t, S_t, \alpha_t)$ given the market-clearing conditions of an equilibrium).

We have said that the appropriate interpretations of S_{t+1} and e_t are that these are the period-ahead spot and one-period forward exchange rates respectively. We now derive a relationship between the two that is of relevance to the efficient markets hypothesis. Write $E_t x_t$ for $\int x_t dF(w_t | w_{t-1})$ to make the notation more compact. From (1) and (2) we have

$$(11) \quad U_{y_t} - \theta_t^* = \frac{e_t P_t^*}{p_t} (U_{x_t} - \theta_t).$$

Then from (3') and (4') we get

$$(12) \quad E_t v_{N_t} = e_t E_t v_{M_t}$$

and (3') and (5') give us

$$(13) \quad E_t v_{Z_t} = q_t E_t v_{M_t}$$

Hence,

$$(14) \quad E_t v_{Z_t} = \frac{q_t}{e_t} E_t v_{N_t}$$

Now (6) and (3'') imply

$$(15) \quad v_{M_{t-1}} = \frac{\lambda_t}{p_t} + \frac{\theta_t}{p_t}$$

so by (1)

$$(16) \quad v_{M_{t-1}} = \frac{U_{x_t}}{p_t}$$

From (6) and (4'')

$$v_{N_{t-1}} = s_t \left(\frac{\lambda_t}{p_t} + \frac{\theta_t}{p_t} - \mu_t \right)$$

and using (1) and $p_t > 0$ gives

$$(17) \quad v_{N_{t-1}} = s_t \left(\frac{U_{x_t}}{p_t} - \mu_t \right).$$

and using (5'') and (1) we get a similar expression for $v_{Z_{t-1}}$:

$$(18) \quad v_{Z_{t-1}} = \frac{e_t + d_t}{p_t} (U_{x_t} - \theta_t).$$

Now let's combine (12) with the expressions for v_{N_t} and v_{M_t} given by (16) and (17). Substitution of these in (12) yields

$$E_t S_{t+1} \left(\frac{U_{x_{t+1}}}{P_{t+1}} - \mu_{t+1} \right) = e_t E_t \frac{U_{x_{t+1}}}{P_{t+1}} \quad (11)$$

or,

$$(19) \quad E_t \frac{U_{x_{t+1}}}{P_{t+1}} (S_{t+1} - e_t) = E_t \mu_{t+1} S_{t+1} \quad (12)$$

Combining (13) with (13) and (16) in similar fashion we get

$$E_t \frac{q_{t+1} + d_{t+1}}{P_{t+1}} (U_{x_{t+1}} - \theta_{t+1}) = q_t E_t \frac{U_{x_{t+1}}}{P_{t+1}} \quad (13)$$

Dividing both sides by q_t and rearranging,

$$(20) \quad E_t \left[\left(\frac{q_{t+1} + d_{t+1}}{q_t} - 1 \right) \frac{U_{x_{t+1}}}{P_{t+1}} \right] = E_t \left(\frac{q_{t+1} + d_{t+1}}{q_t} \right) \frac{\theta_{t+1}}{P_{t+1}} \quad (14)$$

And, similarly, combining (13) with (17) and (18),

$$E_t \left(\frac{q_{t+1} + d_{t+1}}{P_{t+1}} \right) (U_{x_{t+1}} - \theta_{t+1}) = \frac{q_t}{e_t} E_t S_{t+1} \left(\frac{U_{x_{t+1}}}{P_{t+1}} - \mu_{t+1} \right) \quad (15)$$

Multiplying through by q_t/e_t and rearranging,

$$(21) \quad E_t \left\{ \left[e_t \left(\frac{q_{t+1} + d_{t+1}}{q_t} \right) - S_{t+1} \right] \frac{U_{x_{t+1}}}{P_{t+1}} \right\} \\ = E_t \left\{ e_t \left(\frac{q_{t+1} + d_{t+1}}{q_t} \right) \frac{\theta_{t+1}}{P_{t+1}} - S_{t+1} \mu_{t+1} \right\} \quad (16)$$

One can see that $(21) = (19) + (20)$.

Now consider (19). Define the conditional covariance

$\text{cov}_t(x_t, y_t)$ according to

$$E_t x_t y_t = E_t x_t E_t y_t + \text{cov}_t(x_t, y_t). \quad (17)$$

Applying this to (19) gives

$$E_t \left[\frac{U_{x_{t+1}}}{P_{t+1}} (S_{t+1} - e_t) \right] = E_t \mu_{t+1} E_t S_{t+1} + \text{cov}_t(\mu_{t+1}, S_{t+1})$$

If, in equilibrium, $M_{t-1} > M_t^0$ always, then $\mu_t = 0$ always and then

$$E_t \left[\frac{U_{x_{t+1}}}{P_{t+1}} (S_{t+1} - e_t) \right] = 0$$

which is an orthogonality condition on $\frac{U_{x_{t+1}}}{P_{t+1}}$ and $S_{t+1} - e_t$ that is testable by the Generalized Method of Moments (GMM) estimation technique popularized by Hansen and Singleton (1982) and Hansen (1982).

Typically, though, this equation is instead rewritten using the definition of $\text{cov}_t(x_t, y_t)$ above. We can rearrange this and apply the definition of cov_t again and obtain the expression for e_t as the sum of the expected spot $E_t S_{t+1}$ and a risk premium:

$$(22) \quad e_t = \frac{E_t \left[\frac{U_{x_{t+1}}}{P_{t+1}} S_{t+1} \right]}{E_t \left[\frac{U_{x_{t+1}}}{P_{t+1}} \right]} = E_t S_{t+1} + \frac{\text{cov}_t \left(\frac{U_{x_{t+1}}}{P_{t+1}}, S_{t+1} \right)}{E_t \frac{U_{x_{t+1}}}{P_{t+1}}}$$

where the last term on the RHS has the interpretation of a consumption-risk premium (a covariance between the future return of a risky asset and the future return on random consumption). Typically, this equation, or very similar ones, is the one that gets to be implemented in the empirical research on the efficiency hypothesis in a time-varying CAPM context (Hansen and Hodrick 1983, Frankel and Engel 1984, Engel and Rodrigues 1986, 1987) for now if we put $S_{t+1} = E_t S_{t+1} + \epsilon_{t+1}$ where $\epsilon_{t+1} \sim \text{i.i.d.}(0, \sigma_\epsilon^2)$ then we have

$$(23) \quad e_t = S_{t+1} + \frac{\text{cov}_t \left(\frac{U_{x_{t+1}}}{P_{t+1}}, S_{t+1} \right)}{E_t \left(\frac{U_{x_{t+1}}}{P_{t+1}} \right)} - \epsilon_{t+1}$$

or

$$(23'') \quad S_{t+1} - e_t = \frac{-\text{cov}_t \left(\frac{U_{x_{t+1}}}{P_{t+1}}, S_{t+1} \right)}{E_t \left(\frac{U_{x_{t+1}}}{P_{t+1}} \right)} + \epsilon_{t+1}$$

which can be estimated, for instance, as a random coefficients model in the manner of Hildreth and Houck (1963), say, which is essentially a kind of GLS procedure, or as an error-components model which also entails a GLS procedure. (OLS is not consistent here for if we take

differences between the risk premium $\frac{\text{cov}_t \left(\frac{U_{x_{t+1}}}{P_{t+1}}, S_{t+1} \right)}{E_t \left(\frac{U_{x_{t+1}}}{P_{t+1}} \right)}$ and its

mean, if it exists, these differences will mix with the error term ϵ_{t+1} and need not be serially uncorrelated.)

The empirical importance of the way currency has been accorded a role as liquidity in this model is that the above equation may no longer be the appropriate estimating equation for forward market efficiency. For μ_t need not be zero, in fact from (6), (1), and (2),

$$\mu_t = 0 \quad \text{if and only if}$$

$$\frac{U_{x_t}}{P_t} = \frac{U_{y_t}}{S_t P_t^*}$$

i.e., if and only if

$$\frac{U_{x_t}}{U_{y_t}} = \frac{P_t}{S_t^{p^*}}$$

when $\mu_t > 0$ (iff $\frac{U_{x_t}}{P_t} > \frac{U_{y_t}}{S_t^{p^*}}$) equation (19) becomes

$$\begin{aligned} (24) \quad e_t &= \left[E_t \frac{U_{x_{t+1}}}{P_{t+1}} \right]^{-1} \left[E_t \frac{U_{x_{t+1}}}{P_{t+1}} E_t S_{t+1} + \text{cov}_t \left(\frac{U_{x_{t+1}}}{P_{t+1}}, S_{t+1} \right) \right. \\ &\quad \left. - E_t \mu_{t+1} E_t S_{t+1} - \text{cov}_t (\mu_{t+1}, S_{t+1}) \right] \\ &= \left(1 - \frac{E_t \mu_{t+1}}{E_t \frac{U_{x_{t+1}}}{P_{t+1}}} \right) E_t S_{t+1} + \text{cov}_t \left(\frac{U_{x_{t+1}}}{P_{t+1}}, S_{t+1} \right) \\ &\quad - \text{cov}_t (\mu_{t+1}, S_{t+1}) \\ &= \left(1 - \frac{E_t \mu_{t+1}}{E_t \frac{U_{x_{t+1}}}{P_{t+1}}} \right) E_t S_{t+1} + \text{cov}_t \left(\frac{U_{x_{t+1}}}{P_{t+1}} - \mu_{t+1}, S_{t+1} \right) \end{aligned}$$

Now putting $S_{t+1} = E_t S_{t+1} + e_{t+1}$ as before gives us, instead,

$$\begin{aligned} (25) \quad e_t &= S_{t+1} - \frac{E_t \mu_{t+1}}{E_t \frac{U_{x_{t+1}}}{P_{t+1}}} S_{t+1} + \text{cov}_t \left(\frac{U_{x_{t+1}}}{P_{t+1}} - \mu_{t+1}, E_t S_{t+1} + e_{t+1} \right) \\ &\quad - \left(e_{t+1} - \frac{E_t \mu_{t+1}}{E_t \frac{U_{x_{t+1}}}{P_{t+1}}} e_{t+1} \right) \end{aligned}$$

Equation (25) suggests a number of things, first of which is that the forward rate need not be an unbiased predictor of the future spot rate not just because of the risk premium $\text{cov}_t \left(\frac{U_{x_{t+1}}}{P_{t+1}} - \mu_{t+1}, \varepsilon_{t+1} \right)$

but also because of the term $\frac{E_t \mu_{t+1}}{E_t \frac{U_{x_{t+1}}}{P_{t+1}}} S_{t+1}$. I am inclined to

interpret this term as the "liquidity premium" from having to give up liquidity today in order to speculate on tomorrow's spot rate-- this interpretation is suggested by the relationship between μ_t and the shadow prices of liquidity θ_t and θ_t^* given in (6).

This liquidity premium can be time-varying for a number of reasons: (a) there is the component S_{t+1} which need not be stationary, (b) the process F_t could have been nonstationary had we relaxed the Markovian assumption that $F_t(w_t) = P(w_t | w_{t-1})$, or (c) the equilibrium itself may well be nonstationary-- μ_{t+1} or $U_{x_{t+1}}$ or P_{t+1} may, in the equilibrium of this model follow a nonstationary process (like an AR(1) process, for example), which could also cause the term $E_t \mu_{t+1} / E_t (U_{x_{t+1}} / P_{t+1})$ to drift over time.

The second thing we notice in (25) is that we also have a potentially time-varying risk premium, but not so simple a one as before for now variations in the liquidity premia with time enter

via μ_{t+1} as an adjustment to $\frac{U_{x_{t+1}}}{P_{t+1}}$, so that we must be

careful to interpret the risk premium as the covariation between

ε_t and the marginal utility of home consumption net of the

liquidity value of additional domestic currency brought into the goods

trading session. ~~Thirdly,~~ the error term in (25) can also be time-varying

unless $E_t \mu_{t+1} / E_t (U_{x_{t+1}} / P_{t+1})$ is stable.

It is possible to rewrite (25) in terms of observables for
by (6) and (1) and (2),

$$(26) \quad \mu_t = \frac{U_{x_t}}{P_t} - \frac{U_{y_t}}{S_t P_t^*}.$$

Using (26) to substitute for μ_{t+1} in (25) gives

$$\begin{aligned} e_t = S_{t+1} - & \left[1 - \frac{E_t \left(\frac{U_{y_{t+1}}}{S_{t+1} P_{t+1}^*} \right)}{E_t \left(\frac{U_{x_{t+1}}}{P_{t+1}} \right)} \right] S_{t+1} - \text{cov}_t \left(\frac{U_{y_{t+1}}}{S_{t+1} P_{t+1}^*}, \epsilon_{t+1} \right) \\ & - \left[\frac{E_t \left(\frac{U_{y_{t+1}}}{S_{t+1} P_{t+1}^*} \right)}{E_t \left(\frac{U_{x_{t+1}}}{P_{t+1}} \right)} \right] \epsilon_{t+1} \end{aligned}$$

We are interested in an equation with S_{t+1} as the LHS variable.

This equation is

$$(27) \quad S_{t+1} = \frac{E_t \left(\frac{U_{x_{t+1}}}{P_{t+1}} \right)}{E_t \left(\frac{U_{y_{t+1}}}{S_{t+1} P_{t+1}^*} \right)} e_t - \text{cov}_t \left(\frac{U_{y_{t+1}}}{S_{t+1} P_{t+1}^*}, \epsilon_{t+1} \right) \frac{E_t \left(\frac{U_{x_{t+1}}}{P_{t+1}} \right)}{E_t \left(\frac{U_{y_{t+1}}}{S_{t+1} P_{t+1}^*} \right)} - \epsilon_{t+1}$$

where $\epsilon_t \sim \text{i.i.d. } (0, \sigma_\epsilon^2)$ as before. This can be implemented
empirically as a regression

$$S_{t+1} = \alpha_t \beta_t + \beta_t e_t - \epsilon_{t+1}$$

$$\text{where } \beta_t = \frac{E_t \left(\frac{U_{x_{t+1}}}{P_{t+1}} \right)}{E_t \left(\frac{U_{y_{t+1}}}{S_{t+1} P_{t+1}^*} \right)}$$

$$\alpha_t = - \text{cov}_t \left(\frac{U_{y_{t+1}}}{S_{t+1} P_{t+1}^*}, \epsilon_{t+1} \right)$$

β_t here was written out as time-varying, but in a stationary

equilibrium under the Markov assumption on F_t it will not be so, hence $\beta_t = \beta$ in such a case. α_t , however, can be time-varying even if F_t is stationary, so the estimation procedure used will have to be some GLS procedure since time-varying parameters are involved whose random parts may be serially correlated and will get mixed in with ϵ_{t+1} .⁷ The above equation must then be tested against the

null that $\beta_t \geq 1$ since $\mu_t = \frac{U_{x_t}}{P_t} - \frac{U_{y_t}}{S_t P_t^*} \geq 0$ implies

$$\frac{E_t(U_{x_{t+1}}/P_{t+1})}{E_t(U_{y_{t+1}}/S_{t+1}P_{t+1}^*)} \geq 1. \quad \text{Rejection of such a null would imply a}$$

rejection of the model and all auxiliary assumptions made in order to get equation (27). The next step would be to test against

$H_0: \beta = 1$, for a rejection of this would tell us that liquidity premia of the kind suggested by the model are important in explaining deviations from the simple efficiency hypothesis. Failure to reject this, however, would suggest that liquidity premia do not explain much that is not already explained by risk premia.

There are other various possibilities with the econometrics that arise when we give form to U and F to get a closed-form expression for S_{t+1} in terms of the shocks (w_{t+1}, w_t, \dots) . To do this would amount to solving for a particular general equilibrium of the model, which requires additional assumptions to get but is something we deem valuable and will be done in the future as a natural continuation of the results implied by our model.

Notes

¹ See Strotz (1955), Kydland and Prescott (1977), Calvo (1978), Encarnacion (1981) among others for details on the time-inconsistency phenomenon.

² There are a number of problems associated with the equilibrium of a model in which there are an incomplete set of Arrow-Debreu securities. Radner (1972) showed that if short sales of securities are bounded, equilibrium exists but then Hart (1975) points out the dependence of equilibrium on the bound that Radner chooses arbitrarily. He points out that without such an assumption the usual proof of existence of equilibrium fails because for certain sequences of goods prices the return structure of securities may change in dimension and create discontinuities in the consumer's budget correspondence. Later Cass (1984), Werner (1985), were able to obtain existence results for securities whose return structure did not depend on goods prices and Chae (1986) extended the existence result for securities with returns denominated in state-dependent "index" commodities ("unit" is Chae's term). The more troublesome problem is the optimality of an incomplete markets equilibrium and a discussion is in Hart (1975). The conclusion is that generally such equilibria are Pareto inefficient save under very restrictive definitions of Pareto efficiency.

³ The most ad hoc way is to include currency as an argument in the utility function. The least ad hoc way that is also general equilibrium is the use of an overlapping generations structure but in such models money can be rate-of-return dominated by bonds. The Baumol-Tobin approach has been partial equilibrium although some work extending it to general equilibrium situations has been done in Jovanovic (1982) and Grossman and Weiss (1982). The cash-in-advance approach is less ad hoc than currency-in-the-utility-function, but it is somewhat ad hoc in the sense that the liquidity services of money that results from the assumption of decentralized trading in goods is accepted at the level of (5.a)-(5.b) rather than explicitly derived from the structure of trading in the decentralized market.

⁴ Kareken and Wallace point out this problem in the context of an overlapping-generations model but Sargent (1987) has an adaptation of this to be cash-in-advance model.

⁵ Take (5.a) as an example. Let $\Delta_t \in M_t^0 - P_t X_t \geq 0$. The quantity equation is $M_t^0 v_t = P_t X_t$ where v_t is the velocity of money at t . Then $M_t^0 v_t = M_t^0 - \Delta_t$ or $v_t = 1 - \frac{\Delta_t}{M_t^0}$. In models where $M_t^0 - P_t X_t = 0$ always (which happens if information on w_t were to be available when asset markets are still open) $\Delta_t = 0$ always and $v_t = 1$ always.

⁶ On the existence of v (Sargent, 1987): let V be the space of continuous bounded functions that map elements of a set Ω into \mathbb{R} . Let $\Delta: V \times V \rightarrow \mathbb{R}$ be the metric on V defined as

$$\Delta(\bar{v}, \tilde{v}) = \sup_{\ell \in \Omega} |\bar{v}(\ell) - \tilde{v}(\ell)|$$

for $\bar{v}, \tilde{v} \in V$. Then (V, Δ) is a complete normed vector space or Banach space.

Let $T: V \rightarrow V$ be given by

$$\begin{aligned} T(v)(\ell) &= \max_{z \in \mathbb{R}^k} U(z, \ell) \\ &\quad + \beta \int v(\ell') dF \\ \text{s.t. } \ell' &\leq g(\ell, z) \end{aligned}$$

where U is real-valued, continuous, monotonic, concave, and bounded; and the set $\{(\ell', \ell); \ell' \leq g(\ell, z), z \in \mathbb{R}^k\}$ is compact and convex. Then T is a contraction on (V, Δ) , i.e., for any $\bar{v}, \tilde{v} \in V$ there is a number $\delta \in \mathbb{R}$, $\delta \in [0, 1)$ for which

$$\Delta(T\bar{v}, T\tilde{v}) \leq \delta \Delta(\bar{v}, \tilde{v}).$$

That T is a contraction follows from a theorem known as Blackwell's theorem which states that any operator T on the metric space (Ω, Δ) that satisfies

(Monotonicity) For any $x, y \in \Omega$, $x \geq y$ implies $T(x) \geq T(y)$

(Discounting) Let c be the constant function with real value c^0 at all $\ell \in \Omega$. For any $c^0 > 0$ and $\ell \in \Omega$, $T(x + c) \leq T(x) + \beta c$ for some $\beta \in [0, 1)$.

is a contraction with $\delta = \beta$.

The Banach fixed-point, or contraction mapping theorem then states that for such T defined on (Ω, Δ) there is a $v \in V$ for which (FP) $v = Tv$ for any $\ell \in \Omega$.

It can be shown that v is also concave by an argument in Lucas (1973). When U is differentiable then v can also be shown to be differentiable and for this see Benveniste and Scheinkman (1979).

⁷ See Cumby (1987) for a model of time-varying conditional covariances in a context that is isomorphic to ours.

TIMING

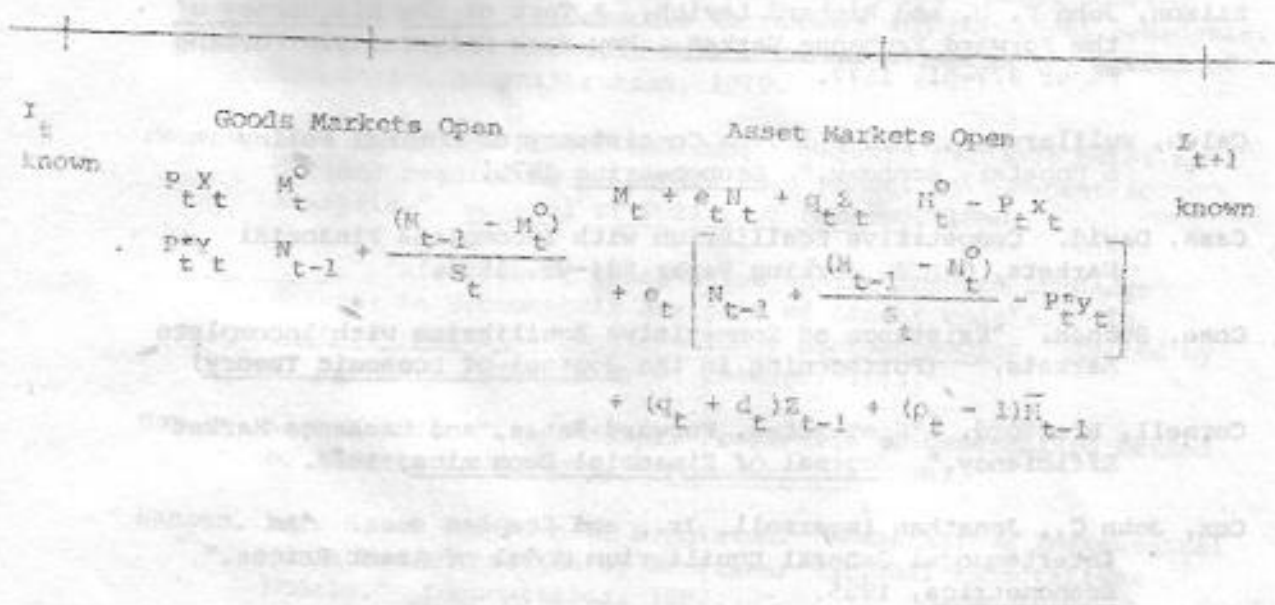


FIGURE 1

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