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~~Interpreting the Basic Rational Expectations~~
Macroeconomic Model

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Abstract

The conclusion that systematic monetary policy is ineffective seems necessary in the basic rational expectations macroeconomic model. This rests on an implicit normalization that puts the natural employment level L^n at the classical full employment point which interpretation, however, requires irrational workers. A more self-consistent interpretation defines L^n in terms of the condition that production meets demand at the going price while firms maximize expected profit. The resulting implications are closer to standard Keynesian theory: fully anticipated monetary policy can, and fully anticipated demand changes must, affect output and employment.

Interpreting the Basic Rational Expectations Macroeconomic Model

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I. Introduction

The central conclusion of the rational expectations (r.e.) literature, viz. that systematic monetary policy is ineffective,^{1/} is now known to be not necessary if one departs from the simple specifications of what might be called the basic r.e. macroeconomic model. If one allows for nonlinearities (Shiller (1978) and Snower (1984)), stochastic parameters (Dickinson, Driscoll and Ford (1982), intertemporal substitution effects with overlapping generations (Azariadis (1981)), or nonnormal distributions of shift variables (Otani (1985)), the ineffectiveness result need no longer hold. However it still seems to hold in the basic model. In this paper we will take a closer look at its rationale and argue that a coherent interpretation of the model leads to a different conclusion.

II. The Basic Model

Lucas (1973), Taylor (1985), and Attfield, Demery and Duck (1985) have formulated slightly different versions of a basic r.e. model. Following Taylor mostly, it might be stated as follows.

Assume identical firms and only one good, but let there be N separate markets each with the same number of firms. Because of information lags the current price P_i in market i ($i = 1, \dots, N$) is not known elsewhere during the current period. For convenience we will write $x = \log X$ if X is a variable but we will also refer to x as the variable itself. Accordingly we will say that $p_i = \log P_i$ is the price in the i th market and $p = \sum p_i / N$

is the average price, short for the logarithm of the geometric mean $(p_1 \dots p_N)^{1/N}$.

Let the supply function in the i th market be given by

$$y_i^s = y^n + c (p_i - E_i p) \quad (1)$$

where y^n is the "natural" or normal level of output, c is a positive constant, and $E_i p = E(p|p_i)$ is the expectation of p given p_i . Suppliers are interested in knowing p , which is taken as a cost proxy,^{2/} but they can only have $E(p|p_i)$. The actual p_i is determined by (1) and the demand function^{3/}

$$y_i^d = y^* + a (m - p_i) + \delta_i \quad (2)$$

where δ_i is a random variable whose expected value $E\delta_i = 0$, y^* is normal demand net of real balance effects expressed through $a(m - p_i)$, a is a positive constant, and the exogenous money stock m is stochastic. Putting $y_i^s = y_i^d$,

$$p_i = (a + c)^{-1} (y^* - y^n + cE_i p + am + \delta_i). \quad (3)$$

Now suppose that from past observations it can be assumed that p_i and p are jointly normally distributed, $p_i = p + \epsilon_i$ with ϵ_i random, and $E\epsilon_i = 0$ so $E p_i = E p = \hat{p}$, and $\text{cov}(p, \epsilon_i) = 0$. It follows that $\text{cov}(p_i, p) = \sigma_p^2$, and the standard formula

$$E(p|p_i) = E p + \frac{\text{cov}(p_i, p)}{\text{var } p_i} (p_i - E p_i)$$

then gives

$$E_i p = \hat{p} + b(p_i - \hat{p}) \quad (4)$$

where $b = \sigma_p^2 / (\sigma_p^2 + \sigma_c^2)$ assuming $\text{var } \epsilon_i = \sigma_c^2$ for all i . Equation (1) can therefore be written as

$$y_i^s = y^n + \gamma (p_i - \hat{p}) \quad (5)$$

putting $\gamma = c(1 - b)$. Substituting (4) into (3) one gets

$$p_i = (a + \gamma)^{-1} (y^* - y^n + \gamma \hat{p} + a m + \delta_i) \quad (6)$$

which is compatible with the previous assumption that $p_i = p + \epsilon_i$ taking

$$\epsilon_i = (a + \gamma)^{-1} \delta_i \quad \text{and} \quad \sum \delta_i = 0.$$

Let supply in the average market or in the aggregate be denoted by y^s and demand by y^d . Writing $y = y^s = y^d$, (5) and (2) give

$$y = y^n + \gamma (p - \hat{p}) \quad (7)$$

$$y = y^* + a (m - p). \quad (8)$$

The expected values $\hat{y}$, $\hat{p}$ and $\hat{m}$ of the corresponding variables must therefore satisfy

$$\hat{y} = y^n + \gamma (\hat{p} - \hat{p}) \quad (7')$$

so $\hat{y} = y^n$, and

$$\hat{y} = y^* + a (\hat{m} - \hat{p}) \quad (8')$$

whence

$$\hat{p} = a^{-1} (y^* - y^n) + \hat{m} \quad (9)$$

From (8),

$$p = a^{-1} (y^* - y^n) + m \quad (10)$$

Using (9) and (10) in (7),

$$\begin{aligned}
 y - y^n &= \gamma (p - \hat{p}) \\
 &= \gamma [a^{-1} (y^n - y) + m - \hat{m}] \\
 &= a\gamma (a + \gamma)^{-1} (m - \hat{m}).
 \end{aligned}$$

(11)

According to (11), $y = y^n$ if $m = \hat{m}$, and only an unexpected difference $m - \hat{m}$ can make output y differ from y^n . This much is certainly correct in the model. However, more conclusions have been drawn that are striking and controversial.^{4/} The claim that a fully anticipated change in monetary policy that changes $\hat{m}$ will have no effect on output is correct only if, in (11), y^n is invariant with respect to changes in $\hat{m}$. The stronger claim that "only random changes in aggregate demand can affect the level of real output; predictable, systematic changes in aggregate demand will affect prices but not output" ^{5/} is correct only if y^n is invariant with respect to changes in y^* so that in (9) and (10), an increase in y^* merely increases $\hat{p}$ and p without affecting y^n and y .

(8) It is quite clear that the supply function is crucial. Its specification and the natural rate y^n deserve a close look.

III. The Supply Function

To get at a rationale for equation (1), suppose first the case where there is no uncertainty about costs. With only one good in the economy, let W be the money wage and labor L_i the only variable input in market i . Since the model of Section II takes the average price as a cost proxy, (1) can be written as

$$y_i^s = y^n + c (p_i - w). \quad (12)$$

If the firm maximizes profit $P_i Y_i - W L_i$ subject to a production function $Y_i = Y(L_i)$ with the usual properties $Y'(L_i) > 0$ and $Y''(L_i) < 0$, it is necessary that $Y'(L_i) = W/P_i$. Consider the normalization

$$Y'(L_i) = 1 \text{ at } Y = Y^n. \quad (13)$$

Accordingly if $P_i = W$ hence $p_i = w$, the optimal output $y_i^s = y^n$, and if $P_i = W$, output is greater than the normal level Y^n in conformity with (12).

Turning to the uncertainty case, suppose now that W is a random variable whose density is $g(W)$, and write $\hat{W} = \int_0^\infty W g(W) dW$ for the expected money wage. If the firm maximizes expected profit

$$\pi(P_i, L_i) = \int_0^\infty [P_i Y(L_i) - WL_i] g(W) dW$$

the preceding paragraph can be repeated word for word, $\hat{W}$ and $\hat{w} = \log \hat{W}$ replacing W and w respectively. Taking expected profit maximization as axiomatic, (13) thus provides a rationale for (5), reading $\hat{p}$ there as a proxy for $\hat{w}$.

IV. The Normal Level of Output

Omitting subscripts henceforth to speak of the representative firm or the economy as a whole, and denoting the normal employment level by L^n , (13) implies $Y'(L^n) = 1$. To determine Y^n it therefore suffices to define L^n . The "new classical" literature does this by the condition that the labor market clears.

In Fig. 1 the curve labelled L^d is the locus of points satisfying $Y'(L) = \hat{W}/P$; i.e. given $\hat{W}$ and P , employment of

$$L^d = f(\hat{W}/P) \quad (14)$$

workers maximizes expected profit, provided of course that their output does get sold at the price P . L^s is the labor supply curve. Suppose $Y'(L) = 1$ at $L = L_c$, where L_c is the classical full employment level given by

$L^d = L^s$, which makes L_C the normal level of employment, $(\hat{W}/P)_C = 1$ the normal (expected) wage, and $Y(L_C)$ the normal rate of output. This normalization at L_C has the curious consequence, however, that if actual employment L is greater than L_C and the economy is on the L^d curve as required by (14)—the economy is say at V —one is forced to say that workers have been fooled into supplying labor in excess of L_C at a less than normal wage. Since rational workers know from the shape of the L^d curve that $L > L_C$ would be hired only if $\hat{W}/P < 1$, surely they would not supply any $L > L_C$. Even at the level of a particular market i or of an individual firm, rational workers would know that employment greater than normal would be offered only if $P_i > \hat{P}$ (see (5)), which means an expected wage lower than that required to supply the normal amount of labor.

Also, if $L < L_C$ so the economy is say at U , one is forced to explain the existing unemployment as the result of voluntary decisions to search for higher paying jobs. But it is obvious from the L^d curve that $L < L_C$ only if $\hat{W}/P > 1$, which means that the expected wage is in fact already higher than normal. Since the unemployed would be willing to work for less than the current expected wage, there is simply no rationale for search activities in these conditions, and we conclude that the usual interpretation that makes L_C the normal employment level is incompatible with rational behavior on the part of workers.

V. An Alternative Interpretation

There is a different way of defining L^n which is more self-consistent. Taking $\hat{W}$ and the expected money stock $\hat{M}$ as exogenous, let other unstated demand parameters be given. Supposing that the employment level is L^d , the

price level is P , and the realized money stock is M , there would be a corresponding demand for output. Let

$$\hat{L}^k = h(\hat{L}^d, \hat{W}/P, M/P) \quad (15)$$

be the amount of labor needed to produce that output, and if $M = \hat{M}$ in (15), write

$$\hat{L}^k = h(\hat{L}^d, \hat{W}/P, \hat{M}/P). \quad (15')$$

In Fig. 1 the depicted $\hat{L}^k$ curve is the locus of points $(\hat{L}^k, \hat{W}/P)$ determined by (14) and (15').^{6/}

Consider the normalization $Y'(L) = 1$ at $L = L_K$ which is defined by $L^d = \hat{L}^k$. This makes L_K the normal level of employment, $(\hat{W}/P)_K = 1$ the normal wage, and $Y^n = Y(L_K)$ the normal output rate. Observing from (14) and (15') that L^n satisfies

$$L^n = h(L^n, \hat{W}/P, \hat{M}/P) \text{ and } L^n = f(\hat{W}/P) \quad (16)$$

it must be the case under rational expectations that $\hat{P}$ satisfies

$$L^n = h(L^n, \hat{W}/\hat{P}, \hat{M}/\hat{P}) \text{ and } L^n = f(\hat{W}/\hat{P}) \quad (16')$$

and therefore $\hat{P} = \hat{W}$ as required by the W proxy role of P .

As drawn, $L_K < L_C$ since $L_K = L_C$ would be fortuitous and $L_K > L_C$, though possible, seems less usual. Notice that if $M = \hat{M}$ and the economy were to be at U , some of the output produced would be unsold at that price level— $\hat{L}^k$ being less than L^d there—which would require price and employment reductions until point E is reached.

The employment level L that will be realized obtains from (14) and (15) by putting $L = \hat{L}^k = L^d$. Since $M > \hat{M}$ means a larger real balance effect

hence an L^k curve to the right of the $\hat{L}^k$ curve, an M larger than expected implies higher than normal P , L and Y in consonance with the model of Section II. More interesting, firms are profit maximizing, the output produced is exactly the amount demanded at the going price so the goods market clears, and therefore although unemployed workers may want employment at the prevailing wage, there would be no reason for firms to hire a larger work force.

Since L^n as defined in this section depends on the exogenous $\hat{M}$ and $\hat{W}$, the L^n level depends on these parameters. If policy revisions induce equal proportionate increases in $\hat{M}$, $\hat{W}$ and $\hat{P}$, evidently (from (16')) L^n would be quite unchanged. However, although it could be argued that changes in $\hat{M}$ should produce equiproportionate changes in $\hat{P}$ --this is necessary in the model of Section II in order for y^* and y^n to remain the same (see (7') and (8'))--there is no reason why the same relationship should hold between $\hat{M}$ and $\hat{W}$. Even accepting that $\hat{M}'/\hat{P}' = \hat{M}/\hat{P}$ (writing $\hat{M}'$, $\hat{P}'$ and $\hat{W}'$ for the new values), one may have $\hat{W}'/\hat{P}' < \hat{W}/\hat{P}$ in which case, looking at (16'), the new $L^{n'}$ must be larger than the previous L^n . This happens if the net result of the changes in the parameters is a rightward shift of the $\hat{L}^k$ curve, which means a new normalization $Y'(L^{n'}) = 1$ appropriate to the new parameters. Thus in Section II, y^n is not invariant with respect to changes in $\hat{m}$, and therefore the monetary policy ineffectiveness conclusion does not hold.

It is also clear that even with the same $\hat{M}$ and $\hat{W}$, changes in other unstated demand parameters (which determine the position of the $\hat{L}^k$ curve) will affect the L^n level. In brief, as should be expected, demand factors must have an effect on the normal rate of employment and output.

VI. Conclusion

We have argued that the basic r.e. model implicitly involves a production function and an expected profit maximization assumption, but a simple normalization at the normal level of employment L^n yields the model. The usual interpretation puts L^n at the classical full employment level; however, this has implications contrary to rational behavior on the part of workers. We have proposed to define L^n in terms of the condition that the output produced meets demand at the going price. In this alternative view, business firms hire only that number of workers whose output can be sold, and they maximize expected profit doing so. In contrast to the conclusions drawn in the r.e. literature, the results are closer to standard Keynesian theory: fully anticipated monetary policy can, and fully anticipated demand changes must, affect output and employment.

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Notes

1. The "most famous proposition advanced by the rational expectations school [is] that with regard to monetary policy, only unexpected changes in the stock of money affect the level of output" (Dornbusch and Fischer (1984), p. 567).
2. "The [average] price level should be an indicator of the firm's input costs" (Taylor (1985), p. 397).
3. See Attfield, Demery and Duck (1985), p. 53.
4. See e.g. Buiter (1980).
5. Attfield, Demery and Duck (1985), p. 43.
6. Except for $\hat{W}$ in place of W , Fig. 1 is similar to Fig. IVb in Edwards (1959).

References

- Attfield, C.L.F., Denery, D. and Duck, N.W. (1985). Rational Expectations in Macroeconomics. Oxford: Blackwell.
- Azariadis, C. (1981). 'A reexamination of natural rate theory.' American Economic Review, vol. 71, pp. 946-960.
- Buiter, W.H. (1980). 'The macroeconomics of Dr. Pangloss: a critical survey of the new classical macroeconomics.' Economic Journal, vol. 90, pp. 34-50.
- Dickinson, D.G., Driscoll, M.J. and Ford, J.L. (1982). 'Rational expectations, random parameters and the non-neutrality of money.' Economica, vol. 49, pp. 241-248.
- Dornbusch, R. and Fischer, S. (1984). Macroeconomics, 3rd ed. New York: McGraw-Hill.
- Edwards, E.O. (1959). 'Classical and Keynesian employment theories: a reconciliation.' Quarterly Journal of Economics, vol. 73, pp. 407-428.
- Lucas, R.E., Jr. (1973). 'Some international evidence on output-inflation tradeoffs.' American Economic Review, vol. 63, pp. 326-334.
- Otani, K. (1985). 'Rational expectations and non-neutrality of money.' Weltwirtschaftliches Archiv, vol. 121, pp. 203-215.
- Shiller, R.J. (1978). 'Rational expectations and the dynamic structure of macroeconomic models.' Journal of Monetary Economics, vol. 4, pp. 1-44.
- Snower, D.J. (1984). 'Rational expectations, nonlinearities, and the effectiveness of monetary policy.' Oxford Economic Papers, vol. 36, pp. 177-199.
- Taylor, J.B. (1985). 'Rational expectations models in macroeconomics.' Frontiers of Economics (ed. K.J. Arrow and S. Honkapohja). Oxford: Blackwell, pp. 391-425.

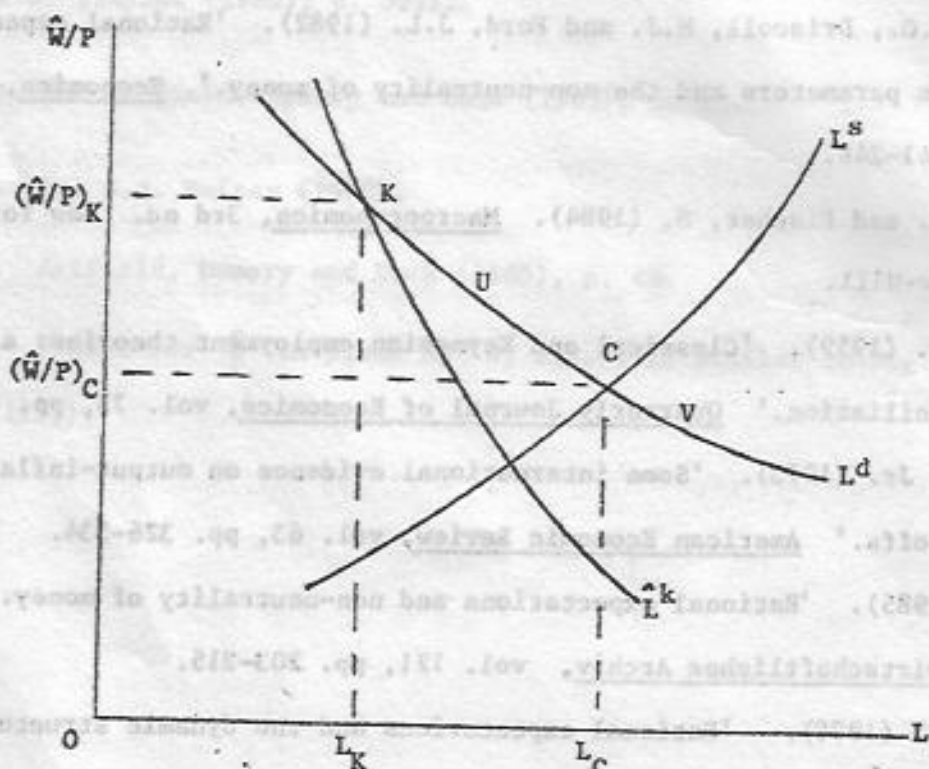


Fig. 1