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A GENERAL EQUILIBRIUM MODEL OF EXCHANGE MARKET
INTERVENTION WITH VARIABLE STERILIZATION

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Abstract

This paper presents a general equilibrium model that incorporates three of the most prominent features of the recent floating exchange rate period: (1) rational asset markets; (2) "managed" exchange rates with partial (and variable) sterilization; and (3) the use by policymakers of both purely discretionary and systematic policies. The model is characterized by rational expectations, sticky prices, imperfect capital substitution, separate policy functions for domestic credit and reserves, and variable sterilization. The most important implications of the model are that (1) monetary, exchange rate, and variable sterilization policies may be used to pursue tradeoffs between internal and external objectives; and (2) these tradeoffs are nonlinear functions of the policies chosen (e.g., partial sterilization is not a simple linear interpolation of complete and zero sterilization).

I. Introduction.

This paper presents a dynamic general-equilibrium model of a small open economy in which the monetary authority has both internal and external targets. The model synthesizes a number of separate extensions of the Dornbusch (1976) model of exchange-rate overshooting and is characterized by rational expectations, sticky prices, and imperfect capital substitution. The most novel aspect of the model, however, is the explicit specification of both monetary and exchange-rate policies. This is done with separate policy functions for domestic credit and reserves, which are linked by a money supply production function. Each policy function permits both endogenous (systematic) and exogenous (purely discretionary) interventions. The model also permits variable sterilization, whereas previous models are characterized by either zero or complete sterilization. Thus, state variables (exchange rates/interest rates and prices) are explicit functions of sterilization policy.

Why should this model be of interest? It is our contention that previous models have not fully responded to three of the most salient features of the floating rate period that began in the early 1970s: (1) volatile (real and nominal) exchange rates, a phenomenon consistent with rational asset markets; (2) "managed" exchange rates with partial sterilization, a phenomenon that suggests that policymakers not only have both internal and external targets, but also pursue tradeoffs between them; and (3) the use of endogenous as well as exogenous policy interventions, a phenomenon that is consistent with the assumption that policymakers rarely

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know in advance what exogenous disturbances will occur and, therefore, will typically exhibit systematic rather than purely discretionary policies.

The first of these features, volatile exchange rates, has received the greatest attention. The pioneering work in this literature is the Dornbusch (1976) model of exchange-rate overshooting in response to an unanticipated increase in the money supply. Dornbusch assumes perfect capital mobility and substitutability, which together imply uncovered interest rate parity; a regressive expectations scheme consistent with perfect foresight; stock equilibrium in the money market; and sluggish price adjustment that depends on transitory excess demand. Others have extended the Dornbusch model to account for imperfect substitution between domestic and foreign assets (Branson 1984), for imperfect capital mobility (Frenkel and Rodriguez 1982), and for activist monetary policy (Pappell 1985). Eaton and Turnovsky (1983) consider both imperfect asset substitution and discretionary monetary policy.

Much less attention has been focused on the latter two issues -- exchange-rate intervention with partial sterilization and the use of both endogenous and exogenous policy actions. Branson (1984) examines discretionary intervention, but makes no distinction between sterilized and nonsterilized intervention and does not model systematic intervention ("leaning against the wind" for example). Blundell-Wignall and Masson (1985) consider only fully-sterilized exchange-market intervention. Our model addresses these two issues by incorporating both internal and external policy targets and systematic and discretionary policies. The tradeoff between targets is highly nonlinear function of the policies chosen.

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II. Model Structure and Dynamics

The model is composed of three explicit markets (the goods, money, and foreign exchange markets) and one implicit market (the bond market). The model is fully described by equations 1.1 through 1.12 below. All parameters are positive, and all variables, except for domestic and foreign interest rates, are in logarithmic form. Long-run equilibrium values are identified with a tilde; foreign variables with an asterisk. Table 1 provides specific variable definitions.

(1.1)	$y^d = k_0 + k_1 y - k_2 i + k_3(e-p+p^*)$	Aggregate Demand
(1.2)	$y = y^d$	Goods Equilibrium
(1.3)	$dp/dt = \pi(y^d - \tilde{y}) = \pi(y - \tilde{y})$	Price Adjustment
(1.4)	$m^d - p = a_1 y - a_2 i$	Money Demand
(1.5)	$r = r_0 - ui[(e-p+p^*) - (\tilde{e}-\tilde{p}+p^*)]$	Reserve Policy
(1.6)	$c = c_0 - w_1 r + w_2(i-\tilde{i})$	Credit Policy
(1.7)	$m = h_1 c + h_2 r$	Money Supply
(1.8)	$m = m^d$	Money Equilibrium
(1.9)	$i = i^* + E(de/dt) - (1/b)f^d$	Net Private Foreign Asset Demand
(1.10)	$f = f_0 + f_1 nfa - f_2 r$	Net Private Foreign Asset Supply
(1.11)	$f = f^d$	Foreign Exchange Equilibrium
(1.12)	$E(de/dt) = de/dt$	Perfect Foresight

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Equations 1.1 and 1.2 define the goods market equilibrium with income being demand-determined in the short run. Real aggregate demand (equation 1.1) is a function of real income, the interest rate, and the real exchange rate. Equation 1.3 expresses price-adjustment as a function of the gap between contemporaneous and long-run equilibrium income, with an adjustment parameter equal to π.

Equations 1.4 through 1.8 comprise the monetary sector of the model. Real money demand (equation 1.4) is a function of income and the interest rate. The reaction function for reserves (equation 1.5) expresses reserves as a function of the gap between contemporaneous and long-run equilibrium real exchange rate (see, for example, Pappell (1985) and Turnovsky (1985)), and the reaction function for domestic credit (equation 1.6) specifies domestic credit as a function of the level of reserves and the gap between the contemporaneous and long-run equilibrium interest rate (see, for example, Black (1985)). These policy functions permit systematic reactions as an endogenous response to real exchange rates, interest rates, and the level of reserves, as well as discretionary policies as an exogenous change in r_0 or c_0. Aggregate money supply (equation 1.7) is linked to reserves and domestic credit through a money supply production function (see, for example, Makin (1981)), and continuous money market equilibrium is imposed by equation 1.8.

The domestic money market is linked to international financial markets by equations 1.9 through 1.12. The (inverted) net private foreign asset demand function (equation 1.9) embodies the assumption of perfect capital mobility with imperfect capital substitutability. This equation can be derived using either a mean-variance approach (Black (1985)) or an optimization model (Turnovsky (1985)). Equation 1.10 is the net private

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foreign asset supply identity, and equation 1.11 imposes continuous asset equilibrium. Equation 1.12 imposes perfect foresight on exchange rate expectations.

The structure of the model implies that the exchange rate is determined as part of the general (real and monetary) equilibrium of the system. Only in a limiting case, therefore, can one assert that the exchange rate is an exclusively monetary phenomenon. The model does contain the monetary and portfolio-balance models as special cases. The monetary approach to flexible exchange rates is characterized by fully flexible prices (π equal to infinity), purchasing power parity (k_3 equal to infinity), perfect capital mobility with perfect capital substitutability (b equal to infinity), exogenous money supply (w_2 equal to u, both equal to zero), and an exogenous income. In this case the goods and foreign exchange markets are irrelevant in the determination of the exchange rate; instead, the exchange rate is determined in the money market alone. The portfolio-balance model (as in Frankel (1979, 1983)) is the same as the monetary approach except that it assumes imperfect capital substitutability (b less than infinity). In this case, the exchange rate is determined in the asset markets -- the money and foreign exchange markets.

Steady-State Solution

The steady-state is described by e equal to $\tilde{e}$, p equal to $\tilde{p}$, dp/dt equal to zero, and E(de/dt) equal to de/dt, which are both equal to zero. These conditions imply that:

(2.1) $\tilde{e} = \tilde{p} - p^* - k_0/k_3 + (k_2/k_3)\tilde{i} + [(1-k_1)/k_3]y$

(2.2) $\tilde{p} = \tilde{m} - a_1\tilde{y} + a_2\tilde{i}$

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$$(2.3) \qquad \tilde{i} = i^* - (1/b)(f_0 + f_1 nfa - f_2 r_0)$$

$$(2.4) \qquad \bar{m} = h_1 c_0 + (h_2 - h_1 w_1) r_0$$

Note that $\tilde{y}$ is assumed to be exogenously determined.

The system is neutral with respect to exogenous changes in domestic credit in the sense that the exchange rate response $(d\tilde{e}/dc_0)$, the price response $(d\tilde{p}/dc_0)$, and the money supply response $(d\tilde{m}/dc_0)$ all equal h_1, which is the parameter on domestic credit in the money supply production function. This neutrality implies that the long-run equilibrium real exchange rate $(\bar{e}-\tilde{p}+p^*)$ is invariant to exogenous changes in domestic credit (c_0) and that, in this case, changes in the exchange rate are equal to changes in the (logarithmic) difference between the domestic and foreign price levels (that is, relative purchasing power parity (PPP) holds in response to an exogenous change in domestic credit).

The system is nonneutral, however, with respect to shocks that affect the domestic long-run equilibrium interest rate. With income $\bar{y}$ fixed, an increase in the domestic interest rate $\tilde{i}$ requires the exchange rate $\bar{e}$ to change by a greater proportion than the price level $\tilde{p}$ to obtain long-run goods market equilibrium. This means that the real exchange rate $(\tilde{e}-\tilde{p}+p^*)$ is not invariant with respect to interest rate shocks and that, in this case, changes in the exchange rate $\bar{e}$ are not equal to changes in the relative price level $(\tilde{p} - p^*)$ -- relative PPP does not hold in response to shocks to the domestic interest rate $\tilde{i}$. Only in the limiting case where the net private foreign asset demand is perfectly elastic (b equals infinity in equation 1.9) is the system neutral in the long-run with respect to interest rate $(\tilde{i})$ shocks.

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Dynamic Properties

The dynamics of our model are governed by the differential equations for the two state variables: the exchange rate and the price level. The expected change in the exchange rate ($E(de/dt)$) is equal to the actual change (de/dt) by the perfect foresight assumption, and the differential equation for the exchange rate is derived from the entire model. The differential equation for price (dp/dt), however, is an assumed feature of the model and specified as equation (1.3).

Because $E(de/dt)$ is derived from the entire model, it is useful first to derive the semireduced forms of the static equations. Using equations 1.1, 1.2, and 1.4 through 1.8, we obtain the following semireduced-form equations for the interest rate, income, and money that are consistent with goods and money market equilibrium:

(3.1) $\quad i = \tilde{i} + (1/V)[a_1 k_3/(1 - k_1) + Hu](e - \tilde{e})$
$\qquad\qquad + (1/V)[1 - a_1 k_3/(1 - k_1) - Hu](p - \tilde{p})$

(3.2) $\quad y = \tilde{y} + (1/V)[Ak_3/(1 - k_1) - Hu(k_2/(1 - k_1)](e - \tilde{e})$
$\qquad\qquad + (1/V)[-k_2/(1 - k_1) - Ak_3/(1 - k_1) + Hu(k_2/(1 - k_1)](p - \tilde{p})$

(3.3) $\quad m = \tilde{m} - Hu[(e - \tilde{e}) - (p - \tilde{p})]$
$\qquad\qquad + (h_1 w_2/V)\{[a_1 k_3/(1 - k_1) + Hu](e - \tilde{e})$
$\qquad\qquad\qquad + [1 - a_1 k_3/(1 - k_1) - Hu](p - \tilde{p})\}$

$\qquad$ where $\quad H = h_2 - h_1 w_1 > 0$
$\qquad\qquad\qquad A = h_1 w_2 + a_2$
$\qquad\qquad\qquad V = A + a_1 k_2/(1 - k_1) > 0$

Similarly, equations 1.9 through 1.11 determine the interest rate

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consistent with foreign exchange market equilibrium:

(3.4) $i = \tilde{i} + E(de/dt) - Fu[(e - \bar{e}) - (p - \tilde{p})]$

where $F = (1/b)f_2$

This equation, together with the interest rate consistent with goods and money market equilibrium (eq. 3.1) and imposing the perfect foresight assumption (eq. 1.12), yields the equation for the dynamic path of the exchange rate:

(4) $de/dt = z_{11}(e - \bar{e}) + z_{12}(p - \tilde{p})$

where $z_{11} = (1/V)[a_1 k_3/(1 - k_1) + Hu + VFu]$

$z_{12} = (1/V)[1 - a_1 k_3/(1 - k_1) - Hu - VFu]$

Given the assumption that H is nonnegative but not greater than h_2, then z_{11} is positive. The coefficient z_{12}, however, can be of either sign, implying that the slope of the $de/dt = 0$ locus (and therefore the slope of the stable trajectory leading toward the new steady-state) can be either positive or negative. Thus, the sign of z_{12} determines the nature of the system's transitional dynamic path.

Using equation 3.2, we can also solve the price adjustment equation (dp/dt in equation 1.3) in terms of $(e - \bar{e})$ and $(p - \tilde{p})$:

(5) $dp/dt = z_{21}(e - \bar{e}) + z_{22}(p - \tilde{p})$

where $z_{21} = (\pi/V)[Ak_3/(1 - k_1) - Huk_2/(1 - k_1)]$

$z_{22} = (\pi/V)[-k_2/(1 - k_1) - Ak_3/(1 - k_1) + Huk_2/(1 - k_1)]$

If z_{21} is greater than zero, then z_{22} is less than zero, and the slope of the $dp/dt = 0$ locus is positive. This assumption rules out perverse income and

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exchange rate responses. Note also that it is the slope of the $de/dt = 0$ locus, not the slope of the $dp/dt = 0$ locus, which determines the slope of the stable trajectory and, therefore, the nature of the path of the system.

Dynamic equations 4 and 5, along with the semireduced-form static equations 3.1 through 3.3, describe the motion of the system over time, contingent on a set of initial conditions. These dynamic equations can be simplified using the following matrix form:

$$(6) \qquad \begin{bmatrix} de/dt \\ dp/dt \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} (e - \tilde{e}) \\ (p - \tilde{p}) \end{bmatrix}$$

The characteristic equation for the dynamic system expressed by matrix equation 6 is:

$$(6.1) \qquad R^2 - (z_{11} + z_{22})R + (z_{11}z_{22} - z_{21}z_{12}) = 0$$

and the characteristic roots are:

$$(6.2) \qquad R_1, R_2 = \{(z_{11} + z_{22}) \pm [(-(z_{11} + z_{22}))^2 - (z_{11}z_{22} - z_{21}z_{12})]^{1/2}\}/2$$

$$\text{where } \det(Z) = R_1 R_2 = (z_{11}z_{22} - z_{21}z_{12})$$

$$= -(\pi/V)[Fuk_2/(1 - k_1) + k_3/(1 - k_1)] < 0$$

$$tr(Z) = R_1 + R_2 = z_{11} + z_{22} \gtrless 0$$

and Z is the coefficient matrix in equation 6. Because the determinant of Z is negative, the two roots are real and opposite in sign, regardless of the sign of the trace. Hence, the model yields a saddlepoint equilibrium.

Given the roots R_1 and R_2, the general solution to equation 6 is of the following form:

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(7.1) $e(t) = \bar{e} + b_{11}x_1\exp(R_1 t) + b_{12}x_2\exp(R_2 t)$

(7.2) $p(t) = \tilde{p} + b_{21}x_1\exp(R_1 t) + b_{22}x_2\exp(R_2 t)$
     where $b_{1j} = 1$ $(j = 1, 2)$
          $b_{2j} = (R_j - z_{11})/z_{12}$
          $x_j = B_{j1}(e(0) - \bar{e}) + B_{j2}(p(0) - \tilde{p})$

B_{ji} $(i = 1, 2)$ is the element in the jth row and the ith column of the inverse of the matrix containing the b_{ij}'s; $e(0)$ and $p(0)$ are the values of the exchange rate and the price level after the initial jump in the exchange rate; and $\bar{e}$ and $\tilde{p}$ are the new long-run equilibrium values.

Starting from an initial steady state ($e = \bar{e}(0)$ and $p = \tilde{p}(0)$), any shock that affects the long-run equilibrium price level will yield a new saddlepoint equilibrium where $e = \bar{e}$ and $p = \tilde{p}$. With R_1 negative and R_2 positive, the model is explosive when x_1 is zero and x_2 is nonzero, but convergent when instead x_2 is zero and x_1 is nonzero. Thus, equations 7.1 and 7.2 demonstrate the saddlepoint behavior of the system.

We are now in a position to derive the bounded solution to the dynamic system expressed by matrix equation 6. The assumption that the price level is sticky implies that its initial value (at $t = 0$) is predetermined. With the price level sticky, the system cannot jump to the new long-run equilibrium. Instead, the exchange rate jumps, placing the system onto the saddlearm of the saddlepoint, which is the stable trajectory to the new long-run equilibrium. This stabilizing jump is determined from the condition that the coefficients associated with the unstable root be zero. This is ensured by the condition:

(8.1) $\quad x_2 = 0 = B_{21}(e(0) - \tilde{e}) + B_{22}(p(0) - \tilde{p})$

$$\text{where } B_{21} = (z_{11} - R_1)/(R_2 - R_1)$$

$$B_{22} = z_{12}/(R_2 - R_1)$$

which implies that:

(8.2) $\quad (e(0) - \tilde{e}) = - [z_{12}/(z_{11} - R_1)](p(0) - \tilde{p})$

At the instant a shock occurs and with the price level sticky, the exchange rate jumps to place the system on the stable arm of the saddlepoint and transmits the impact of the shock to other variables.[1]

Equation 8.2 is the key equation of the model. It determines not only the extent of the jump in the exchange rate, but also the nature of the adjustment toward the new long-run equilibrium. Specifically, when z_{12} is positive (negative) the exchange rate will overshoot (undershoot) its new long-run equilibrium value, and during adjustment the exchange rate and price level move in opposite (the same) direction.

After the jump, the system moves continuously along the stable arm of the saddlepoint. Thus, given equations 7.1, 7.2, and 8.2, the solutions for the exchange rate and price level that yield a convergent path are given by:

(9.1) $\quad e(t) = \tilde{e} + b_{11}x_1\exp(R_1 t)$

(9.2) $\quad p(t) = \tilde{p} + b_{21}x_1\exp(R_1 t)$

$$\text{where } x_1 = - [z_{12}/(z_{11} - R_1)](p(0) - \tilde{p}) = e(0) - \tilde{e}$$

Given these solutions, the solutions for other variables are easily obtained.[2]

From equations 9.1 and 9.2 it is clear that as t approaches infinity,

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both the exchange rate and the price level approach their new steady-state values $\tilde{e}$ and $\tilde{p}$. The dynamics of the exchange rate along the saddlepath, based on equation 9.1, is given by:

(10) $E(de/dt) = de/dt = R_1(e - \tilde{e})$

With z_{12} positive (negative), the exchange rate overshoots (undershoots) its new long-run equilibrium value, and if the exchange rate initially depreciates it will appreciate (depreciate) during the transition toward the new equilibrium.[3]

Similar to other sticky-price models, the system adjusts in two stages to a disturbance that affects $\tilde{p}$. First, the exchange rate jumps to place the system onto the path converging toward the new long-run equilibrium. This jump in the exchange rate transmits the shock to other variables. Thus, the dynamics of the system at impact are determined by the instantaneous response of the exchange rate. Second, after the system jumps, the price level begins to move and propels all other endogenous variables (including the exchange rate). Thus, the transitional dynamics of the system after impact are governed by the slowly evolving price level.

III. Monetary, Foreign Exchange, and Sterilization Policies.

We are now in a position to explore the effects of discretionary monetary and foreign exchange policies, as well as the interactive role of variable sterilization. Endogenous policies are already explicitly specified in the model (equations 1.5 and 1.6). We begin here with discretionary monetary and foreign exchange policies and conclude with an examination of the system's response to an exogenous change in the foreign interest rate, which highlights the role of variable sterilization policy in conditioning

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the behavior of the system. The degree of sterilization is measured (inversely) by H, which equals h_2 minus $h_1 w_1$. H equal to h_2 corresponds to complete nonsterilization, and H equal to zero to full sterilization. In the case of full sterilization, w_2 is also equal to zero, since the monetary authority cannot fix the level of the money supply and pursue interest rate stabilization policies at the same time.

Discretionary Monetary Policy

Consider an unanticipated change in domestic monetary policy in the form of a permanent increase in the autonomous component of domestic credit, c_0. The associated steady-state effects of the monetary shock, derived from equations 2.1 through 2.4, are:

(11.1) $\quad d\tilde{e}/dc_0 = \tilde{h}_1$

(11.2) $\quad d\tilde{p}/dc_0 = h_1$

(11.3) $\quad d\tilde{i}/dc_0 = 0$

(11.4) $\quad d\tilde{m}/dc_0 = h_1$

and $d\tilde{y}/dc_0$ is zero. Thus, in the long run, the system is neutral with respect to an increase in c_0 in the sense that $\tilde{e}$, $\tilde{p}$, and $\tilde{m}$ all increase by the same proportion, h_1, and $\tilde{y}$ and $\tilde{i}$ remain unchanged.

The impact effects, using equations 8.2 and 3.1 to 3.3, are:

(11.5) $\quad de(0)/dc_0 = h_1[1 + (z_{12}/(z_{11} - R_1))]$

(11.6) $\quad dy(0)/dc_0 = h_1\{(1/V)(k_2/(1 - k_1))$
$$+ (1/V)[(Ak_3/(1 - k_1)) - Hu(k_2/(1 - k_1))]$$
$$[1 + z_{12}/(z_{11} - R_1)]\}$$

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(11.7) $\quad di(0)/dc_0 = h_1(1/V)\{-1 + (a_1k_3/(1 - k_1) + Hu][1 + z_{12}/(z_{11} - R_1)]\}$

$\qquad\qquad\quad = h_1\{R_1[z_{12}/(z_{11} - R_1)] - Fu[1 + z_{12}/(z_{11} - R_1)]\}$

(11.8) $\quad dm(0)/dc_0 = h_1 - Hu(de(0)/dc_0) + h_1w_2(di(0)/dc_0)$

where $[1 + z_{12}/(z_{11} - R_1)]$ is positive regardless of the sign of z_{12}, implying that the short-run exchange rate always jumps in the same direction as $\tilde{e}$ and $\tilde{p}$.

Following an increase in c_0 and with the price level sticky, the exchange rate depreciates instantly to place the system on the convergent path (see equation 11.5). With foreign prices (p^*) exogenously fixed, the real exchange rate depreciates as well, causing net exports and real income to increase.

We are now able to characterize the transitional dynamics of the system in response to an increase in c_0 under the assumption that the exchange rate overshoots, which occurs when z_{12} is positive. In the money market, the resulting increase in income tends to increase money demand. The money supply increases directly with c_0 but decreases in response to endogenous intervention and interest rate stabilization (see equation 11.8). Nevertheless, the initial increase in domestic credit (c_0) more than offsets the combined endogenous decrease in money supply and endogenous increase in money demand to create excess money supply at the initial domestic interest rate and price level. With continuous money market equilibrium, the domestic interest rate must initially fall ($di(0)/dc_0$ is negative), but will then rise along the convergent path ($[di(0)-d\tilde{i}]/dc_0$ is negative). The initial decline in the interest rate is greater in absolute value than the decline in the net return on foreign assets (see equations 3.1 and 3.4).[4] To compensate the

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holders of domestic assets, $E(de/dt)$ must be negative (eq. 10). That is, the short-run exchange rate must overshoot its new long-run equilibrium value so that after the initial depreciation there will be continuous exchange rate appreciation along the convergent path. Otherwise, foreign exchange market equilibrium cannot hold (see equation 3.4) on the path toward the new long-run equilibrium. The extent of overshooting, however, is dampened by intervention, as shown by the subtractive term $Fu[1 + z_{12}/(z_{11} - R_1)]h_1$ in equation 11.7.

In the borderline case where z_{12} is zero, the increase in c_0 again leads to excess money supply and a decline in the domestic interest rate, but the decline in the interest rate equals the decline in the net return on foreign assets. Consequently, $E(de/dt)$ must be zero: the exchange rate neither overshoots nor undershoots its new long-run equilibrium value.

What happens if the sterilization and interest-stabilization parameters (H and w_2) are large enough to make z_{12} negative? The instantaneous exchange rate depreciation (remember that $e(0)$ and $\tilde{e}$ move in the same direction) must be less than the long-run depreciation: the exchange rate undershoots its long-run equilibrium value. In this case, endogenous intervention reinforces undershooting, whereas in the previous case (z_{12} positive) it tends to dampen overshooting. There are three possibilities for the interest rate, however, depending upon whether an increase in c_0 again creates excess money supply at the initial domestic interest rate and price level. If so, then the domestic interest rate falls to maintain money market equilibrium, but by less than the decline in the net return on foreign assets. $E(de/dt)$ is positive to compensate the holders of foreign assets, and the domestic interest rate will rise along the convergent path after its

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initial decline.

If the increase in c_0 does not lead to any disequilibrium in the money market at the initial domestic interest rate and price level, then there will be no change in the domestic interest rate. Because the net return on foreign assets decreases, however, E(de/dt) must still be positive to maintain foreign exchange market equilibrium, and the exchange rate undershoots its new long-run equilibrium value.

If, on the other hand, an increase in c_0 causes money demand to increase enough to create excess money demand at the initial domestic interest rate and price level, the domestic interest rate initially rises to maintain money market equilibrium. Because the domestic interest rate increases and the net return on foreign assets decreases, E(de/dt) must be positive to compensate the holders of foreign assets and maintain foreign exchange market equilibrium. The exchange rate again undershoots its new long-run equilibrium value, and the domestic interest rate falls on the path toward the new long-run equilibrium after its initial rise.

Figure 1a (1b) illustrates the case where the exchange rate exhibits overshooting (undershooting). The adjustment toward the new steady state is characterized by an exchange rate appreciation (depreciation), an increase in the price level, an increase (an increase, no change, or a decrease) in the domestic interest rate, a decrease in income, and a decrease (either a decrease or an increase) in money supply. These adjustments continue until e equals $\tilde{e}$ and p equals $\tilde{p}$. Significantly, the response of the system to an exogenous change in domestic credit (c_0) is conditioned by intervention and sterilization policies, as well as by endogenous monetary policies.

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Discretionary Foreign Exchange Policy

Discretionary intervention takes the form of variations in the autonomous component of reserves (r_0). The money supply effects of intervention may be fully-sterilized, nonsterilized, or partially sterilized. As demonstrated below, the effects of intervention depend not only on the subsitutability of domestic and foreign assets (b), but also on sterilization policy (H).

The steady-state effects, using equations 2.1 to 2.4, are:

(12.1) $d\tilde{e}/dr_0 = H + (a_2 + k_2/k_3)F$

(12.2) $d\tilde{p}/dr_0 = H + a_2 F$

(12.3) $d\tilde{i}/dr_0 = F$

(12.4) $d\tilde{m}/dr_0 = H$

The system is not homogeneous in the long-run with respect to changes in r_0, in the sense that $d(\tilde{e} - \tilde{p} + p^*)/dr_0$ and $d(\tilde{m} - \tilde{p})/dr_0$ are not zero. An increase in reserves leads to an increase in $\tilde{i}$, and with long-run equilibrium income exogenous, also causes $\tilde{e}$ to change by a greater proportion than $\tilde{p}$ to obtain long-run goods market equilibrium.

The impact effects of an increase in r_0, using equations 8.2 and 3.1 to 3.3, are:

(12.5) $de(0)/dr_0 = [H + (a_2 + k_2/k_3)F] + (H + a_2 F)[z_{12}/(z_{11} - R_1)]$

(12.6) $dy(0)/dr_0 = (H + a_2 F)(1/V)\{[k_2/(1 - k_1)]$
$+ [Ak_3/(1 - k_1) - Huk_2/(1 - k_1)][1 + z_{12}/(z_{11} - R_1)]\}$

(12.7) $di(0)/dr_0 = F + (H + a_2 F)(1/V)\{-1 + [a_1 k_3/(1 - k_1) + Hu]$
$[1 + z_{12}/(z_{11} - R_1)]\}$
$= F + (H + a_2 F)[R_1(z_{12}/z_{11} - R_1) - Fu(1 + z_{12}/(z_{11} - R_1))]$

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(12.8) $dm(0)/dr_0 = H - Hu\{d[(e(0) - \tilde{e}) - (p(0) - \tilde{p})]/dr_0\}$
$$+ h_1 w_2 \{d[i(0) - \tilde{i}]/dr_0\}$$

Again, the limiting case of H equal to h_2 implies that monetary authorities abstain completely from sterilization policies, whereas H equal to zero and w_2 equal to zero correspond to full sterilization.

Discretionary intervention, like an increase in c_0, can lead to exchange rate overshooting, but endogenous policies can dampen or reverse the overshooting. Undershooting occurs if both H and w_2 are large enough to make z_{12} negative. The adjustment of the system in the case of overshooting (undershooting) is illustrated in Figure 2a (2b). Adjustments of the exchange rate and price level are similar to those depicted in Figures 1a and 1b for discretionary monetary policy, except that $\tilde{e}$ changes by a greater proportion than $\tilde{p}$. Unlike a change in domestic credit (c_0), a partially or completely sterilized intervention can affect the long-run equilibrium interest rate ($\tilde{i}$). The exchange rate will:

(a) overshoot ($z_{12} > 0$) if the domestic interest rate (i) decreases, but the net return on foreign assets increases; or (ii) does not change or increases by less than the increase in the net return on foreign assets.[5]

(b) neither overshoot nor undershoot ($z_{12} = 0$) if the increase in the domestic interest rate equals the increase in the net return on foreign assets.

(c) undershoot ($z_{12} < 0$) if the domestic interest rate increases by more than the net return on foreign assets, which may increase, decrease, or remain the same.

With imperfect capital substitution and nonzero sterilization,

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intervention can affect not only the money supply, but also the long-run equilibrium interest rate. With perfect capital substitutability (b equal to infinity in equation 1.9): (a) a fully sterilized intervention has no effect in either the short or long run; and (b) a completely nonsterilized intervention has exactly the same effect as a change in domestic credit — changes in r_0 and c_0 are perfect substitutes (adjusted for scale).

How does sterilization policy (H) condition the effects of intervention? First, consider the effect of H on the jump in the exchange rate under the assumptions that z_{12} is positive (exchange rate overshooting) and that k_2 is zero (the demand for output is completely interest-inelastic):

(13.1) $d[de(0)/dr_0]/dH = [1/V(z_{11} - R_1)]$
$$\{[a_1 k_3/(1 - k_1) + h_1 w_2 Fu - VR_1]$$
$$[1 + z_{12}/(z_{11} - R_1)] + (1/2)[Hu + a_2 Fu]$$
$$[1 - (tr(Z)/((-tr(Z))^2 - det(Z))^{1/2})]\}$$

where $0 < tr(Z)/((-tr(Z))^2 - det(Z))^{1/2} < 1$

which is positive. Because the increase in $\tilde{e}$ increases with H and de depends on $d\tilde{e}$, de will be larger as H increases (as sterilization decreases). The effect of H on the extent of exchange rate overshooting/undershooting is given by:

(13.2) $d[d(e(0) - \tilde{e})/dr_0]/dH = (H + a_2 F)\{d[z_{12}/(z_{11} - R_1)]dH\}$
$$+ [z_{12}/(z_{11} - R_1)]\{d[H + a_2 F]/dH\}$$
$$= [z_{12}/(z_{11} - R_1)] - (H + a_2 F)[u/V(z_{11} - R_1)]$$
$$\{1 + (1/2)[a_{12}/(z_{11} - R_1)]$$
$$[1 - tr(Z)/((-tr(Z))^2 - det(Z)^{1/2})]\}$$

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which is ambiguous in sign regardless of the sign of z_{12} because less sterilization leads both to an increase in the spot exchange rate and the long-run equilibrium rate. Different degrees of sterilization can lead to different adjustment paths for the economy, both by affecting the steady state and by altering the initial jump in the exchange rate. Hence, the transitional dynamics in particular tend to vary with sterilization policy.

With complete sterilization (H equal to zero), the monetary authority targets both money and the exchange rate and can no longer use domestic credit to influence the domestic interest rate. In this case w_2 must also equal zero. For both the long-run equilibrium exchange rate and price level, the effects of intervention vary inversely with the degree of sterilization. Unsterilized intervention is the most effective in influencing $\bar{e}$, but it also has the greatest effect on $\tilde{p}$. In contrast, fully sterilized intervention has the least effect on both. Significantly, however, the relative magnitudes of the changes in $\tilde{e}$ and $\tilde{p}$ vary with the degree of sterilization. The greater the degree of sterilization, the greater the change in $\tilde{e}$ relative to $\tilde{p}$ in response to exchange market intervention. Partial sterilization ($0 < H < h_2$) reflects a compromise between domestic monetary policy and foreign exchange policy: the money supply is only partially controlled. Changes in $\tilde{e}$ through intervention are obtained at the cost of greater changes in $\tilde{p}$ as sterilization declines.

Changes in the Foreign Interest Rate

The role of sterilization policy is demonstrated even more starkly in the case of an unanticipated increase in the foreign interest rate (i^*). The steady-state effects, using equations 2.1 to 2.4, are:

(14.1) $d\tilde{e}/di^* = a_2 +/k_2 k_3$

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(14.2) $d\tilde{p}/di^* = a_2$

(14.3) $d\tilde{i}/di^* = 1$

(14.4) $d\tilde{m}/di^* = 0$

The impact effects, using equations 8.2 and 3.1 to 3.3, are:

(14.5) $de(0)/di^* = [a_2+k_2/k_3] + a_2[z_{12}/(z_{11} - R_1)]$

(14.6) $dy(0)/di^* = a_2(1/V)\{k_2/(1 - k_1)$
$$+ [Ak_3/(1 - k_1) - Huk_2/(1 - k_1)][1 + z_{12}/(z_{11} - R_1)]\}$$

(14.7) $di(0)/di^* = 1 + a_2(1/V)\{-1 + [a_1k_3/(1 - k_1) + Hu]$
$$[1 + z_{12}/(z_{11} - R_1)]\}$$

(14.8) $dm(0)/di^* = -Hu\{d[(e(0) - \tilde{e}) - (p(0) - \tilde{p})]/di^*\}$
$$+ h_1w_2\{d[i(0) - \tilde{i}]/di^*\}$$

As one can see from comparing equations 1.1 through 1.8 to equations 12.1 through 12.8, an increase in the foreign interest rate has effects qualitatively similar to those of an exogenous increase in reserves (r_0), except that the former (i) has nonneutral effects even if asset substitution is perfect (b equals infinity) and (ii) always leads to excess money demand and a jump-increase in the domestic interest rate.[6] Specifically:

(a) The exchange rate overshoots $(z_{12} > 0)$ when the domestic interest rate increases by less than the increase in the net return on foreign assets;

(b) The exchange rate neither overshoots nor undershoots $(z_{12} = 0)$ when the domestic interest rate and the net return on foreign assets increase

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by the same amount; and

(c) The exchange rate undershoots ($z_{12} < 0$) when the domestic interest rate increases by more than the net return on foreign assets, which may increase, decrease, or remain the same.[7]

To see how endogenous intervention and the degree of sterilization (variations in H) affect the transitional dynamics of the system in response to a change in the foreign interest rate, we again assume that z_{12} is positive (exchange rate overshooting) and that k_2 is zero (demand for output is perfectly interest-inelastic). The effect of sterilization on the size of the jump in the exchange rate is:

$$(15.1) \quad d[d(e(0) - \tilde{e})/di^*]/dH = d[de(0)/di^*]/dH$$
$$= -a_2[u/V(z_{11} - R_1)]\{1 + [z_{12}/(z_{11} - R_1)]$$
$$[1/2][1 + (tr(Z)/((-tr(Z))^2 - det(Z))^{1/2})]\}$$

which is negative. Thus, decreased sterilization (higher H) tends to dampen the jump in the exchange rate, decreasing the extent of overshooting. Because the deviation of $e(0)$ from $\tilde{e}$ is smaller, it follows that the speed of adjustment is slower. This can be seen from:

$$(15.2) \quad d(-R_1)/dH = -(u/2V)[1 - (tr(Z)/((-tr(Z))^2 - det(Z))^{1/2})]\}$$

which is negative -- the speed of adjustment ($-R_1$) is slower. Hence, the the size of the instantaneous exchange rate adjustment ($e(0) - \tilde{e}$) and the subsequent speed of adjustment are directly related.

Decreases in sterilization (increases in H) also cause the increase in the domestic interest rate to become larger:

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(15.3) $\quad d[di(0)/di*]/dH = a_2[u/V(z_{11} - R_1)]\{[VFu - R_1] [1 + z_{12}/(z_{11} - R_1)]$

$$+ (1/2)[Hu + a_1 k_3/(1 - k_1)]$$

$$[1 - \{tr(Z)/((-tr(Z))^2 - det(Z))^{1/2}\}]\}$$

which is positive.

To summarize, then, as the degree of sterilization decreases (H increases), the jump in the domestic interest rate increases, but the system's speed of adjustment, the initial jump in the exchange rate, and the short-run deviation of the exchange rate from long-run equilibrium all decrease. In the limiting case of full sterilization (H equals zero and w_2 equals zero), the money supply is exogenous, and the degree of exchange rate overshooting is more severe than when sterilization is partial ($0 < H < h_2$). The money supply is endogenous with less than complete sterilization and, in this case, absorbs part of the pressure of adjustment, reducing the degree of exchange rate overshooting. In contrast, a policy of fully sterilized intervention tends to increase the degree of disequilibrium in the money market (at the initial interest rate and price level) and, consequently, necessitates a larger exchange rate adjustment.

It can also be shown that w_2 (the endogenous response of credit policy to deviations in the spot interest rate from long-run equilibrium) has qualitative effects similar to those of H (sterilization policy). Hence, w_2 reinforces the effects of H. As the foreign interest rate increases, the the domestic interest rate increases, but in the overshooting case di is less than d$\tilde{i}$. With $d(i(0) - \tilde{i})$ negative, domestic credit will fall by the amount $w_2 d(i(0) - i)$, and money supply will fall by $h_1 w_2 dc$. Thus, with the price level initially fixed and a jump-increase in income, di is smaller when w_2 is small. A decrease in H or w_2 reduces the increase in i and increases the degree of exchange rate adjustment.

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Therefore, particular short-run tradeoffs between exchange rate and interest rate movements are established by the values of H and w_2 . Thus, the monetary authority may be able to pursue different domestic interest rate and exchange rate targets in the short run. In particular, the monetary authority may choose to follow a partial sterilization policy ($0 < H < h_2$), which is a compromise between domestic credit and foreign exchange policy or, equivalently, between exchange rate and interest rate changes.

IV. Concluding Remarks.

Unlike previous studies, this paper fully incorporates three of the most prominent features of the recent floating exchange rate period: (1) rational asset markets; (2) "managed" exchange rates with partial (and variable) sterilization; and (3) the use by policymakers of both purely discretionary and systematic policies. The model is characterized by rational expectations, sticky prices, imperfect capital substitution, separate policy functions for domestic credit and reserves, and variable sterilization. The most important implications drawn from the model are that monetary, exchange rate, and variable sterilization policies may be used to pursue tradeoffs between internal and external objectives and that these tradeoffs are nonlinear functions of the policies chosen. The tradeoff between internal and external objectives implied by various degrees of partial sterilization, for example, is not a linear interpolation of the tradeoffs implied by complete and zero sterilization. Our model is silent, however, on the desirability of particular internal and external objectives, an issue that might be clarified by extending the model to include variable real income.

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Footnotes

[1] The impact effects for the other variables (i, y, and m) can be derived by substituting equation 8.2 into equations 3.1 to 3.3.

[2] The time paths of i, y, and m can be derived by substituting equations 9.1 and 9.2 into equations 3.1 to 3.3.

[3] When z_{12} is positive (z_{12} is negative) and the exchange rate initially appreciates and overshoots (undershoots) its new equilibrium value, then during transition the exchange rate is depreciating (appreciating), that is $de/dt > 0$ ($de/dt < 0$).

[4] The net return on foreign assets tends to decrease regardless of the sign of z_{12} the real exchange rate initially increases and the long-run equilibrium domestic interest rate remains the same (see equations 11.2, 11.3, 11.5, and the right-hand side of equation 3.4).

[5] The change in the net return on foreign assets (which equals $\{F - Fu[1 + z_{12}/(z_{11} - R_1)](H + a_2F)\}dr_0$ in this case) is positive when z_{12} is either positive or zero and ambiguous when z_{12} is negative (see equations 12.2, 12.3 and 12.5, and the right-hand side of equation 3.4).

[6] The term $(1 - a_2/V)$ in equation 14.7 equals h_1w_2/V and is, therefore, positive.

[7] The change in the net return on foreign assets (which equals $\{1 - a_2Fu[1 + z_{12}/(z_{11} - R_1)]\}di^*$ in this case) is positive when z_{12} is positive or zero and ambiguous when z_{12} is negative (see equations 14.2, 14.3 and 14.5, and the right-hand side of equation 3.4).

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Table 1 List of Variables

| Variable	Description
c	logarithm of domestic credit (official net domestic assets)
c_0	logarithm of autonomous component of domestic credit
e	logarithm of exchange rate (domestic price of foreign currency)
f	logarithm of private net foreign asset supply, in foreign currency
f^d	logarithm of private net foreign asset demand, in foreign currency
i	domestic interest rate
i^*	foreign interest rate
k_0	autonomous component of aggregate demand
m	logarithm of money supply
m^d	logarithm of money demand
nfa	logarithm of net foreign assets, in foreign currency
p	logarithm of domestic price level
p^*	logarithm of foreign price level
r	logarithm of international reserves (official net foreign assets), in foreign currency
r_0	autonomous component of reserves, in foreign currency
y	logarithm of real income (output)
y^d	logarithm of real aggregate demand
~	denotes steady-state (long-run equilibrium) value
π	parameter for speed of price adjustment
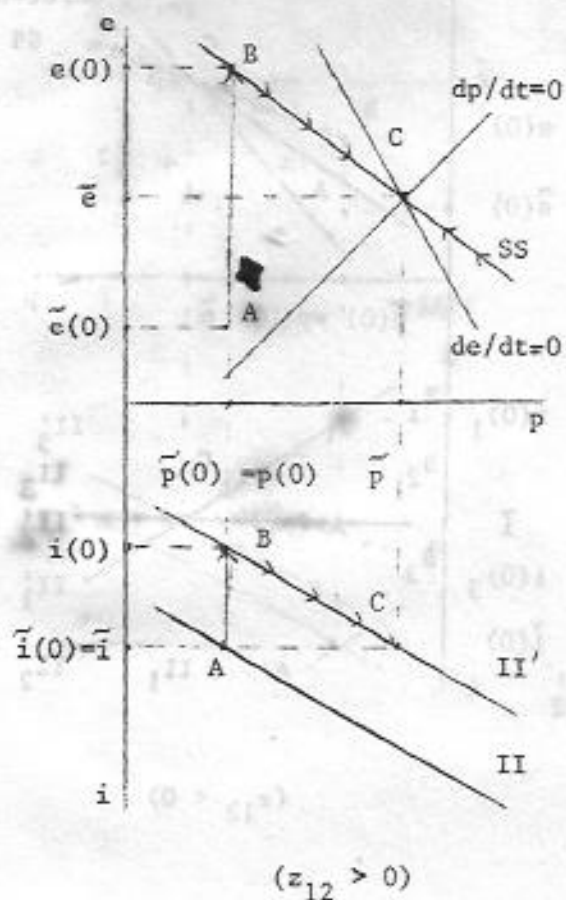
Fig. 1(a) Monetary Policy
          and Overshooting

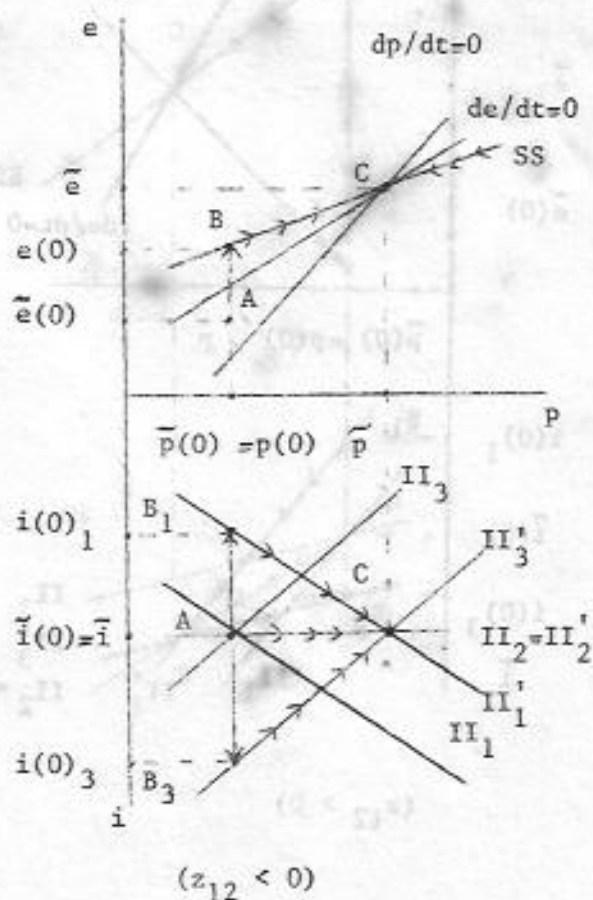Fig. 1(b) Monetary Policy
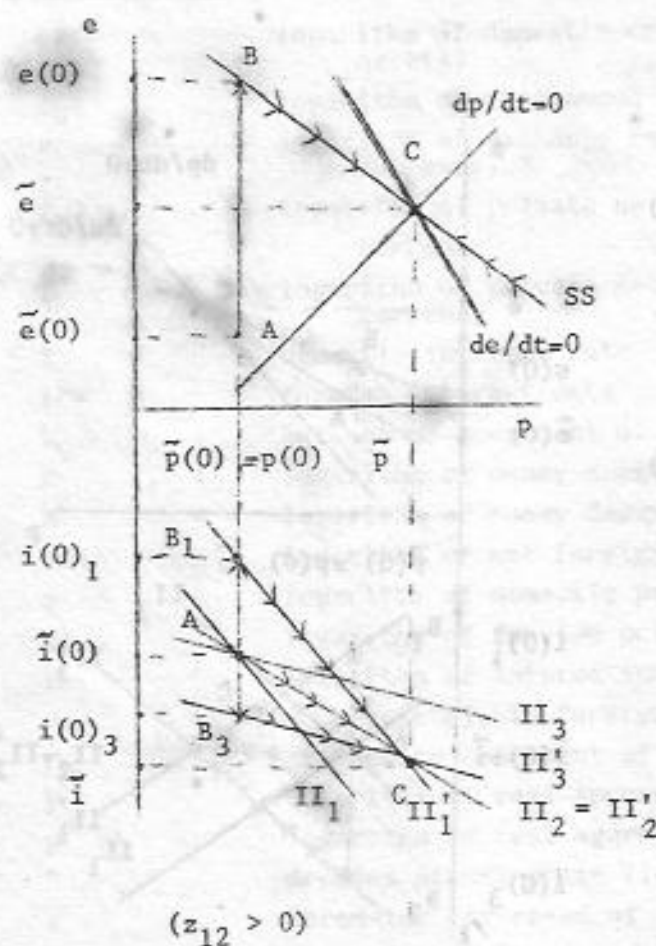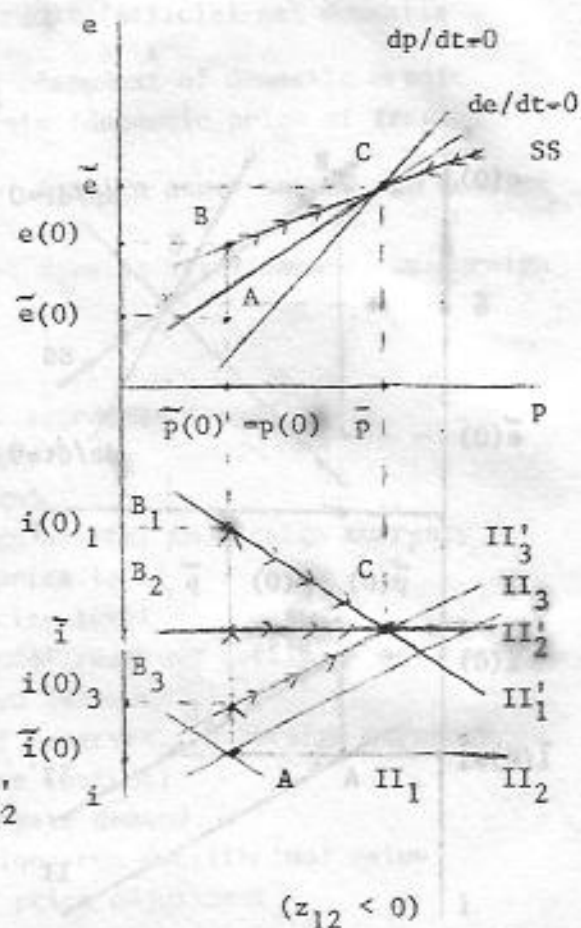   and Undershooting

Fig. 2(a) Intervention and
            Overshooting

Fig. 2(b) Intervention and
            Undershooting