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A MODEL OF INCOME AND INCOME INEQUALITY IN THE PROCESS OF GROWTH

by

Manuel F. Montes

ABSTRACT

This paper proposes a symbolic model that parameterizes some of the elements of Kuznets' Inverted-U hypothesis. The model incorporates the following important factors in the development process: savings rate, the capital-output ratio, the speed of labor absorption and the rate of growth of profits. Every inverted-U path requires a fixed vector of these parameters so that a change in any of these parameters displaces an economy to another inverted-U path. This result cautions against a mechanical understanding of the income-income inequality relationship in the development process.

This paper identifies some important parameters of the inverted-U process that are called the investment coefficient, the trickle-down coefficient, and the concentration coefficient and explains how these coefficients are derived from the more basic factors. The model therefore permits the analysis of the impact of different development policies on the inverted-U process. A short discussion of the empirical aspects of the model is also provided.

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1.0 Introduction

This paper proposes a symbolic model that parameterizes some of the elements of Kuznets' Inverted-U Hypothesis on relationship between income and income distribution in the course of development. This relationship is characterized as a Lyapunov function summarizing two laws of motion between total income and relative income inequality. The usefulness of this approach is that it identifies the parameters inherent in the process in which relative income inequality increases at the beginning of growth, reaches a peak, and then starts to decline in the later stages of growth. These parameters are shown to be dependent on a few economic and institutional parameters of the particular society embarking on capitalist growth. In particular, the savings rate, the capital-output ratio, the speed of labor absorption and the rate of growth in profits are found to be important and any change in these parameters could result in a change in the Kuznets path the society will "ride on" in the course of its development.

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The value of this paper is not that it proposes a "new theory" about the relationship between income and income inequality but that it "models" many of the existing hypotheses about the Kuznets curve in a manner in which their economic significance can be studied. To my knowledge, previous attempts to model the relationship have relied heavily on the mechanical or statistical properties of systems with two or more groups that evolve unevenly. Thus Fields [1975] and Robinson [1976] derive parabolic expressions between income and income inequality based mainly on these mechanical considerations.[1/]

While, the two-sector approach is important because it is easily related to many two-sector "facts" about development---such as urban-rural differentials, the model proposed here is easily related to other ideas in the literature---such as the savings rate and the "trickle down" effect.

The basic model assumes that the principal parameters are fixed through the development process. (The effect of changes in these parameters are illustrated in a separate section.) While this assumption contradicts the historical record of growth of many countries, the model succeeds in demonstrating that capitalist growth is inherently inequality widening at the start and inequality reducing after a certain level of development is achieved. This demonstration relies on the economics of investment and labor absorption instead of the mechanics of the quadratic nature of the statistical variance.

The assumed fixity of institutional factors in the model argues against a mechanistic attitude to the Kuznets curve.[2/] An implication of the model is that changes in these factors generate new development paths and one cannot naively design development strategy to maximize growth (and neglect equity) in the hope that the worsening trend in income inequality will automatically reverse itself in the future. According to the model changes in property rights (as might be induced in changes in investment incentives) always forces an economy into a new Kuznets curve which may have a later turning point.

The model generates a functional form directly derived from the underlying process that can be adopted for empirical tests. The resulting functional form does not impose symmetry between the rising and the falling portions of the curve, a pattern that is implicit in the parabolic functions used in previous empirical work. (For example Paukert [1973].) The proposed model is strictly applicable only to particular societies and has no natural interpretation with cross section data. Because of data limitations and severe estimation restrictions, our empirical test is beyond the scope of this paper.

The model is presented in the next (the second) section. The theoretical implications of the model are discussed in the third section. The fourth section discusses extensions and possible uses of the model. The last section provides the concluding comments.

2.0 The Level of Income and Relative Income Inequality

Kuznets [1955] observed that relative income inequality seemed to be declining in the advanced countries---specifically the United States, England, and Germany---since the 1870s. This observation, coupled with a simulation exercise which demonstrated the inequality widening process of investment dependent growth gave birth to the hypothesis that inequality tends to rise at the start of development, reaches a peak and commences a steady decline afterward. The pattern of an inverted-U in a graph with income on the horizontal axis and an index of income inequality on the vertical summarizes this hypothesis.

Kuznets [1955] original discussion on the economic processes behind the pattern have been extended by many researchers. (For example, see the recent study by Mizoguchi and Takayama [1984] which analyzes the most recent and probably shortest development process in the post-war period.) Oshima's [1983] model presents many of the elements in the process. At the beginning of development both the "between" and "within" income inequality rises as new techniques and products are introduced which only the relatively skilled and/or propertied members of society can exploit. Savings and property incomes become increasingly unevenly divided. Continued reinvestment, however, increases labor absorption until full employment is achieved, approximately at the income level when relative income inequality reaches its peak. After full employment, increased mechanization and increasing productivity on a broad front eliminate the "within" inequality and reduce

the "between" inequality. Hence, relative income inequality falls thereafter.

The model proposed here does not seek to capture the richness of the theories that have been presented in this field. Instead, it attempts to the more modest goal of identifying some summarizing parameters inherent in these theories of the inverted-U relationship between an index of development, represented by per capita income, y, and an index of relative income inequality, θ.

Let $\theta \geq 0$ can be an income distribution index with the conventional specification that a higher value of θ is associated with a greater degree of income inequality. The field of income distribution is awash with discussion about the relative usefulness and sensitivity of different income distribution indices, their decomposition, and proposals for even other indices. The particular inequality index is used here as the ratio of property income (including profits) to the wage bill. Because profit/property incomes become an increasing importance in the upper income brackets, this index is closely related to the Kuznets Index. We appeal to the existing literature regarding equivalences between different indeces to provide some generality to the present model.

2.1 Rate of Growth of per Capita Income

The first element of the model is derived from the idea that the rate of growth of per capita income is positively related to an increase in the per capita income going to the upper class whose marginal propensity to save is higher. This relationship had been derived by Kaldor [1955-56] and Passinetti [1962] as an extension of the Harrod-Domar growth model which demonstrates that the growth rate of income depends on the investment rate.

Let a be the incremental capital-output ratio so that

$$(1) \qquad \dot{y} = \frac{1}{a}\dot{k}$$

where $\dot{y}$ is the derivative of per capita income and $\dot{k}$ is the time derivative of per capita capital stock. $\dot{k}$ is therefore the rate of investment.

Per capita savings, s, needed to carry out this investment, is the sum of the savings of the lower class and the upper class. Letting ζ be the share of profit in total income, s_1, the average propensity to save out of wage income, and s_2 the average propensity to save out of profit, the equality of per capita investment to per capita saving requires that:

$$(2) \qquad \dot{k} = s = s_1(1 - \zeta)y + s_2\zeta y.$$

Substituting (2) for $\dot{k}$ in (1) results in:

$$(3) \qquad \dot{y} = \frac{1}{a}[s_1(1 - \zeta) + s_2\zeta]y$$

Passinetti's analysis of this equation has shown that while logic requires that $s_1 \neq 0$, the dynamics of the analysis are unchanged if it were assumed that no savings are generated from lower class income. Making this simplifying assumption (i.e., that $s_1 = 0$) and letting $\alpha = \frac{s_2}{a}$ changes (3) into:

(4) $\dot{y} = \alpha\zeta y.$

The variable ζ in (4) is not quite the variable θ, the inequality index of this paper. Let Π stand for per capita profit/property income (total profit/property income divided by total population), and v be per capita wage income (total wage income divided by total population). Therefore the following relationships hold:

$$y = \Pi + v$$

$$\zeta = \frac{\Pi}{y} = \frac{\Pi}{\Pi + v}.$$

Let $\theta = \frac{\Pi}{v}$.

Thus, the variables ζ and θ are monotonically related in the following manner:

$$\zeta = \frac{\theta}{\theta + 1}$$

When $\zeta = 0$ then $\theta = 0$. The variable θ is always greater than or equal to ζ. While ζ achieves a maximum at 1, θ can flexibly take any non-zero value because it reflects the ratio of profit to wage income.

The model requires that instead of (4) the income growth rate equation be:

(5) $\dot{y} = \alpha\theta y.$

where we have substituted θ for ζ in (4). The practical reason for this adjustment is the solvability of the resulting system. Because $\theta \geq \zeta$, the rate of income growth, $\dot{y}$, is always higher for (5) than for (4). The substitution can be justified from the consistent finding from the total factor productivity literature that increases in inputs (including capital input that is the focus here) cannot account for all of the measured growth in actual economies.

The parameter α is the ratio of the average propensity to save from profits to the incremental capital-output ratio.

Equation (5) expresses the "trade-off" often expressed in development planning---at any given level of per capita income, faster growth is possible the wider is income inequality. This is induced by the fact that investment in capitalist growth is carried out by the saving classes. The size of the trade-off between inequality and growth is expressedly contained in the size of α which is larger the greater the marginal propensity to save of the upper class or the smaller is the incremental capital-output ratio. I propose to call α the investment coefficient.

2.2 Rate of Growth of Inequality

The equation for the rate of growth of inequality makes direct use of the idea that profits, Π, are the most significant elements of income of the upper class. Let w be the wage rate, ℓ the number of wage earners and v the total wage bill. Then

$$(6) \qquad \theta \equiv \frac{\Pi}{v} = \frac{\Pi}{w\ell}$$

Differentiating (6) with respect to time results in:

$$(7) \qquad \dot{\theta} = \frac{\dot{\Pi} w\ell - \Pi(\dot{w}\ell + w\dot{\ell})}{(w\ell)^2}$$

which simplifies to:

$$(8) \qquad \dot{\theta} = \frac{\dot{\Pi}}{w\ell} - \frac{\Pi}{w\ell}\left(\frac{\dot{w}}{w} + \frac{\dot{\ell}}{\ell}\right)$$

$$= \frac{\dot{\Pi}}{\Pi}\theta - \theta\left(\frac{\dot{w}}{w} + \frac{\dot{\ell}}{\ell}\right)$$

Equation (8) requires some explanation. The first term reflects the concentrating effect of property accumulation with increasing profit rates. The impact of increasing monopolization would be captured by this term. Each social structure (expressed in inheritance practices and anti-trust laws, for example) permits a different rate of profit growth, $\frac{\dot{\Pi}}{\Pi}$. Take such a structure as constant, and let $\frac{\dot{\Pi}}{\Pi} = \gamma$, some parameter, which we might call the concentration coefficient of the society.

The second term is determined by what happens in labor productivity growth, $\frac{\dot{w}}{w}$ if labor markets are competitive, and labor absorption, $\frac{\dot{\ell}}{\ell}$. When $\frac{\dot{w}}{w} = 0$, only the labor absorption term, $\frac{\dot{\ell}}{\ell}$, applies. This mode of growth corresponds to the Fei-Ranis [1964] and Lewis [1954] models of economic growth. For high population density societies with seasonal full employment, Oshima [1963] has pointed out that $\frac{\dot{w}}{w}$ could be positive even at the early stages of development. According to Equation (8), labor productivity growth and labor absorption reduce the rate of growth of inequality so that the second term could be associated with the famous (or infamous) "trickle down" dimension of development.

The rate at which an economy spreads the fruits of its development among the lower income households is again heavily dependent on many institutional factors such as technological choices (labor-saving versus labor-using) and the relative power of labor unions---at the same time that is dependent on the scale of economic activity itself ("the pie is larger"). It would be parsimonious to hypothesize that

$$(9) \qquad \frac{\dot{w}}{w} + \frac{\dot{\ell}}{\ell} = \beta y ;$$

that is, the trickle down process is some proportional function of the other state variable in the system, per capita income. The coefficient β might be called a society's "trickle down" coefficient.

Applying these simplications to equation (8), yields

$$\dot{\theta} = -\beta\theta y + \gamma\theta, \tag{10}$$

the second basic equation is our growth model. To understand the dynamics of the model, it remains now to solve the two-equation model, (5) and (10).

2.3 The Lyapunov Solution

First of all, the model has no rest points except when $\theta = 0$, the situation of no inequality.[3/] When $\theta = 0$, $\dot{y} = \dot{\theta} = 0$; otherwise, even if the institutional parameters α, β, and γ remain unchanged, per capita income and inequality will be evolving according to (5) and (10). All equilibrium points are degenerate; as long as some positive amount of inequality exists, the model will go somewhere else.

The convenience of the models (5) and (10) is that it is of the form easily solved by the qualitative (Lyapunov) method first applied by Voltera [1926]. (A modern reference is Luenberger [1979].) Dividing Equation (5) by (10) results in:

$$\frac{\dot{y}}{\dot{\theta}} = \frac{\alpha\beta}{-\beta\theta y + \gamma\theta}. \tag{11}$$

As long as θ and y are not equal to zero, Equation (11) simplifies to:

$$\alpha\dot{\theta} + \beta\dot{y} - \gamma\frac{\dot{y}}{y} = 0 \tag{12}$$

an equation whose terms can be separately integrated through time to get:

(13) $\alpha\theta + \beta y - \theta \ell n y = C$

where C is a constant of integration. For every value of C, define the function:

(14) $L(y, \theta) = \alpha\theta + \beta y - \gamma \ell n y$

as a possible summarizing function.

The function $L(y, \theta)$ is a Lyapunov function because it is continuous, it achieves a minimum at the equilibrium points (i.e. points where $\theta = 0$) and never increases along any trajectory. In fact, the time derivative of $L(y, \theta)$ is zero,[4/] revealing that the constant C is actually a constant of motion. That is, along any trajectory that fulfills equations of motion (1) and (2), the Lyapunov function maintains a constant value. This means that any trajectory, any $y - \theta$ pair consistent with (5) and (10) must lie on the path of the function $L(y, \theta)$.

A graph of the family of functions $L(y, \theta)$ is given in Figure 1. Each curve represents a different value of C. The arrows represent the direction of motion. At per capita income levels below $\hat{y}$, both income inequality and the level of income increase. At per capita income levels above $\hat{y}$, inequality falls while income continues to increase

Figure 1

FAMILY OF FUNCTIONS WITH DIFFERENT CONSTANTS OF MOTION

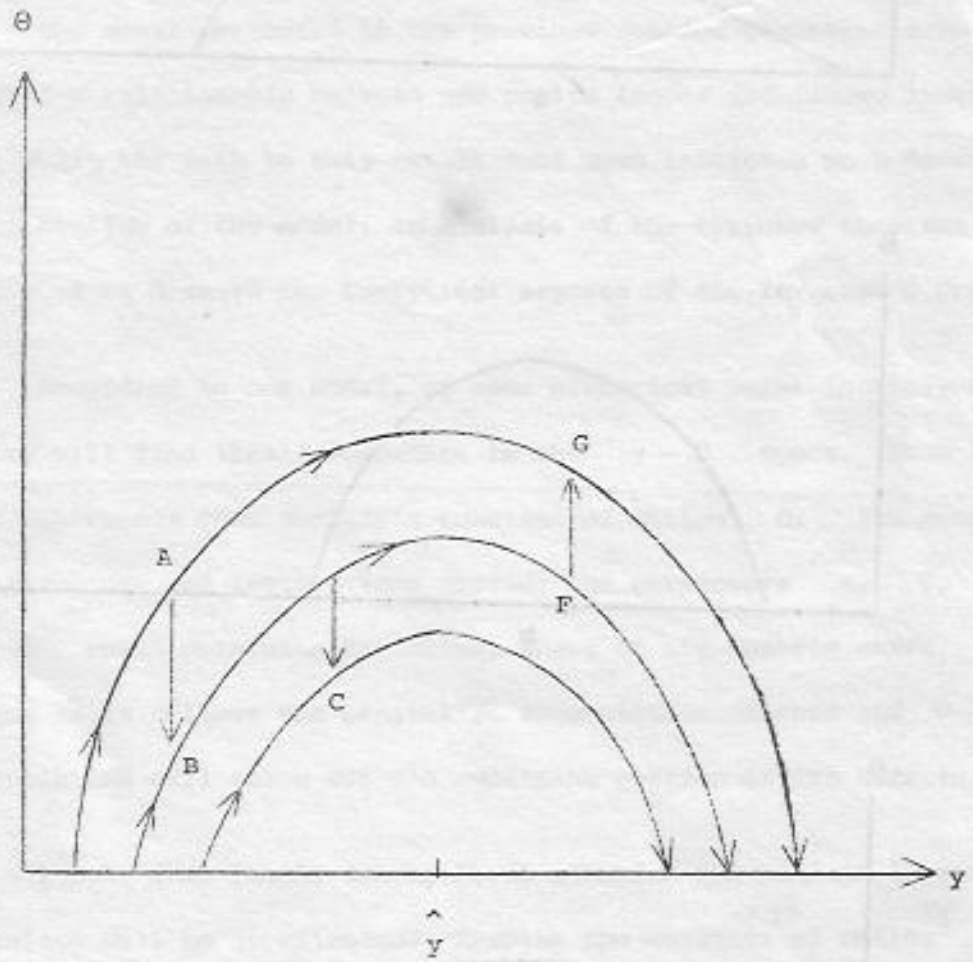


Figure 2

POSSIBLE SHAPES OF THE INCOME INEQUALITY FUNCTION

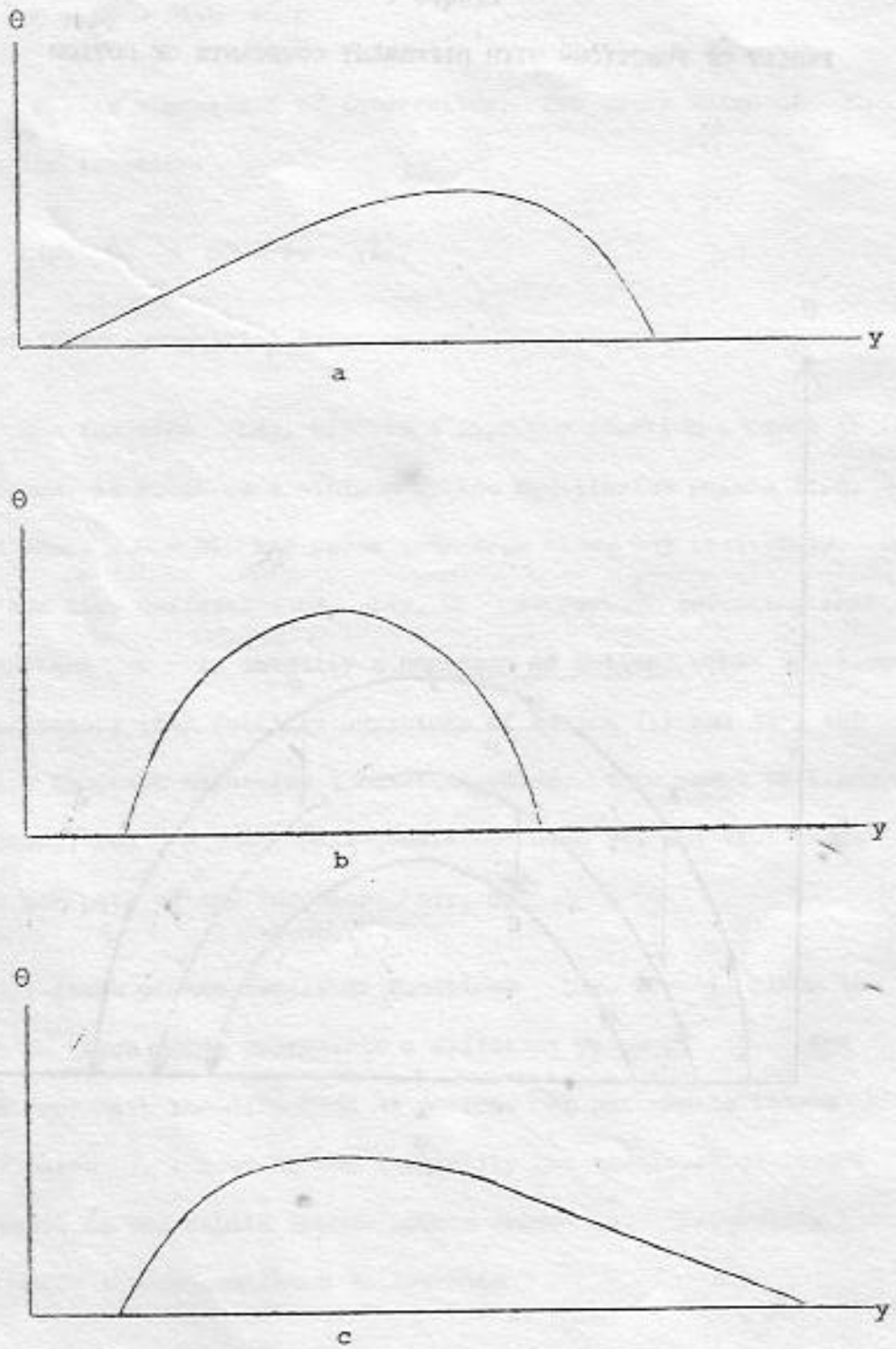


Figure 2 illustrates the fact that this function has greater flexibility of shape than the parabolic form which has been used in countless studies (for example see Ahluwalia [1976] and Paukert [1973]). The function does not require symmetry; its shape is determined by the vector of constants α, β, γ, and C.

3.0 Analytical Implications of the Model

The model presented in the previous section generated a smooth inverted-U relationship between per capita income and income inequality. While the path to this result must have inflicted much damage to the realism of the model, an analysis of the Lyapunov function now permits us to discuss the analytical aspects of the inverted-U process.

According to our model, at some historical point in time, a society will find itself somewhere in the $y - \theta$ space. Such a point determines that society's constant of motion C. Its economic structure and institutions provide the parameters α, β, and γ which determine the actual shape of its Kuznets curve. As long as it follows the capitalist accumulation process and $\theta > 0$, its evolution will trace out the remaining portion of its Kuznets curve.

First, what is the income level at which the maximum income inequality will be experienced? Keeping the constant of motion C fixed, we find maximum θ from (14) to occur at:

$$(15) \quad \hat{y} = \frac{\gamma}{\beta}.$$

Threshold income, $\hat{y}$, is higher the larger the concentration coefficient or the smaller is the trickle down coefficient. It turns out to be independent of the investment coefficient,[5] the rate at which profits are transformed into growth. According to our model, the time of arrival to $\hat{y}$ would be shortened by a higher investment coefficient but the value of $\hat{y}$ at which the downturn occurs is determined only by the rate at which growth in y is allocated between increasing concentration and increasing incomes in the lower income classes.

The maximum income inequality, $\hat{\theta}$, is determined by the particular curve the society is riding on---which depends in turn on the constant of motion of that particular society and the parameters α, β, and γ:

$$(16) \quad \hat{\theta} = \frac{1}{\alpha}(C - \gamma + \gamma \ln(\gamma/\beta))$$

The following *ceteris paribus* statements may be made about $\hat{\theta}$:

1. Lower incremental capital-output ratios and higher rates of investment reduce the maximal inequality level.

2. As long as $\frac{\gamma}{\beta} > 2.71828...$, the natural exponent, an increase in the concentration coefficient γ unambiguously increases maximal inequality.[6]

3. An increase in the trickle down coefficient, β, reduces maximal inequality.

4. An increase in C, the constant of motion, increases the maximal income inequality level.

The value of C is a historically and geopolitically determined constant. Previous "choices" of society, reflecting cultural values about income inequality and growth determine the joint vector C, α, β, γ. Different values of C with the same α, β, and γ values generate "parallel" curves as in Figure 1. Any change in the value of C is a major political, cultural, and economic rearrangement. One such type of drastic rearrangement is a vertical jump or fall from one curve to another. This happens when the distribution of income is changed almost overnight.

The nature of the inverse-U function provides some "iron laws" on growth.

First of all, societies with aspirations for higher terminal income must be prepared to suffer greater, or an agonizingly slow fall in, income inequality in the process. Slow growth can be remedied by a jump to a more unequal curve as in C to D and F to G in Figure 1.

Social rearrangements towards more unequal and inequality worsening curves precipitated, for example, by declining growth are reflected as shifts to a new path to the right of the current one. Within neoclassical economics, a predictive theory of such shifts to new paths is difficult to specify because the idea of a "representative" citizen's preference ordering in the $0 - y$ space is even more tenuous than in the usual case. Do all citizens, for

example, prefer less inequality to more (that is, given $U(\theta, y)$, $U_\theta < 0$ and $U_y > 0$)? Even if that were true, are all citizens willing to bear increasing increments to inequality as per capita income increases (that is, $\frac{d^2\theta}{dy^2} > 0$ for U constant)?

Second, a discontinuous fall in income inequality does not guarantee that income inequality will continue to fall thereafter. The only condition that guarantees falling income inequality is that the threshold income be surpassed. In Figure 1, a fall in inequality from A to B does not prevent inequality from rising further afterwards. However, the maximal income inequality level will be less than if the A to B rearrangement had not been carried out. Because of the analytical links between the parameters of this model and different development strategies are explicit, we can make the following statements:

1. The adoption of strategies that reduce labor absorption or that constrain productivity growth raises the turning point income and the maximum inequality level.

2. The adoption of labor-saving technologies not only reduces labor absorption but also raises the capital-output ratio which raises the maximal inequality level and (by equation (5)) requires higher inequality for any level of income to raise growth rates.

3. Strategies that discourage the domestic reinvestment of savings will increase the maximum inequality.

4. Based on the simple model proposed here, strategies that reduce the concentration coefficient (that is, without changing the

other coefficients) will reduce maximum inequality and turning point income.

5. Drastic social rearrangements cause societies to transfer to different paths, with different rates of growth but the same inverted U-shaped pattern.

4.0 Extensions and Empirical Implications

The fixed parameter model presented above is a partial description of actual development experience precisely because the investment coefficient, (α), the concentration coefficient (γ), and the trickle-down coefficient (β) can be expected to evolve.

A more realistic model, in generalized form may be stated thus

$$
\begin{aligned}
\dot{y} &= f_1(y, \theta_j, \alpha) \\
\dot{\theta} &= f_2(y, \theta_j, \gamma, \beta) \\
\dot{\alpha} &= f_3(y, \theta) \\
\dot{\gamma} &= f_4(y, \theta) \\
\dot{\beta} &= f_5(y, \theta)
\end{aligned} \tag{17}
$$

where now even the values of the parameters depend on the income-inequality state. Prior to any direct empirical confirmation of the model, it has been the experience that the investment coefficient (which is practically equal to the savings rate if foreign borrowings are not significant) rises with per capita income. The theoretical

basis is Engel's Law: As per capita incomes increase, a smaller proportion of income need be devoted to consumption.

How would the growth path of the model change when the investment coefficient increases with per-capita income (assuming β and γ still fixed)? To answer this one must now solve a three-equation model. Instead of an analytical solution, a numerical solution of a three equation model composed of (5), (10) and

$$\dot{\alpha} = \lambda y \tag{18}$$

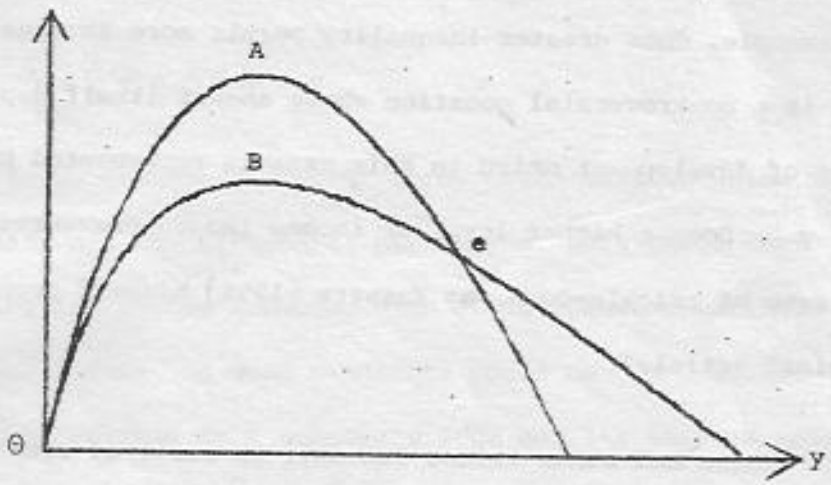
is presented here. It turns out that the model retains its inverted-U shape but compared to another economy with fixed investment coefficient there is a smaller maximum inequality and a longer period of falling inequality.

Figure 3 reports a numerical solution where curve A represents an economy with a fixed investment coefficient and curve B represents an economy in which the investment coefficient monotonically increases from .1 to .2 following (18). Both curves start off from the same income and income inequality value and have the same (fixed) γ and β values.

Notice that the turning point income is the same for both economies; this is due to the fact that turning point income depends only on γ and β, as pointed out earlier. Beyond point e, the economy with rising investment coefficient experiences continued

Figure 3

INCOME INEQUALITY FUNCTIONS



A - Fixed Investment Coefficient

B - Increasing Investment Coefficient

growth while economy A's growth has begun to taper off even though it now enjoys lower inequality. This is again another "iron law" the model seems to exhibit: the higher investment coefficient permits longer growth at higher inequality because investment does increase concentration.

One may speculate on the other relationships implicit on (17). For example, does greater inequality permit more savings or less? This is a controversial question whose answer itself depends on the stage of development which in this case is represented by the level of y. Does a higher level of income induce pressures to increase the rate of trickle-down, as Kuznets [1955] himself proposed in the original article?

These and other issues can only be resolved with an empirical test of the model. A full test would involve estimating (17) which has the complication that the parameters of the first two equations are themselves functions of the variables of the first two equations. As the literature in rational expectations macroeconometrics suggests, the estimation procedure can be quite involved.

This, and an actual estimation of the model is reserved for subsequent work---assuming the existence of appropriate data. What I would like to discuss briefly is the relationship between the model of this paper and <u>previous</u> empirical work.

The proposed model provides an opportunity to carry out an empirical study of Kuznets' hypothesis that transcends curve fitting and pattern recognition. Previous tests (for example, Ahluwalia [1976]) have used a parabolic (or a log-parabolic) specification. These tests have been based on the statistical significance of the square of the income variable. In contrast, the proposed model presents an estimating equation of the form:

$$0 = \frac{C}{\alpha} - \frac{\beta}{\alpha} y + \frac{\gamma}{\alpha} \ell n y + \varepsilon$$

where ε is an econometric error term. From this equation, we cannot identify all the structural parameters only their ratio to α. Identification can be achieved with an independent estimate of one of the parameters. A good candidate would be α itself which was shown to be related to a country's ICOR and its savings rate. Because the model views $\frac{C}{\alpha}$, $\frac{\beta}{\alpha}$, and $\frac{\gamma}{\alpha}$ as derived from structural parameters specific to each country, the model is only applicable to one country at a time. A cross-section estimate which has been common due to data limitation, would have no natural interpretation.

The theory presented in the previous sections seems to present these restrictions on the estimating equation. These are:

1. Only longitudinal tests on the same country are consistent with the model.

2. Because the coefficients reflect the totality of structural relations in the particular society, the error term must be a true error term. The addition of other explanatory variables, such as literacy (a common procedure in this literature), for example, should add no explanatory power to the equation. The effects of such factors as the adult literacy rate should be reflected in coefficients of the model.

3. A separate equation must be estimated for each social regime even for the same country, (this is really a corollary of restriction 2 above) because different regimes mean different time paths.

These restrictions provide a stronger test for the Kuznets hypothesis but they also require more data points for each country.

5.0 Conclusions

The previous section explained the apparently prohibitive amount of data required to test the model against reality. The value of this model is that it provides a means by which one may locate many of the elements that are important in the income and income inequality relationship in one framework. Among these elements are the savings rate, the efficiency of capital, the growth rate of labor productivity, and the rate of labor absorption.

While an empirical test is beyond the scope of this paper, it will be interesting to estimate the values of these parameters for societies for which historical data are available in subsequent work. Such an effort will require intensive empirical work since it will be necessary to group historical data into different regimes which would be represented as different vectors of $(C, \alpha, \beta, \gamma)$.

There are however relatively "loose" ways to use the model. It will be interesting to make rough estimates of the investment, concentration, and trickle-down coefficients for different countries and see if the model's predictions seem to apply.

The final comment one might make is that it is more likely that the actual income-income inequality observations for many countries do not lie on the same Kuznets curve, as it is modeled here. This says that the development experience is a complex result of many determining factors. It is always valuable to try to analyze the direction in which a factor will affect an important development variable. But it will be more difficult to attempt to confirm the expected effect from actual data.

FOOTNOTES

1/ For example, Robinson [1976] characterizes his work thus: "The purpose of this note is to demonstrate that the U hypothesis can be derived from a very simple model with a minimum of economic assumptions.

2/ The institutionalist school accuses many of the believers of the Kuznets curve to have this attitude. (See for example Wright [1973].)

3/ Suppose Θ is positive. The equilibrium requirement is that $\dot{y} = \dot{\Theta} = 0$. The condition $\dot{\Theta} = 0$ immediately requires that $y = \frac{\gamma}{\beta} > 0$ by Equation (10). However by Equation (5), $\dot{y} = 0$ requires that $y = 0$, a contradiction.

4/ Because the time derivative is equal to zero instead of negative, the Lyapunov analysis tells us that the equilibrium is only marginally not asymptotically stable: a perturbation from an equilibrium does not result in a return to it but to a region near it.

5/ It is characteristic of Volterra models to which the model here belongs that the turning point values are determined by the parameters of only one of the two equations---in this case the inequality growth equation. Thus, the independence result would still apply under a slight generalization.

6/ Differentiating Equation (8) with respect to Θ gives us:

$$\frac{\partial \hat{\Theta}}{\partial \gamma} = \frac{1}{\alpha}\left(\frac{1}{\beta} + \ell n \frac{\gamma}{\beta} - 1\right).$$

which is greater than zero as long as $\ell n \frac{\gamma}{\beta} \geq 1$.

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