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MORALE, SYNERGY AND WELFARE

by

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ABSTRACT

Morale is a proverbial hidden variable in models of social games. Its importance is unquestioned but it ordinarily defies operational definition. We attempt here to parametrize its role and trace how it impacts on social production and welfare in a Nash-like bargaining game. Synergy is introduced through a production function with special simple properties. Society attains its highest welfare in the "pure synergy solution". At this point, welfare egalitarianism obtains. Society is worst off within the bargaining milieu in the "pure exchange solution". When some members perceive that they are getting less within the bargaining milieu than they would under an alternative set-up, they may quit the game. Synergy and welfare rise when individuals perceive (a) a strong matching generosity current and (b) a strong synergy current. The present morale crisis in the country is then interpreted in this light.

MORALE, SYNERGY AND WELFARE

An economic crisis when accompanied by a morale crisis becomes a trap that defies purely economic attempts at a solution. Steering out of the economic crisis calls for some perhaps sizeable sacrifice, some galvanizing of the societal muscles, some unity of resolve and action. The morale crisis may however dictate otherwise as cynicism, ambiguity of the desired outcome and whatever else may be encompassed by the phrase "negative mood" prompt individual action. The rapid fall of Kampuchea to Vietnamese invaders attest to the extreme ambiguity of the Cambodians towards Khmer Rouge. The choice between resisting the Vietnamese and risking decapitation by the Khmer Rouge is not clear cut. The analogy may be improper but the main ingredients may be the same.

A crisis ordinarily precipitates a closing of the ranks, a spontaneous formation of concerted effort, a proliferation of instances of uncommon valor that galvanize the ranks whether it be a fighting unit or a nation, it discovers reservoirs of strength it never suspected existed. Synergy comes to life as individuals come to attach a new meaning to the word "we". This happens when members perceive that the solution to the crisis resounds singly and severally to their benefit. When they no longer ask "why", they can be made to go through almost any "how" (Nietzsche). A military invasion is the usual precipitating factor. Israel is the usual case in point. Commercial invasion by the

Japanese has also done wonders to American firms as management and unions set aside petty internal differences to fight off an external threat. Prolonged struggle over my share and yours becomes meaningless when the pie itself stand to vanish.

A morale crisis, however, aborts the development of this galvanizing synergy. The central question that remains unanswered is as usual "why?". Why are we here? Why do we risk life and limb? The individuals may not succeed in finding the answer to this themselves but will look instead to others for indications. When the folks back home became lukewarm to Vietnam, the Yankee lost his grit. When the "Peace Now Movement" demanded withdrawal, the vaunted Israeli fighting machine faltered in Lebanon. That "why" being absent, strength dissipates quickly as imminent defeat fails to inspire horror.

This intangible element called morale is the subject of this enquiry. Most everyone is convinced that it plays a central role in the determination of outcomes of social games and serves as a reservoir to draw from in trying times but it also defies easy and natural parametrization. It is the proverbial familiar hidden variable in economic and other social games. Such a parametization will be attempted in the subsequent pages and its implications on theoretical as well as contemporary concerns will be unravelled. We first consider the theoretical parentage of our own modest attempt.

Nash and the Pure Exchange Game

Students of microeconomicsare familiar with the Edgeworth-Bowley Box allocation game with its initial individual endowments, the individual preference ordering numerically represented and the "core" along the contract curve. The determination of the solution element of the core is the subject of many modern theoretical breakthroughs (Hildenbrand and Kirman, 1976). One and probably the earliest approach towards a solution involves defining and maximizing a social welfare function defined over individual utility functions. The Nash social welfare function (Nash, 1950, 1953; Luce and Raiffa, 1957) defined as

$$(1) \quad W^N = \prod_{i=1}^{n} [U_i(X_i) - U_i(\bar{X}_i)]$$

where X_i is ith bundle after the exchange while $\bar{X}_i$ is the ith initial position, is of special interest because not only does it pick out an element of the core as solution, it also uniquely satisfies some interesting and desirable group choice properties. A 2-person cooperative exchange game will have as its solution the profiles (X_1^*, X_2^*) that maximizes (1). Elaborations on W^N have been done of late by Kaneko and Nakamura (1979). Fabella (1982) introduced the idea of an "exit fee" charged against the individual initial endowment to generate a Nash-like welfare function uniquely satisfying certain desirable properties including strong Paretoness which W^N does not satisfy. The welfare function W^{NL} is

$$(2) \quad W^{NC} = \log \prod_{i=1}^{n} [U_i(X_i) - U_i(\bar{X}_i - e_i)]$$

where e_i is the "exit fee vector" charged individual i for being in the game. Although originally conceived to allow the strong Pareto condition, the exit fee lends itself to a natural economic interpretation. What the exit fee does is that it creates a wedge between initial endowments and fall back positions of the parties in the game which creates room for theoretical maneuver. The exit fee allows one to expand or contract the core of the game and has the potential for motivating income distribution concerns as well as progressive taxation.

A Public Good Approach to Exit Fee

An attractive interpretation revolves around the concept of "public good", P. Now public goods are accessible but not appropriatable. It does not enter X_i. But public goods raise the quality of life within a society; it raises the welfare of every individual participant. If society (and we use the term to refer to the aggregate of cooperating agents) is possessed of public goods, quitting the society means forfeiting not only the welfare enhancing possibilities of cooperative exchange but also forfeiting 'public goods' which are society-specific. This has some implication on the determination of the fallback position. For suppose we apportion no more no less than $\bar{X}_i$ to i in the exchange game, he would still be better off within this society than out because of P, i.e.

$$U(\bar{X}_i, P) - U(\bar{X}_i, 0) > 0 \quad (3)$$

The public goods specific to prestigious institution like Yale or Harvard are such that assistant professors would willingly receive less dollars and be in Yale than receive more being at SUNY, Albany. How do we make the impact of P on the fallback position be explicit? We define the exit fee e_i such that

$$(4) \quad V_i = U_i(X_i) - U_i(\bar{X}_i - e_i) = U_i(X_i, P) - U_i(\bar{X}_i)$$

In other words, we have made the fallback position subsume all the relevant information involving P as far as individual welfare is concerned. By (4), the exit fee e_i is the embodiment of the "public commitment" of individual i; the resources he is willing to go without just to avail of the public goods of society. Note that a rising e_i decreases i's fallback position and results in the deterioration of his final share in the allocation game. In other words, the more he values the public goods of the society, the more likely he is willing to receive less non-public resource as his share in the pie. This is no more than the usual observation of the exploitation of the more concerned. The public corridor ends up being swept by garbage haters more often if not always than by swines. We assume "truthful revelation" of preferences so as to simplify matters.

The Synergy and Exchange Game

The welfare function (2) is now maximized subject ot a resource constraint, the definition of which is the section's concern. To reflect the influence of cooperative synergy, one where the whole exceeds the sum of its parts, we assume the existence of a synergy function F defined over the sum of the exit fees, i.e., $F(\Sigma e_i)$ with the following properties (a) $F(0) = 0$ or nothing comes from nothing, (b) $F' > 1$. This assumes that synergy process is strictly resource generating and (c) F is continuous and twice differentiable. Thus the total allocatable resource (the resource constraint) may be different from the sum of the initial endowments, i.e.,

$$(5) \qquad \sum_{i=1}^{n} X_i = F(\sum_{i=1}^{n} e_i) + \sum_{i=1}^{n} (\bar{X}_i - e_i)$$

The resource constraint (5) has the following lower and upper limits: (i) $e_i = 0, \forall_i$, then $\Sigma X_i = \Sigma \bar{X}_i$ and the game is simply

$$\text{Max } W^{NL} = \log \Pi [U_i(X_i) - U_i(\bar{X}_i)]$$

$$\text{s.t.} = \Sigma X_i = \Sigma \bar{X}_i$$

which is the pure exchange game; (ii) $e_i = \bar{X}_i, \forall_i$ makes the constraint be $\sum_{i=1}^{n} X_i = F(\sum_{i=1}^{n} \bar{X}_i)$ so that maximizing (2) subject to this is the "pure synergy" game. The two games are distinguished one from the other on two counts: (i) the size of the pie and (ii) the bargaining strength of the parties concerned. Note that the quantities considered here (X_i, $\bar{X}_i$, e_i and F) are properly vectors so that F is, say, an m-vector function, i.e., $F = (F_1, F_2, \ldots F_m)$ where m is the number of distinct resources. The constraint function (5) is really an m-element constraint function. We assume further that (a) $U_i(0) = 0$ and (b) every U_i

is (log i) - concave so that the maximizing problem

$$\max \quad W^{NL} = \log \Pi [U_i(X_i) - U_i(\bar{X}_i - e_i)]$$

$$\text{s.t.} \quad \sum_{i=1}^{n} X_i = F(\Sigma e_i) + \sum_{i=1}^{n} (X_i - e_i)$$

has a solution $\{X_i^*\}$ for a given $\{e_i\}$. The profile $\{e_i\}$ is the public commitment vector of the individuals in a society. It is notable that the constitution W^{NL}, if we may consider it as such, has no provision for public commitment. This is not subject to legislation. One can bring a horse to a river, but drinking is another matter. So $\{e_i\}$ is taken as given. The individuals however are free to determine their individual e_i's depending on their strategic concerns. Given $\{e_i\}$ we can find $\{X_i^*\}$ and $W^{NL}(e_i)^*$, i.e., the maximum social welfare given $\{e_i\}$. We show the following:

<u>Claim 1</u>: $W^{NL}(\{\bar{X}_i\})^* \geq W^{NL}(\{e_i\})^*$, $\forall\{e_i\}$

<u>Proof</u>: By our synergy assumption, $\forall\{e_i\}$

$$F(\Sigma\bar{X}_i) \geq F(\Sigma e_i) + \Sigma(\bar{X}_i - e_i)$$

since the right hand side increases monotonically as $\{e_i\}$ rises. This is true because for each e_i, $F' - 1 > 1$ and this implies a monotonically increasing property for the right hand side. Furthermore, the right hand side is maximize at $F(\Sigma\bar{X}_i)$ so the statement follows.

Since the constraint's level set is furthest from the origin at $F(\bar{X}_1, \bar{X}_2)$, it follows that the highest welfare level it affords is a weak maximum among the set of welfare maxima. Q.E.D.

Remark 1:

The constraint set is a set of straight lines of dimensions less than the dimension of the domain of the objective function. Pushing one straight line outwards may not improve the welfare level of society if the other constraints keep their original positions but it will never decrease an afforded welfare level. Thus the weak maximum.

Claim 2:

At $W^{NL}(\{\bar{X}_i\})^*$, $U_i(X_i^*) = U_j(X_j^*)$, $\forall_{ij}$.

Proof:

At $W^{NL}(\{X_i\})^*$, $\{e_i\} = \{\bar{X}_i\}$ which means that the objective function is $W^{NL} = \log \Pi U_i(X_i)$ to be maximized subject to $\Sigma X_i = F(\Sigma\bar{X}_i)$. Since $U_i(\bar{X}_i) = U_j(\bar{X}_j) = 0$ for all, i,j, W^{NL} is clearly maximized at $U_i(X_i^*) = U_j(X_j^*)$ $\forall$, i.j. Q.E.D.

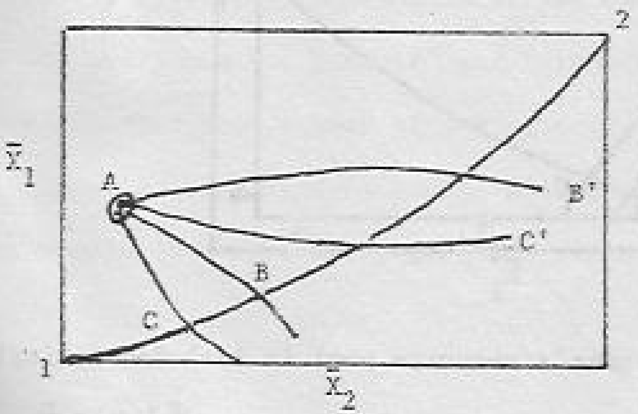
Remark 2:

The situation described by claim 2 is one of "welfare egalitarianism", i.e., every one is equally as happy. This does not imply resource egalitarianism except in a society of clones characterized by $U_i(X_i) = U_j(X_j) \rightarrow X_i = X_j \; \forall ij$.

Remark 3:

It is clear that the greater is the public commitment profile $\{e_i\}$ of the members, the larger is the allocatable resource pool as a result of synergy. At one extreme $\{e_i\} = \{\bar{X}_i\}$, the resource pool is at maximum. At the other $\{e_i\} = \{0\}$ and the resource pool is just $\Sigma\bar{X}_i$. This has interesting implications for the core of the game as well. Let the Edgeworth Bowley representation of the two-party, two-good pure exchange game be

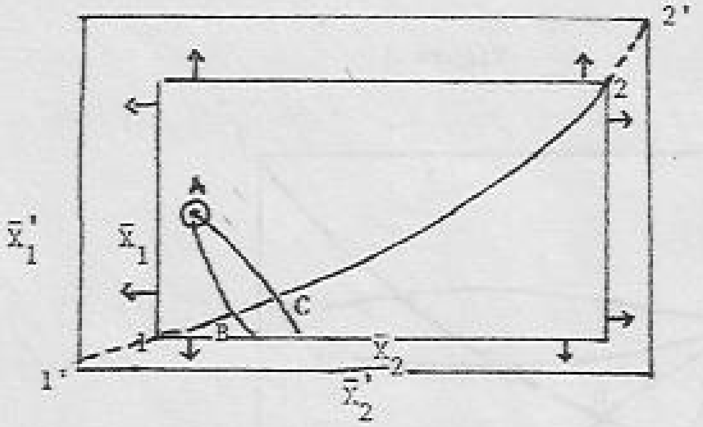
Figure 1



with the initial allocation at A, and the contract curve the curve connecting corners 1 and 2. The core of this game is interval CB on the contract curve if AC and AB are the initial welfare levels. The solution should be an element of the core in this game. Now the core solution can be terribly inequitable if A happens to be close to corner 1 implying that 2 is very wealthy (relatively). The core will then locate close to corner 1. This will not be the case of course if the initial welfare levels are given by AC' and AB' (the broken lines). The core is B'C' closer to corner 2. This eventuality is taste-contingent and we know that taste is a poor explanation for phenomenon.

The pure synergy game enlarges as we said the resource pool but also enlarges the core of the game. The following shows this.

Figure 2



The original exchange game's resource pool is included strictly in the resource pool of the pure synergy-exchange game, $\bar{X}_i' > \bar{X}_i$. The contract curve now connects 1' and 2'. But the core is now the whole of the contract curve since the fallback positions are zero for both. A more equitable resource distribution has a greater possibility of being chosen by society. The solution is patently susceptible to false revelation of preferences. An individual can falsely declare a low level of satisfaction to get awarded a bigger share but we will not deal with this here.

Remark 4:

There are instances in a society's existence when the pure synergy game becomes natural. The imminence of an external invation effectively makes the fallback position of members (except those of fifth columnists) the point zero. Society squeezes out every synergistic juice. Moreover a concommittant egalitarianism ensues as gentleman and peasant alike man the ramparts.

Remark 5:

A society gripped by moral malaise and completely imbued with the ethos of "every man for himself" and "grab as grab can" will degenerate into a pure exchange game devoid of any synergy. When members begin to disparage the value of society's public goods, when demoralization sets in, the trip to a pure exchange game is assured and the total pie shrinks.

Towards a Higher Synergy

It is of importance to enquire just how a society moves from a lower to a higher synergy level or vice-versa how a society evolves into a pure exchange game. Since our method is comparative statics, we will submit to a few modifications for convenience. First, we assume only two individuals 1 and 2 and only one commodity. Thus we move from vector magnitudes to scalar magnitudes. The initial set of exit fees is $(e_1, e_2) = (a, b)$ to avoid indices. The Lagrangean of the social problem is

$$L = \log [U_1(X_1) - U_1(\bar{X}_1 - a)] + \log [U_2(X_2) - U_2(\bar{X}_2 - b)] +$$

$$\lambda [F(a + b) + (\bar{X}_1 - a) + (\bar{X}_2 - b)]$$

The 1° conditions are

(i) $U_1'/V_1 - \lambda = 0$

(ii) $U_2'/V_2 - \lambda = 0$

(iii) $F(a + b) + (\bar{X}_1 - a) + (\bar{X}_2 - b) - X_1 - X_2 = 0$

where V_i is as we defined it in (4). For given (a, b), (i) - (iii) we will generate (X_1^*, X_2^*).

Greed and Counter Greed

It is in the nature of morale that one member's actuations elicit counter actuations from the other member. Under normal circumstances, generosity on the part of one member draws out matching generosity on the part of the other member greed produces countergreed. Now although exit fees are parameter for a society, they are strategy instruments for the members. By decreasing "b", 2 stands to corner a larger share of the pie. But 1 does not take 2's maneuvers lying down and may retaliate by decreasing "a". This interdependence of strategy is necessitated by the interdependence of outcomes. One, if one chooses, may consider the game as a prisoners' dilemma game where the outcomes are also interdependent and the temptation to renege is always present. We are concerned with the conditions that induce more rather than less cooperation and thus conduce to more synergy. We thus assume that 1 responds to generosity by matching generosity and to greed by another greed using the exit fee as the strategic instrument:

$$\text{(a)} \qquad a = a(b) \qquad a' > 0$$

The problem is then posed for member 2: should he or shouldn't he become more generous? Generosity here is gauged by the level of b, i.e., if it rises, he is more generous because (a) he loses bargaining power and (b) he raises the size of the pie to the advantage of the other member. He will become more generous only if his share in the subsequent pie is still bigger than his share before. This depends on (i) the degree of enlargement of the pie and (ii) the matching generosity response to 1". If 1 does not match generosities, 2's bargaining power loss may take a heavy toll on his subsequent share.

He may actually do the reverse and become greedy. The Jacobian matrix of (i) - (iii), J, has the following entries:

$$J_{11} = (V_1U_1'' - U_1'U_1')/V_1^2 < 0$$

$$J_{12} = 0$$

$$J_{13} = -1$$

$$J_{21} = 0$$

(7) $$J_{22} = (V_2U_2'' - U_2'U_2')/V_2^2 < 0$$

$$J_{23} = -1$$

$$J_{31} = -1$$

$$J_{32} = -1$$

$$J_{33} = 0$$

The total differential of (i) - (iii) with respect to b gives

$$A_1 = U_1'U_1'a'/V_1^2 > 0$$

(8) $$A_2 = U_2'U_2'/V_2^2 > 0$$

$$A_3 = (a' + 1)[1 - F'] > 0$$

The determinant of J is $(-J_{22} - J_{11}) > 0$.

Comparative Statics

Our concern is the behavior of X_1^* and X_2^* as 2 becomes more generous. For X_1^* we have

$$(9) \qquad \frac{dX_1^*}{db} = \frac{A_2 + A_3J_{22} - A_1}{|J|}$$

For X_2^* we have

$$(10) \qquad \frac{dX_2^*}{db} = \frac{A_1 + A_3J_{11} - A_2}{|J|}$$

These total effects are decomposable into more primitive component effects which are

(a) Pie size effect

$$\left.\frac{dX_1}{db}\right/_{V_1,V_2} = \frac{A_3J_{22}}{J} > 0$$

where $/_{V_1,\, V_2}$ means V_1 and V_2 are held constant. Thus X_1^* rises unambiguously as pie size rises.

(b) Nash effect against 2

$$\frac{dX_1^*}{db}\bigg/_{V_1,P} = \frac{A_2}{|J|} > 0$$

where V_1 and pie size P is kept constant. Thus 1 tends to gain unambiguously from 2's loss of bargaining power.

(c) Nash effect against 1

$$\frac{dX_1}{db}\bigg/_{V_2,P} = -\frac{A_1}{|J|} < 0$$

where V_2 and P are held constant. Thus 1 stands to lose some of his initial gain if he matches generosity. We can then rewrite (9) and (10) as

$$(9') \quad \frac{dX_1^*}{db} = \frac{dX_1^*}{db}\bigg/_{V_1,V_2} + \frac{dX_1^*}{db}\bigg/_{V_1,P} + \frac{dX_1^*}{db}\bigg/_{V_2,P}$$

$$(10') \quad \frac{dX_2^*}{db} = \frac{X_2^*}{db}\bigg/_{V_1,V_2} + \frac{dX_2}{db}\bigg/_{V_2,P} + \frac{dX_2^*}{db}\bigg/_{V_1,P}$$

Although the decomposition helps intuition reckon with what happens, there are not by themselves our concern. Recall that what we want to establish are the conditions that conduce towards more synergy.

Sufficient Conditions for Greater Synergy

Consider (10) first. Substituting in the relevant expressions we have the numerator of dX_2^*/db being:

$$\left(\frac{U_1'}{V_1}\right)^2 a' + (A' + 1)(1 - F')\left(\frac{V_1U_1'' - U_1'U_1'}{V_1^2}\right) - \left(\frac{U_2'}{V_2}\right)^2$$

Using (i) and (ii) of the 1° conditions we get

$$(11)\quad \left(\frac{U_1'}{V_1}\right)^2 a' + (a' + 1)(1 - F')\ \frac{V_1V_1''}{V_1^2} - \left(\frac{U_1'}{V_1}\right)^2 [(a' + 1)(1 - F') + 1]$$

For $dX_2^*/db > 0$, we need only for the bracketed expression to be negative. We can also combine the first and last expression to get

$$\left(\frac{U_1'}{V_1}\right)^2 [(a' + 1)(1 - F') - a' + 1]$$

and we need only for the bracketed expression to be negative. Thus

Claim 3:

If either

(a) $|(a' + a)(1 - F')| > 1$

(b) $|(a' + 1)(1 - F') - a'| > 1$

then greater synergy follows

These conditions induces 2 to raise b and since a is locked in with b in an increasing fashion, a also rises. Thus greater synergy is assured.

Remark 6:

Greater synergy is more likely when the extent of matching generosity is large (a') and when the productivity of the synergy function is high (F'). A sufficiently large a' or F' will ensure growth in synergy.

Claim 4:

If either (a) [$|a'| < 1$, $F' < 1$] or [$a' < -1$, $F' > 0$], society becomes a pure exchange game.

Proof:

Consider (11). If $F' < 1$, the second expression is negative. So is the third. Decomposing the third expression and comparing with the the first, we have

$$\left(\frac{U_1'}{V_1}\right)^2 (a' - 1) \quad < \quad 0 \qquad\qquad \text{if} \quad a' < 1$$

Thus $dX_2^*/db < 0$. If $-1 < a' < 0$, the first expression is also negative.

Q.E.D.

The nonexistence of synergy $(F' < 1)$ and weak or perverse generosity matching $(|a'| < 1)$ are the main ingredients of a morale crisis. It induces 2 here to reduce his public commitment in the strategic attempt to raise his share of the pie but, in the process, he only succeeds in decreasing the pie size.

The conflict that is spawned by the coexistence of an economic crisis and a morale crisis is that whereas the economic crisis calls for increasing (a, b) which is our parametrization of the call for greater sacrifice leading to higher resource generation, the morale crisis prompts individuals to go the opposite direction. In other words, the morale crisis destroys the very foundation of a solution to the economic crisis. Society or the leadership is unable to concert individual efforts towards higher efficiency.

But since a morale crisis does not develop in a vacuum but springs from the individual member's perception of both past and present reality (a', F'), it constitutes the ultimate act of censure by society of its leadership. When Cambodians in large numbers welcomed the Vietnamese, they were in some way exercizing their power of recall on the Khmer Rouge.

Application to Contemporary Reality

An economic crisis accompanied by a morale crisis becomes a low level trap. Whereas steering out of the economic crisis requires higher productivity thru hard work, greater investment thru saving and austerity, the morale crisis serves to limit both. Social observers are unanimous in declaring the mood now to be negative rather than galvanized. People hold on to the little they have in the fear of losing them altogether. We have with us a morale crisis as well.

That the response to the official call for greater sacrifice and austerity is one of skepticism is not irrational. While the perception today of the possibilities of synergy is negative, there is no reason to believe that in reality the opposite is true. Corruption and blatant opportunism which decreases F' and a' do not seem to have abated. There seems no reason to believe that the resources generated by greater sacrifice won't be mismanaged. The economic crisis seems to have earned its architects commendations by reappointment rather than reproof. In other words, people do not seem convinced that the inefficiencies that precipitated the crisis have ceased. One often hears the following: "I am willing to go without, if all of us (usually meant for the government and the leadership) will go without; not otherwise". The perception seems to be that while the people suffer, the leadership is unwilling to do its share which increases "a". Thus they resort to actuations like buying dollars, salting them, hoarding commodities, clamoring for wage increases, requesting exemption from Pag-ibig, etc., all of which constitute their way of forcing accountability where this is nonexistent. With the morale crisis, the economic crisis is a trap. That is why many observers feelthat the morale crisis (crisis of leadership, crisis of confidence, the political crisis or however it is called) has to be dealt with first. With a morale crisis, the banishment of the architects of the economic crisis which will alter people's perception of a' and F' would certainly help. This gives a semblance of accountability and a warning against mismanagement. But this does not seem to be the case. As long as the people continue in this mood, calls for greater sacrifice will fall on deaf ears and we are bound to grace the economic doldrums for a long time. This is all very tragic but eminently logical.

The analysis also has implications for the question of a "suitable" alternative to the present leadership. It is often claimed that unless such a suitable alternative is found, the situation will only go from bad to worse. The definition of suitability is left unclear throughout the discussion. It may refer to national stature. It may refer to a strong constituency and then again it may refer to every personal endowment that can possibly contribute to success. In the presence, however, of a morale crisis, this concern may be off the mark. Let us suppose there are two choices: A already in power who is, ability wise, 20% more endowed than B. But A by his presence alone is a demoralizing influence while B causes morale to rise 20% more among the members of society. Operationally, this means that the e_i of all members is 20% higher with B than with A. The societal production will be higher with B than with A. In a sense, with a morale crisis, the people "make" the leader. Thus, even with a less talented B, the economic crisis may be solved with greater dispatch. This perception underlies the persistent clamor for a change in leadership.

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