Discussion Paper 8406 October 1984

# BLOCK-RECURSIVENESS OF THE HOUSEHOLD PRODUCTION MODEL UNDER RISK

by

Raul V. Fabella

## ABSTRACT

Block-recursiveness of the household production model is a convenient property to have in the analysis of household production relations. We show here that, with perfectly competitive rural markets, block-recursiveness holds when production risk is additive but not when it is multiplicative. Block-recursiveness does not hold under price risk, additive or otherwise. This has some troubling implications for approaches that require homogeneous functions.

# BLOCK-RECURSIVENESS OF THE HOUSEHOLD PRODUCTION MODEL UNDER RISK

Raul V. Fabella

It is difficult to exaggerate the role "block-recursiveness" plays in recent major studies of farming households (see Lau and Yotopoulos (1979) for a summary of some recent works). Indeed it is hardly possible nowadays to find an analysis of rural household production relations with no explicit reference to this property. Two main currents have contributed to the widespread popularity of the concept: (a) the popularity of the household production model which recognizes that decisions involving production and consumption by farming households are made in one and the same spatio-temporal continuum and thus are intimately related; (b) the extensive use of duality theory in the analysis of the production relations of agricultural enterprises.

In general, the interdependence of the decision blocks (consumption and production) that is inherent in the spirit of the household production model renders duality theory's cost and profit function approaches less than theoretically appropriate. If households allow consumption concerns to detract them from seeking maximum profit from their commercial operations, the estimated profit and cost functions would correspond to decision problems different from those of concern to the household. The approaches could however be used to investigate whether profit maximizing exists. This latter use is of importance to this particular paper.

Consider a rural farming household with consumption and production decisions to make. The set of decisions concerning consumption we call the "consumption block." The set that has to do with production we call the "production block." The intersection of the two blocks may or may not be empty. If the intersection between the two sets is nonempty, we call them "dependent;" otherwise they are "independent." If the solution of second (production) block should first be determined (independent of the decisions in the first block), before first block solutions are found, we have a "block-recursive system." The important consideration is that the subsequent first block decisions do not feed back into the second block to induce an adjustment in the second block. This "absence of feedback" characterizes "block-recursiveness."

The conditions for "block-recursiveness" are the concern of this paper. Sasaki and Maruyama (1966) and Jorgenson and Lau (1969) both independently found the existence of a perfectly competitive labor market sufficient for the existence of block-recursiveness in the rural household model under deterministic conditions. Kuroda and Yotopoulos (1980) considered a household production model with child labor. If child labor is utilized in adult-specific production activities, block-recursiveness no longer obtains. If however, child labor is utilized for child-specific production activities only, block independence will hold. This is the extent of the literature in this area. Surely a feature so important deserves more attention.

Block-Recursiveness in a Simple Household Model: Different Market Regimes

Consider a rural household with the following well-behaved twice continuously differentiable utility function

$$(1) \qquad U = U(c, X, L^T - L_h)$$

where

$c$ = the domestic consumption of farm-produced commodity, say, rice

$X$ = the market purchased commodity

$L^T$ = the total available household time

$L_h$ = the household labor time used for home production

$L^T - L_h = 1$ = leisure

The household production function which is assumed strictly concave twice continuously differentiable over $K$ and $L$ is:

$$(2) \qquad Q = f(K, L)$$

where

$K$ = the amount of capital used; we assume perfect competition in the capital market

$L$ = the amount of labor used: $L = L_h + L_m$ where $L_h$ is household and $L_m$ is bought in labor market

The profit function is:

(3) $\Pi = pf(K, L) - wL - rK$

Note that it is possible for household labor to be sold in the labor market, $L_m < 0$. The budget constraint in the purchase of market commodities is then

(4) $P_x X = P[f(K, L) - c] - wL_m - rK$

where:

$-wL_m$ = cancels out since this amount goes back to the household

$P_x$ = price of $X$

$w$ = wage of labor

$r$ = price of capital

$P$ = price of the home produced farm product

Note that by the way we wrote the production relation, hired labor and household labor are perfect substitutes. There are no free riding and motivation problems and so no extra cost to hired labor. Maximizing (3) subject to (4) gives the following first order conditions:

$$\text{(i)} \quad U_c - \lambda P = 0$$

$$\text{(ii)} \quad U_x - \lambda P_x = 0$$

$$\text{(iii)} \quad P(f - c) - wL_m - rK - P_x X = 0$$

$$\text{(iv)} \quad -U_e + \lambda P f_L = 0$$

$$\text{(v)} \quad \lambda\left(Pf_L - w - L\frac{\partial w}{\partial L}\right) = 0$$

$$\text{(vi)} \quad \lambda[Pf_K - r] = 0$$

where $\lambda$ is the Lagrange undetermined multiplier. Note that the consumption decision block ((i), (ii), (iii), (iv)) is not independent of the production block ((v), (vi)). We see from (iv) that $f_L$ enters the determination of leisure consumption and this would thus be affected by solutions in the production block. Likewise, $f$, $L_m$ and $K$ enter (iii). The two equations (v) and (vi) can not be solved for optimal $L^*$ and $K^*$ without regard to the solutions of the consumption block. In other words, the production block is not independent of the consumption block, and the two blocks must be solved simultaneously for the unknowns.

It is clear from (v) and (vi) above that the crucial expression for block recursiveness is the labor market description (dw/dL). (i) Imperfect Labor : (dw/dL) $\neq$ 0. Let the household find $L^*$ and $K^*$ that satisfy (v) and (vi). $L^*$ implies a certain level of hired labor $L^*_n$ which fixes $L^*_m$. $L^*_m \geq 0$, $L^*_h \geq 0$. Suppose to start with that $L^*_h = 0$. Now solve (i) - (iv) for ($X^*$, $C^*$, $L^*_h$, ). If $L^*_h = 0$ from the consumption block, then the system is solved. But this coincidence is rare. Let $L^*_h > 0$ from the consumption block, i.e., household would want family labor utilized in production. Then some $L^*_m$ should be bumped off to accommodate $L^*_h$. Then influx of hired labor into the market now lowers the wage rate via (dw/dL) and a new $L^*$, $K^*$ has to be found to satisfy the new (v) and (vi). This new $L^*$, $K^*$ will induce a new $f^*$ and the solutions for (i) - (iv) will now again change which will feed back into the production block. In other words the two decision blocks have to be solved simultaneously. There is no block-recursiveness.

(ii) Perfectly Competitive Labor Market: (dw/dL) = 0

Conditions (v) and (vi) will now become

( v') $Pf_L - w = 0$

(vi') $Pf_K - r = 0$

Let the household find $L^*$, $K^*$ satisfying (v') and (vi'). Let us suppose, without loss of generality, that $L_h^{*p} = 0$. Let (i) to (iv) be solved for $(X^*, c^*, L_h^*)$. Let $L_h^{*c} > 0$ from the consumption block. To accommodate $L_h^{*c} > 0$, some $L_m^*$ is bumped off to the market. But $(dw/dL) = 0$, so the market wage rate is unchanged. The potential feedback from consumption to production dissipates in the market. Block-recursiveness is the order of the day. Thus a perfectly competitive market acts as a sinkhole of feedback via wages.

(iii) Institutionally Fixed Wage

The wage rate is now fixed by sociocultural forces (Ranis and Fei, 1964) beyond the control of the farmer. Again the condition $(dw/dL = 0)$ holds. It is now the sociocultural environment that acts as the infinite sinkhole and prevents the feedback mechanism to operate. This third case is meant to correct the impression given by current literature that block-recursiveness is possible only under a perfectly competitive labor market and thus would never obtain where there is surplus labor and positive wage.

## Risk and the Farming Household

In the foregoing, we will assume that either there is perfect competition in the labor market or that there is an institutionally fixed wage. We will however introduce a consideration organically part of the farmer's life - risk. The element of risk is doubly important for the farmer as (a) the nature of the operation exposes the production process to the vagaries of nature, (b) crop insurance is either absent or primitive, and (c) farmers in LDCs are generally too dangerously close to subsistence to ignore risk. To make the structure manageable we assume that (1) the risk is sufficiently reflected by random variables of normal distribution. We will consider two risk regimes: risk in production and risk in product price.

A. <u>Block-Recursiveness Under Production Risk</u>

Let $\tilde{f}(K, L)$ mean that production is a random variable with a normal distribution and a finite mean and variance. From (1), (2) and (4), we have

$$(5) \quad U = U(C, P\hat{f}(K, L) - wL - rK - Pc, L^T - L_h).$$

Taking the Taylor series expansion of (5) around the mean of $f$, $\bar{f}$, and taking the expectation of the expansion gives:

$$(6) \quad EU = U(c, P\bar{f}(K, L) - wL_m - rK - Pc, L^T - L_h) + U_{xx}(c, P\bar{f} - wL_m - rK - Pc, L^T - L_h)\ \sigma_x^2$$

the higher moments disappear under the normality assumption.
Note that

$$\sigma_x^2 = [E(X)^2 - (EX)^2] = P^2 [E(f)^2 - (Ef)^2]$$

There are now still four choice variables $c$, $L_h$, $L_m$ and $K$. The farmer is assumed to maximize expected utility (6)*. The first order conditions are:

$$\text{(i)} \quad U_c - U_x P + \frac{\sigma_x^2}{2} [U_{xxc} - U_{xxx} P] = 0$$

$$\text{(ii)} \quad -U_{L^T - L_h} + U_x P\bar{f}_L + \frac{\sigma_x^2}{2} [U_{xxx}\ P\bar{f}_L - w) - U_{xxL^T - L_h}] = 0$$

$$\text{(iii)} \quad U_x (P\bar{f}_L - w) + \frac{\sigma_x^2}{2} U_{xxx} (P\bar{f}_L - w) + \frac{U_{xx}P^2}{2} \operatorname{cov}(ff_L) = 0$$

$$\text{(iv)} \quad U_x (P\bar{f}_K - r) + \frac{\sigma_x^2}{2} U_{xxx} [P\bar{f}_K - r] + U_{xx} P^2 \operatorname{cov}(f, f_K) = 0$$

---

*This assumption does not sit well with a lot of people. Even if $U_{xx} < 0$ so that the farmer is risk averse and views gains and losses differently, the choice rule may not be strong enough.

since

$$\frac{\partial \sigma_x^2}{\partial h} = P^2\, 2[E(ff_h) - E(f)\,E(f_h)] = 2P^2 \operatorname{cov}(f, f_h)$$

$h = L, K$. Rewriting (ii), we have (iii') $(P\bar{f}_L - w)[U_x + \frac{\sigma_x^2}{2} U_{xxx}] =$

$$\frac{-U_{xxx}P^2}{2} \operatorname{cov}(f, f_L)(P\bar{f}_L - w) = \frac{R_v P^2 \operatorname{cov}(f, f_L)}{2}$$

where

$$R_v = \frac{-U_{xx}}{U_x + \sigma x^2 \frac{U_{xxx}}{2}}$$

is a variance-dependent index of risk aversion. Rewriting (iv) we have

$$\text{(iv')} \qquad P\bar{f}_K - r = \frac{R_v P^2 \operatorname{cov}(f, f_K)}{2}$$

## Decreasing Absolute Risk and $R_v$: A Curiosum

We will first focus on this new and rather unfamiliar risk aversion measure $R_v$. It has the following properties:

a) If absolute risk aversion is nonincreasing, i.e., $\partial R_A/\partial X \leq 0$, then $R_v < R_A$. This is so since $U_{xxx} > 0$

so the denominator of $R_v$ is strictly greater than that of $R_A$.

b) If absolute risk aversion is nonincreasing, then $R_v \to 0$, as $\sigma_x^2 \to \infty$. This is so since $U_{xxx} > 0$.

c) If $U_{xxx} < 0$ with $U_x > |\sigma_x^2 U_{xxx}|$, then $R_v$ rises as $\sigma_x^2$ becomes large.

The Arrow assumption of decreasing absolute risk aversion (DARA) is very much a part of the literature on behavior under risk. Unlike his assumption of "increasing relative risk aversion" (IARA), DARA is less controversial and enjoys a large following. Now the necessary condition for DARA is $U_{xxx} > 0$. If this is the case, then we have a curious case where the larger the variance becomes, the more the farmer throws caution into the wind.

The difference between $R_v$ and $R_A$ is that $R_v$ is also affected by the risk structure of the decision problem besides being affected by the utility structure. It is one viewpoint to say that one's attitude towards risk is independent of the risk structure. It is not necessarily the only nor the true one. That one becomes more conservative the greater is the variability seems to be a very innocuous observation. Neither the absolute risk aversion index nor the relative risk aversion index displays this property. These indices did gain prominence in the wake of interest in risk but this is not the same as saying that they are wedded to attitudes towards risk. For example the name "Arrow-Pratt measure of absolute risk aversion" is used to identify the expression $U''(C)/U'(C)$ in the solution

$$\frac{\dot{\lambda}'}{\lambda} = \frac{-U''(C)}{U'(C)} \dot{C}$$

to the optimal control problem maximizing the utility of the consumption stream. But no risk is involved in this class of problems. More properly $R_A$ should be called "measure of directional assymetry." $R_v$ on the other hand is properly a measure of risk aversion.

We can now easily show the following results:

Proposition 1:

If production risk is additive, then given our assumptions, the household production model is block-recursive.

Proof:

let $\hat{f} = g + \hat{e}$, $\hat{e}$ is normally distributed with $E(\hat{e}) = 0$

The variance $\sigma_x^2$ is then

$$\sigma_x^2 = P^2[E(g + \hat{\varepsilon})^2 - (E\{g + \hat{\varepsilon}\})^2]$$

$$= P^2[E(g^2 + 2g\hat{\varepsilon} + \hat{\varepsilon}^2) - [(E_g)^2 + 2EgE\hat{\varepsilon} + (E\hat{\varepsilon})^2]$$

$$\sigma_x^2 = P^2[Eg^2 - (Eg)^2 + 2g\,E(\hat{\varepsilon}) - 2g\,E(\hat{\varepsilon})^2 + E(\hat{\varepsilon})^2 - (E(\hat{\varepsilon})^2]$$

$$= P^2[E(\hat{\varepsilon})^2 - (E(\hat{\varepsilon})]^2$$

since $g$ is nonrandom. Differentiating with respect to $L$ we have

$$\frac{\partial \sigma_x^2}{\partial L} = 0 = \text{cov}\,(f, f_L)$$

Thus (iii') reduces to

$$P\bar{f}_L - w = 0$$

$$P\bar{f}_K - r = 0$$

and $L^*$ and $K^*$ are reached independent of considerations in the consumption side of the household. Furthermore, since the labor market is either perfectly competitive or institutionally constrained any potential feedback dissipates in the market Q.E.D.

Additive randomness is a common assumption in econometrics. Most all linear estimation models assume additive error term. For the farmer, additive risk comes in the form of vagaries of nature (typhoon, floods, drought, worm and insect infestation) and man-made disasters such as social unrest, wars, etc. The farmer has absolutely no control over these chance events. The random component of production does not covary with the productivity of the variable inputs.

Proposition 2:

If the production risk is multiplicative of finite variance, then under our assumptions, the household production model is non-block-recursive.

Proof:

Let $\hat{f} = \hat{\varepsilon} g(K, L)$ so that $p\hat{f} = p\hat{\varepsilon} g(K, L)$

It is clear that

$$\sigma_x^{\ 2} = p^2\, g(K, L)^2\ \sigma_\varepsilon^{\ 2}$$

$$\frac{\partial \sigma_x^{\ 2}}{\partial L} = p^2\ \sigma_\varepsilon^{\ 2}\ 2g_L \neq 0$$

$$\frac{\partial \sigma_x^{\ 2}}{\partial K} = p^2\ \sigma_\varepsilon^{\ 2}\ 2g_K \neq 0$$

Thus the right-hand side of (iii') and (iv') will not be zero with finite variance and the optimum $(L^*, K^*)$ will be determined simultaneously with the consumption variables. Q.E.D.

Multiplicative risk comes in the form of new seed varieties, new-fertilizer packages, new cultivation methods with innovation in general and labor disturbance. Productivity may be demonstrably higher but so may risk. Thus the model has something very specific to say about farm household conservation: farmers resist innovations not because of the natural risks ordinarily associated with farming; they resist innovations because these innovations are perceived to increase the farming risks.

Proposition 3:

If the farm household exhibits DARA, then the household production model becomes more approximately block-recursive as risk increases.

Proof:

It is clear that as $\sigma_x^2$ rises $R_v$ falls if $U_{xxx} > 0$ which is a necessary condition for DARA. Thus the right-hand side of (iii') and (iv') will approach zero as $\sigma_x^2$ rises indefinely. Q.E.D.

This is the curious result when DARA is assumed. This can be understood in this way. As variability rises indefinitely, the capacity of the farmer to affect the risk diminishes so that beyond a certain point he won't care. Thus the crucial link here is the farmer's belief in his capacity to devise a homemade insurance in the form of conservation in the use of inputs. If he believes very little can be done in this area, he will seek insurance somewhere else (viz., extended family system, share tenancy arrangements, etc.).

## II. Block-Recursiveness Under Price Risk

We now consider the situation when price is a random variable. Now price uncertainty is also an issue of great importance to farmers. We know that price covaries with aggregate production and price support schemes are designed partly to alleviate problems arising from this covariation. There are other risks which are unrelated to this covariation. Demand may be disrupted due to wars, technical innovation, natural calamities (such as the disruption of the anchovy cycle due to the disappearance of pyroplanktons in certain waters off Central America). In the following, we will be dealing more with price uncertainty generated from the demand side.

Let $\hat{P}$ be the random price realized by the farmer from the sale of a unit of farm produce. The equation corresponding to (1) is

$$(7) \quad U = U(C, \hat{P}(f - c) - wL_m - rK, L^T - L_h) \quad .$$

Corresponding to (6) we have

$$(8)\quad E(U) = U(c, \bar{P}f(K, L) - wL_m - rk - PC, L^T - L^n) + \frac{Uxx}{2}\sigma_x^2$$

where $\sigma_x^2 = (f\{K, L\} - c)^2 \;/\bar{E}(P^2) - (EP)^2/ = (f - c)^2 \sigma_p^2$

Corresponding to (iii') and (iv') are

$$(v)\quad Pf_L - w = \frac{R_v(f - c)f_L \sigma_p^2}{2}$$

$$(vi)\quad Pf_L - r = R_v \frac{(f - c)\, f_K \sigma_P^2}{2}$$

since $\sigma_p^2$ is assumed independent of levels of L and K used by the farmer.

Proposition 4:

Under price uncertainty, block-recursiveness obtains for the household production model if either:

a) f = c, i.e., the production of x is for subsistence consumption

b) the household exhibits DARA and $\sigma_x^2 \to \infty$

Proof:

Obvious from right-hand side of (v) and (vi) Q.E.D.

It is clear that these situations are not very interesting. In case (a) price variability is not relevant. Price uncertainty in every form translates into multiplicative risk for production so the interesting cases wash out. Let $\hat{p} = p^o + \hat{\varepsilon}$. Then $\hat{p}g = p^o g + \hat{\varepsilon} g$ which is multiplicative in production. Let $\hat{p} = \hat{\varepsilon} p^o$, then $\hat{p}g = p^o \hat{\varepsilon} g$ which is again multiplicative in production.

Block-Recursiveness and Profit Maximization: An Empirical Consideration

We have clearly shown that the conditions for block-recursiveness are identical to the conditions for profit maximization. This means that the test for profit maximization is also a test for block-recursiveness if we assume that farming under any circumstance is a risky business. This tie-up is significant because in effect the model generates hypotheses about risk behavior the empirical tests of which have been already done extensively by many different authors in many different countries. Specifically, if farmers are seen to maximize profit with respect to variable inputs, then farming risks are perceived to be additive risks and not multiplicative ones.

Lau and Yotopoulos (1979) summarizing six studies on agricultural resource allocation observed that at 0.01 level of significance the hypothesis of profit maximization for farmers from Taiwan, Japan, Malaysia, Thailand and Turkey cannot be rejected. Barnum and Squires' (1980) well-known analysis of Muda padi agriculture also fails to reject profit maximization. Ali (1980) also finds that profit maximization cannot be rejected for Misamis Oriental farmers. Thus it appears that farming risks are perceived to be additive. Farmers then do not feel that they can fashion a homemade insurance scheme by being conservative on the variable inputs.

## Homogeneity and Additive Risk

The presence of additive risk constrains the type of production functions possible. It is clear that the most important class of production functions that cannot allow additive risk is the class of homogeneous functions. Let

$$f = g(K, L) + \hat{\varepsilon}$$

where $g$ is homogeneous of degree $r$ and $\hat{\varepsilon}$ is the random term. $f$ itself fails to be homogeneous of any degree. This is problematic since the Duality theorems (Diewart, 1974) assume constant returns to scale (homogeneity of degree 1) in the original production functions. If risks are perceived to be additive, the original production functions will have intercepts and would then be inhomogeneous. Likewise, the Duality theorem for production assumes profit to be maximized. If risks are not additive, profit maximization is not possible unless farmers are risk neutral. But the latter is rejected by Binswanger (1980) in his study of Indian farmers. In fact, he finds that decreasing absolute risk aversion is the prevailing attitude. The relation between additive risk and homogeneous production function needs to be investigated more closely.

Summary

In the foregoing, we have highlighted the following about the household production model under risk:

(a) Additive production risk allows the household production model to be block-recursive (Prop. 1); multiplicative risk does not (Prop. 2);

(b) Price uncertainty stemming from demand conditions does not allow interesting block-recursive cases. This is because price risk translates into multiplicative risk in production;

(c) For all types of uncertainties, a decreasing absolute risk aversion effects approximate block-recursiveness as $\sigma_x^2 \to \infty$.

(d) We also introduced the idea of the risk-dependent risk aversion measure

$$R_v = \frac{Uxx}{Ux + \frac{\sigma_x^2}{2} Uxxx}$$

and compared its properties to the index of absolute risk aversion of Arrow and Pratt.

On the whole, we conclude that farm households will be conservative with respect to input use when they can devise a homemade insurance thru input use.

## REFERENCES


Barnum, H. and Squire, L. (1980). A Model of Agricultural Household: Theory and Evidence, World Bank Staff Occasional Papers, No. 27, Baltimore and London: The Johns Hopkins University Press.

Binswanger, H. (1980). "Attitudes Toward Risk: Experimental Measurement in Rural India," American Journal of Agricultural Economics, 62.

Diewart, W. (1974). "Applications of Duality Theory," in M.D. Intrilligator and D.A. Kendrick (eds.), Frontiers of Quantitative Economics, North Holland Pub. Co., Amsterdam.

Jorgenson, D.W. and Lau, L. (1969). "An Economic Theory of Agricultural Household Behavior," Paper presented at the Fourth Far Eastern Meeting of the Econometric Society, Tokyo, June 26-28.

Kuroda Y. and Yotopoulos, P. (1980). "A Subjective Equilibrium Model of the Agricultural Household with Demographic Behavior," Working Paper No. 80-3, Demographic Impacts of Agricultural Development Policy.

Lau, L. and Yotopoulos, P. (1979). Resource Use in Agriculture: Applications of the Profit Function to Selected Countries, Food Research Institute Studies, Vol. XVII, FRI, Stanford University, Stanford, California.

Moshin, Ali (1982). "Production Decisions and Fertility Behavior of Agricultural Households: A Microeconomic Approach," Unpublished Ph.D. Dissertation, School of Economics, U.P., Diliman, Quezon City.

Ranis, G. and Fei, J.C.H. (1964). Development of Labor Surplus Economy: Theory and Policy, R.D. Irwin, Homewood, Illinois.

Sasaki, K. and Maruyama, Y. (1966). "Impacts of Fixed Inputs on the Farm - An Extension of the Theory of Subjective Equilibrium," Journal of Rural Economics, Vol. 38.