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Price Decisions and Equilibrium

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Abstract

This paper explores a model of the imperfectly competitive firm, facing demand uncertainty, that sets a price on its product so as to maximize the probability of obtaining satisfactory profits. It is a consequence that the price is higher with higher demand or higher costs. With all firms as price setters there is a general price equilibrium where prices are stationary. This entails an equilibrium average level of employment which is likely to be less than full employment. The model accommodates the phenomenon of inflation and increasing unemployment.

Price Decisions and Equilibrium

J. Encarnación*

I. Introduction

This paper develops some implications of the hypothesis that under conditions of demand uncertainty and imperfect competition, the firm sets a price on its product in order to maximize the probability of covering full cost (Robert Hall and Charles Hitch, 1939), defined to include an acceptable return on capital. This objective function gives a significant role to the concept of a satisfactory profit rate which appears widely held among businessmen, and it seems worth exploring. It turns out that the resulting model has more explanatory value than the usual expected profit maximization hypothesis.

Section II relates the two hypotheses to a simple formulation of the firm's utility function. Sections III and IV discuss the firm's price decision and equilibrium prices in the industry. With all firms in the economy as price setters, Section V defines a general price equilibrium where prices are stationary. Section VI concludes the paper.

II. The Firm's Utility Function

Let the price p of its product be the firm's decision variable under demand uncertainty. The usual assumption is that its utility function can be stated as

$$u_2(p) = E(\Pi(p))$$

where $\Pi(p)$, the resulting profit given p , is a random variable and E

the expectation function. (The reason for the subscript in u_2 will be clear in a moment.) In the absence of uncertainty the firm's preference ordering would therefore be determined by the sure profit level corresponding to each p .

Consider instead the lexicographic utility function^{1/}

$$u(p) = (u_1(p), u_2(p))$$

where

$$u_1(p) = \Pr \{ \Pi(p)/K \geq r^* \},$$

the probability that the profit rate given the firm's capital K will be no less than some acceptable rate r^* . In the absence of uncertainty $u_1(p)$ is either 0 (if $\Pi(p)/K < r^*$) or 1 (if $\Pi(p)/K \geq r^*$) so the preference ordering would be the same as that given simply by u_2 . Thus with no uncertainty, the utility functions u and u_2 have identical implications: profit is maximized. In the presence of uncertainty however, u reduces essentially to its first component u_1 since u_2 would become relevant only if the maximization of $u_1(p)$ yielded more than one value of p . Accordingly, u_1 and u_2 may be considered as alternative hypotheses regarding the firm's decision criterion under uncertainty, assuming for simplicity that $\max u_1(p)$ gives only one value of p .

The criterion u_1 combines two ideas in a natural way-- the Hall and Hitch (1939) finding that some satisfactory rate of profit is an important parameter in business decisionmaking, and the Cramér-Roy safety first principle (Andrew Roy, 1952) of minimizing the probability of a "disaster", viewing the latter as a subsatisfactory profit rate. Whether

u_1 is descriptive of the behavior of firms is of course an empirical issue. The results of interviews reported by James Mao (1970) suggest that business executives think of risk in terms of the chances of falling short of some specified rate of return, which is consistent with u_1 but not particularly with u_2 . It seems plausible that the firm's managers would choose to show satisfactory profits always, even if the average profit turns out to be less than what is possible, rather than a higher average profit that involves large profits at times and large losses at other times, since the latter event could cause their dismissal. While u_2 is of course superior as a long run objective from a profit viewpoint, the long run cannot be had without surviving the short, whose probability is thus a high priority concern.

III. The Price Decision

Consider an imperfectly competitive firm whose differentiated product is subject to uncertain demand.^{2/} Let $f(x|p, \theta)$ be the conditional probability density of the random quantity demanded $x \geq 0$ in each time period, given the price $p > 0$ and the demand parameter $\theta > 0$. The parameter θ is a "given" to the firm but depends inter alia on the prices that other firms choose to set on their products. Higher prices in the rest of the industry would make some buyers turn to the firm's product and thus raise θ . Given p and θ we can assume that the distribution defined by f is unimodal. Let

$$(1) \quad \pi = \pi(q, p, \theta) = \int_q^{\infty} f(x|p, \theta) dx, \quad q > 0,$$

so $\pi_q = \partial\pi/\partial q \leq 0$, $\pi_p \leq 0$, and $\pi_\theta \geq 0$. In what would be of interest, strict inequalities hold. Note for instance that $\pi_q = 0$ only if q is

so high that the probability $\pi = 0$ or q so low that $\pi = 1$, either of which would be an extreme case.

Given θ , putting $\pi = \alpha$ ($0 < \alpha < 1$) in (1) defines what we will call a π line relating q and p where the probability $\Pr\{x \geq q \mid p, \theta\} = \alpha$. The family of π lines corresponding to different values of α ($0 < \alpha < 1$) constitutes the (stochastic) demand schedule, which is assumed to originate from a point on the p -axis that depends on θ . One such line, labelled π^0 , is shown in Fig. 1. (Since $\pi(q, p, \theta)$ is not defined for $q = 0$, the originating point does not belong to any π line.) Noting that π lines fan out towards the horizontal q -axis--those to the left are steeper than those to the right--there is greater variability of demand at lower values of p . An increase in θ shifts the originating point upwards and therefore each π line rightwards.

Let $y \geq 0$ denote the firm's output, $C(y)$ the production cost of y , and $g(y) = C'(y)$ in the range where $C''(y) > 0$ so $g(y)$ is an increasing function. We assume that production is instantaneous^{3/}, enabling the firm to produce the output to meet the demand x in each period, subject to

$$(2) \quad y = \min \{x, g^{-1}(p)\}.$$

I.e., output is limited to the range beyond which the marginal cost exceeds whatever price it has chosen. If it happens that $x > g^{-1}(p)$, some part of the then demand will simply be left unfulfilled.

Our basic assumption is that the firm sets a price for its product to maximize $\Pr\{py - C(y) \geq A^* \mid \theta\}$ where $A^* = r^* K$. In view of (2) it is

clear from their meanings that

$$\Pr \{py - C(y) \geq A^* | \theta\} = \alpha$$

if and only if

$$\exists q: pq - C(q) \geq A^* \text{ \& } \pi(q, p, \theta) = \alpha.$$

Thus the price decision problem is to maximize $\pi(q, p, \theta)$ subject to $pq - C(q) \geq A^*$. We rule out the case where θ is so low that $\pi(q, p, \theta) = 0$ for all (q, p) satisfying the constraint--such a firm should soon be out of business--and for simplicity we assume a unique solution. Accordingly, if $\pi^0 = \pi(q^0, p^0, \theta)$ is maximal, the price p^0 is optimal. One has the necessary conditions

$$(3) \quad \pi_p + \lambda q = 0$$

$$(4) \quad \pi_q + \lambda(p - C'(q)) = 0$$

$$(5) \quad pq - C(q) - A^* = 0$$

where $\lambda > 0$ is a Lagrange multiplier. (We will usually omit 0 superscripts denoting solution values of the variables where there is no ambiguity in the context.)

Writing (5) as $p = (C(q) + A^*)/q = k(q)$ defines what we will call the k , or profit constraint, curve. It is straightforward to show that $k'(q) < 0$ and $k''(q) > 0$ where $p > C'(q)$. With $C''(q)$ continuous as usual which implies that $k''(q)$ is continuous, the function k is therefore strictly convex in the relevant range since from (4), $p > C'(q)$ always holds in the problem.

We can assume that θ is bounded to define $p^+ = \sup_{\theta} \{p^0 \mid p^0 = p(\theta)\}$ since p^0 is a function only of θ with $C(y)$ and A^* given. Defining $p^- = \inf_q k(q)$, $p^- \leq p^0 \leq p^+$. Since π lines are shifted to the right by an increase in θ , whence a steeper π line corresponding to a larger value of π must touch the k curve at a higher p , the optimal price is higher with θ unless the price is already at p^+ . Thus we have

Proposition 1. The price that is set by the firm is higher if demand is higher.

This seems intuitively credible as a ceteris paribus statement, but we observe that it is not implied by the u_2 hypothesis.

In order to see the effect of costs on p and π , let the cost function have the sufficiently general form

$$(6) \quad C(y) = a + by + c(y), \quad a > 0, b > 0, c(y) \geq 0, \exists y: c(y) = 0.$$

We will call b the fixed component of unit cost. A higher a , b , or r^* raises the k curve and therefore lowers the probability of meeting the profit requirement. The effect on price is given by total differentiation of (3)-(5):

$$(7) \quad \begin{bmatrix} \pi_{pp} & \pi_{pq} + \lambda & q \\ \pi_{qp} + \lambda & \pi_{qq} & p - C'(q) \\ q & p - C'(q) & 0 \end{bmatrix} \begin{bmatrix} dp \\ dq \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda db \\ q db + da \end{bmatrix}$$

where $A = a + A^*$ and $\pi_{pq} = \partial \pi / \partial q$, etc. Writing D for the determinant of the coefficient matrix in (7) and $D_{\mu\nu}$ for the cofactor of its (μ, ν) th element, $\partial p / \partial A = D_{31} / D$ and $\partial q / \partial A = D_{32} / D$. Since $D = q D_{31} + (p - C'(q)) D_{32}$,

$$(8) \quad q \frac{\partial p}{\partial A} + (p - C'(q)) \frac{\partial q}{\partial A} = 1.$$

It is easy to see that $\partial q / \partial A > 0$, for at the same value of q , $-k'(q)$ is larger if A is greater so a less steep π line must be tangent to a higher k curve at a higher q . If

$$(9) \quad (p - C'(q)) \frac{\partial q}{\partial A} < 1$$

then $\partial p / \partial A > 0$ in (8), and

$$\frac{\partial p}{\partial b} = (\lambda D_{21} + q D_{31}) / D > 0$$

since $D > 0$, from the second-order conditions, and $D_{21} > 0$. Thus if (9) holds, a higher value of a , b , or r^* implies a higher price. Appendix A shows that (9) holds, so we have

Proposition 2. The price is higher if production cost or the acceptable profit rate is higher.

We observe that if u_2 were the decision criterion, the price would be unaffected by higher overhead costs or by a higher fixed component of unit cost, which seems contrary to common observation. Higher costs are often invoked by firms when they post price increases, and it seems unlikely that such a justification can be completely explained away as mere public relations.

IV. Equilibrium Prices in the Industry

The firm of Section III can be considered as firm i producing good i in an n -firm industry, the index i being suppressed in the notation there. Let now $p = (p_1, \dots, p_n)$ and $p_{-i} = (p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_n)$

where p_j is not necessarily equal to p_j^0 ($j = 1, \dots, n$). We assume that θ^i is a continuous function of p_{-i} so $p_i^0 = G^i(\theta^i(p_{-i})) = H^i(p)$ with H^i also continuous. There is then a continuous mapping $p^0 = H(p) = (H^1(p), \dots, H^n(p))$ on the set $S = \{p \mid p^- \leq p \leq p^+\}$, and the Brouwer fixed point theorem implies

Proposition 3. There exists an equilibrium price vector $p^* = H(p^*)$ for the industry.

Since $\theta_j^i = \partial \theta^i / \partial p_j \geq 0$ for $j \neq i$, and $\partial p_i^0 / \partial \theta^i > 0$ from Proposition 1, $G_j^i \geq 0$ ($j \neq i$). We assume that

$$(10) \quad G_{jj}^i \leq 0 \text{ if } G_j^i \geq 0, \quad j \neq i,$$

for otherwise, additional increases in p_j would make p_i^0 increasingly higher, which seems unlikely. If a higher p_j transfers some demand to i , it would only be reasonable to expect such transfers to get progressively smaller as p_j gets higher so the effect on p_i^0 should be attenuated.

Given (10), p^* is unique, for a second equilibrium implies that the G^n surface, say, meets the intersection of the G^1, \dots, G^{n-1} surfaces at a point other than the first equilibrium, requiring G^n to violate (10).

Fig. 2 illustrates the $n = 2$ case. It is evident that as soon as the existing $p \neq p^*$ lies on either the G^1 or the G^2 curve, both prices will converge monotonically to p^* . In the general case, if the existing p satisfies

$$(11) \quad p < p^* \text{ or } p > p^*, \text{ and } p \text{ lies on some } G^i \text{ surface,}$$

we will say it is a "success". Obviously from a success prices can then

only converge monotonically to p^* . Appendix C shows that sooner or later, i.e. with probability one, a success will obtain if the initial price vector is different from p^* . We therefore have

Proposition 4. The industry's price equilibrium p^* is globally stable with probability one.

Suppose p^* is the existing price vector. An exogenous change in cost or demand for some i that lowers its price-- G^i is shifted downward--would then decrease equilibrium prices, and price changes would spread through the industry with each firm responding myopically only to its own cost and demand conditions.^{4/} Accordingly, if firm i lowers its price because of a lower overhead or fixed component of unit cost (due, say, to its discovery of a new technical process), it will have a higher probability of making satisfactory profits, prices in other firms in the industry will be lower because of the shift in demand to i , and they will have only lower probabilities of satisfactory profits. The better competitive position of the cost-reducing firm is thus felt by other firms as a fall in the values of their objective functions, signalling their now weaker competitive positions in the industry. According to the u_2 hypothesis on the other hand, the cost reduction merely increases the expected profits of i without changing its price, there is no effect on the prices or expected profits of other firms, and they get no signals that their market positions have changed. In our view, such an implication of the usual hypothesis is contrary to the workings of actual markets, whereas the price decision model based on u_1 is more in conformity with the facts. (See Appendix B for other observations that seem explainable in terms of the model.)

V. General Price Equilibrium and Employment

Assuming that every firm is a price setter in the sense of Section III, that the money wage rate w is an exogenous parameter determined by collective bargaining, and that the money supply M is given, the argument of Section IV can be extended to the whole economy. We will speak of a general price equilibrium if the prices set by all firms are stationary--there being no reason to change them--and therefore the mean values of the output variables are also stationary. In order to avoid excessive repetition, mean values of all quantity variables (including employment) will be understood where appropriate in what follows, and for notational brevity we will suppress w and M which are fixed unless stated otherwise.

Let now p be the vector of all prices: $p = (p_{(1)}, \dots, p_{(m)})$ where $p_{(i)}$ is the vector of prices in industry $i = 1, \dots, m$; p_{-i} remains the vector p omitting p_i . Denote total employment by $N = \sum N_i$, where N_i is employment in i , which has a positive effect on demand. Writing $p_i^0 = G^i(\gamma^i(p_{-i}), \theta^i(p_{-i}), N) = H^i(p, N)$ now permits input prices in $\gamma(p_{-i})$ to be determinants of p_i^0 because of their effects on cost. From Proposition 2, $G_j^i \geq 0$ if j is an input of i . Allowing for the possibility that $G_j^i < 0$ when i and j are complementary on the demand side, we assume that

$$(10') \quad G_{jj}^i \geq 0 \text{ if } G_j^i < 0, j \neq i,$$

which seems just as plausible as (10). Let $N_i^0 = L^i(p_i, \theta^i(p_{-i}), N)$ be the amount of labor required to produce the output of i , and $N^0 = \sum N_i^0 = L(p, N)$. We can therefore write $(p^0, N^0) = J(p, N)$.

Given the money supply and the existing population, we can assume upper bounds p^+ and N^+ . Defining $N_i^- = \inf_{p, N} L^i(p_i, \theta^i(p_{-i}), N)$, let $N^- = \sum N_i^-$. Since the money wage is given and labor is a necessary input, $p_i = k^i(q_i)$ cannot fall below some positive value no matter the prices of non-labor inputs. Thus we can put $p_i^- = \inf_{p, N} \{p_i^0 \mid p_i^0 = H^i(p, N)\}$ and define $S = \{(p, N) \mid p^- \leq p \leq p^+ \text{ \& } N^- \leq N \leq N^+\}$. The same argument in Section IV then gives

Proposition 5. There exists an equilibrium vector of prices and employment $(p^*, N^*) = J(p^*, N^*)$ involving prices $p^* = H(p^*, N^*)$ and employment $N^* = L(p^*, N^*)$.

In order to have a closer look at the properties of the equilibrium, consider the relation

$$(12) \quad p^e = H(p^e, N)$$

which defines what we will call a quasi-equilibrium price vector p^e given N . (Note that unless $N^0 = N$ in $N^0 = L(p^e, N)$, p^e is not sustainable. If $N^0 > N$ say, the demand for some firms' output must be larger than is being produced with its existing work force and therefore by Proposition 1, its price would have to be raised. Hence the qualifying term "quasi".) Assuming (10), (10'), and

$$(13) \quad \text{if } i \neq j, G_j^i > 0 \text{ everywhere or } G_j^i < 0 \text{ everywhere,}$$

it is clear from an obvious extension of the argument in Section IV that p^e is unique given N . Now since $\partial G^i / \partial N > 0$ for all i , it is also clear that every component of p^e is higher the larger is N . We can therefore define a price index

$$(14) \quad p_0 = h(p^e)$$

that is positively associated with N and write

$$(15) \quad p_0 = \phi(N), \quad \phi'(N) > 0.$$

Given an arbitrary p , not necessarily a quasi-equilibrium p^e , let

$$(16) \quad N^e = L(p, N^e)$$

define a quasi-equilibrium level of employment N^e given p . (Unless $p^0 = p$ in $p^0 = H(p, N^e)$, N^e is in general not sustainable. Evidently if $p^0 \neq p$, some price would be adjusted. Output and employment would then be different.) We will say that p_0 is the price index corresponding to p if p^e in (14) and N^e in (16) satisfy

$$(17) \quad N^e = L(p^e, N^e).$$

In other words, two price vectors have the same price index if they determine the same quasi-equilibrium employment level. Since $\partial L^i / \partial p_i < 0$ for all i , N^e in (17) is smaller if the price index of p^e is higher. Thus we can write

$$(18) \quad N^e = \psi(p_0), \quad \psi'(p_0) < 0.$$

Equations (15) and (18) are shown in Fig. 3 where $p_0^* = h(p^*)$. One sees that the equilibrium (p^*, N^*) is unique--it is only at that point that prices remain stationary. As should be expected, micro price variables can be in equilibrium only if macro quantity variables (output and employment) are in equilibrium, and conversely. The question of stability can be examined by looking at the stability of N^e given p in (16) and the

stability of p^e given N in (12).

We will call $\partial N^0 / \partial N$ the marginal own-effect of employment and assume that $\frac{\partial}{\partial N}$

$$(19) \quad 0 < \partial N^0 / \partial N < 1$$

which is the employment version of the usual assumption regarding the marginal propensity to consume. As in the textbook 45° diagram for the determination of national income, N^e is stable given p if the marginal own-effect of employment is less than one. Hence the horizontal arrows in Fig. 3 indicating possible paths of N , depending on initial conditions, if p is held fixed.

Suppose N is held fixed instead. An extension of the argument in Section IV will show that p^e is then stable. Let us say that p_i and p_j are on the "same side" of p^e if they are both greater than or both less than their corresponding values in p^e , on "opposite sides" if one is greater and the other less, and similarly for vectors of prices. Denote by $\zeta(i)$ the industry in which i belongs, and say now that we have a success if the existing $p = (p_{(1)}, \dots, p_{(m)})$ satisfies (11').

(11') $p_{(\zeta(i))}$ and $p_{(\zeta(j))}$ are on the same side of p^e if (10) holds for i and j , on opposite sides if (10') holds, and in each industry there is an i such that p lies on its G^i surface.

From a success it is clear that prices can only converge monotonically to p^e . Moreover, the argument in Section IV remains valid: (11') will obtain sooner or later if the initial $p \neq p^e$. Thus p^e is stable given N , hence the vertical arrows in Fig. 3.

Diagrams like Fig. 3 are familiar in the literature. In the usual way, the resulting motion of the system depends on the relative strengths of the responses of the variables to disequilibria. Depending on initial conditions, the approach to the equilibrium can be direct or cyclical, but in any event we have

Proposition 6. The prices and employment equilibrium (p^*, N^*) is globally stable with probability one.

There is no reason of course to expect N^* to equal full employment (no matter how this might be defined in a stochastic sense) at the pre-determined money wage w . It has been a recurring question in the literature to what extent a rigid w is essential for a below-full employment equilibrium. To address this question, observe that

$$(20) \quad \frac{\partial N_i^0}{\partial w} = \frac{\partial L^i}{\partial \theta^i} \frac{\partial \theta^i}{\partial w} + \frac{\partial L^i}{\partial p_i} \frac{\partial p_i}{\partial w}$$

may be positive or negative. (From Proposition 1, the first term on the righthand side is positive because of the demand effects of higher wages, but from Proposition 2, the second term is negative because of cost effects on prices.) Thus $\partial N^0 / \partial w$ is indeterminate in sign. A reduction in the money wage may result in less employment instead of more, and although there would be some value of w that maximizes N^* --treating w parametrically-- this could still fall short of full employment. Accordingly, a "correct" w that yields full employment need not exist, and therefore a rigid w is not a necessary condition for a below-full employment equilibrium. It is obviously not sufficient either. An employment equilibrium N^* , which is

implied by a general price equilibrium p^* , is simply due to the fact that firms only hire that number of workers whose output can be sold at equilibrium prices.^{6/} If that output can be produced by a smaller labor force than is available, which would be the a priori expectation, the result is some unemployment.

Suppose such a case. An exogenous increase in general demand--demand for output is higher at the same prices--implies a rightward shift of the N^E curve in Fig. 3. As usual, prices and employment are then higher. Consider instead an exogenous increase in costs (due for example to higher oil prices). This shifts the p_0 curve upwards, so one has then both higher prices and higher unemployment. The so-called stagflation of recent history is thus capable of a simple explanation in terms of our framework.^{7/}

Finally we note that in standard theory which requires that the marginal product of labor be equal to the real wage, the latter is expected to fall in the expansion phase of the business cycle. In the present model there is no necessity for price increases to exceed money wage increases, so real wages may well rise during an expansion as has been shown by the empirical study of Ronald Bodkin (1969).

VI. Concluding Remarks

The hypothesis of this paper is that the imperfectly competitive firm facing demand uncertainty sets a price on its product to maximize the probability of obtaining a satisfactory profit level. Adjusting its price in response only to its own cost and demand conditions, the firm then sets a higher price when demand is higher and also when costs are higher. These

implications of the model, which seem in consonance with observation, do not follow from expected profit maximization.

With all firms in the economy as price setters thus dispensing with a fictitious auctioneer, there is a general price equilibrium--all prices are stationary--which implies an employment equilibrium. In general, the equilibrium is one of below-full employment, accounted for by the fact that firms do not hire more workers than they need, and the need is determined by the output that can be sold at equilibrium prices. As usual, an increase in general demand means higher prices, output and employment. What is less usual, and contrary to standard theory, real wages could then be higher as has been observed in the expansion phase of some business cycles. More important, the framework yields a simple explanation of simultaneous inflation and increasing unemployment.

In a less incomplete account than is given in this paper, one would have an explicit treatment of household decisions, which are in the background and reflected only in the demand schedules of firms. Also, the demand for money is only implicit in the model--a residual (so to speak) from the demand for output. A more complete formulation would include these variables, but we conjecture that they would not require essential changes in the present discussion of a general price equilibrium. It seems obvious, for instance, that a general price equilibrium entails an equilibrium demand for money (as it does an employment equilibrium) for otherwise, prices could not be stationary. However, this should be a subject for more detailed research.

Appendix A

First we show that (9) is satisfied where p^0 is close enough to p^- . Consider a small change $\Delta A > 0$ and the k curves corresponding to A and $A + \Delta A$. From (5), $(p - C'(q))dq + q dp = dA$. At the point (q, p) on the lower k curve the vertical distance between the two curves is $\Delta p = \Delta A/q$ and the horizontal distance $\Delta q = \Delta A/(p - C'(q))$, so (i) $\Delta p/\Delta q = (p - C'(q))/q$ and (ii) $\Delta q/\Delta A = 1/(p - C'(q))$. Let $s(\theta, A)$ be the (absolute) slope of the k curve at the optimal point given θ and A ; then (iii) $s(\theta, A) = (p^0 - C'(q^0))/q^0$ from (3) and (4). Denoting by (q^s, p^s) the point on the higher k curve where the slope equals $s(\theta, A)$, $q^s < q^0 + \Delta A$ from (i) and (iii) since the function k is strictly convex. Now as θ hence p^0 gets smaller, so does $s(\theta, A)$. Observing that $s(\theta, A + \Delta A) < s(\theta, A)$, one has $s(\theta, A) - s(\theta, A + \Delta A) \rightarrow 0$ as $p^0 \rightarrow p^-$, which implies that the optimal point on the higher k curve—denote it by (q', p') —gets arbitrarily close to (q^s, p^s) . Thus if p^0 is low enough, $q' < q^0 + \Delta q$ also. But with (ii) this means that $\partial q/\partial A < 1/(p - C'(q))$, which is (9).

The next step is to see that one never gets $\partial p/\partial A = 0$ even though p^0 is increased. Suppose a p^0 such that $\partial p/\partial A = 0$. This would mean that the slopes of the k curves are equal at that value of p and therefore $q^s = q^0 + \Delta q$. But we have already seen that $q^s < q^0 + \Delta q$, so $\partial p/\partial A = 0$ never holds and the assertion in the text follows.

Appendix B

The purpose here is only to indicate further directions where the model of this paper might have explanatory value. The issues involved are complex and require considerably more research.

The cost parameter b in (6) is the fixed component of unit cost, given the existing plant. Casual observation suggests that it is often technologically possible for b to be lower if the size of the firm were larger, at least up to a point, for there would be economies from larger scale use of material inputs. In Fig. 1 we can imagine a family of k curves corresponding to different sizes of the firm: the larger it is, the higher is $A = a + r^* K$ because of the greater K required (and possibly also a), but the lower is b . Since $p = b + (c(q) + A)/q$, p would be lower (higher) at higher (lower) values of q if A were higher but b lower. Suppose that the set of technical possibilities is sufficiently rich so that through the point (q^0, p^0) of the existing k curve there is another profit constraint curve corresponding to $K' > K$. The constraint curve for K' would then lie above the existing k curve in the range $q < q^0$ but below it for $q > q^0$. This means that with the larger K' plant it would be possible to have a lower p hence a higher π under the same demand conditions.

The firm would therefore want to increase its size to K' where it would make satisfactory profits more often. It is plausible that its ability to grow would be greater if, in the first place, the relative frequency of satisfactory profits were higher. Ceteris paribus the firm is thus more likely to grow (and grow faster) if the required profit rate r^* is less.

In the long run, the firms surviving would be those with the lowest r^* , which might then be called the normal profit rate in the industry. At the same time, free entry of new firms will reduce the mean profit rate \bar{r} of an existing firm if it exceeds the normal rate of profit, so the long run tendency would be for the different \bar{r} 's to converge to the normal rate.

In their empirical study, John Samuels and David Smyth (1968, p. 39) concluded that "Profit rates and firm size are inversely related. [Moreover, the] ... time variability of profit rates and the intra-group variability of profits are both inversely related to firm size." These findings are at least consistent with the present discussion. Larger firms have already gone through a screening process so to speak, so their r^* 's and \bar{r} 's should all be about the same and close to the normal rate. Suppose that because of initial differences there is a wider spread of r^* 's among smaller firms. Those with \bar{r} 's less than their r^* 's are not likely to be observed--they would have left the business--so the observed \bar{r} 's of smaller firms should on average be greater than those of larger firms. Hence the negative correlation between profit rates and firm size. As for the finding that the variance of r is less for larger firms, this would be consistent with their having $r < r^*$ less frequently while at the same time, \bar{r} is close to r^* for them.

Appendix C

If some price is not optimal, it will be adjusted. Only for present purposes, let the time interval between a price change by any firm and the next change by any other firm count as one unit of time. Since p at time t , or $p(t)$, depends not only on $p(t-1)$ but also on the firm(s) making the price move(s) at $t-1$, p as a function of time is a random sequence.

(No restriction can be imposed on the order of moves by the various firms any of which can adjust its price when it wants to.) Consider a given $p \neq p^*$ in S . There is a finite sequence of non-simultaneous moves that

yields a success at some minimum time $\tau = \tau(p)$ with probability say

$\beta = \beta(p) > 0$. Let $\tau' = \max \tau(p)$ and $\beta' = \min \beta(p)$. Starting from an

arbitrary $p \neq p^*$, the probability of a success within τ' periods is

clearly at least $\beta' > 0$. Measuring time in blocks of τ' periods each,

the probability of still no success by the end of block T is no greater

than $(1 - \beta')^T$. But $(1 - \beta')^T \rightarrow 0$ as $T \rightarrow \infty$, proving the assertion.

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Notes

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1. See Peter Fishburn (1974) for a survey of this literature and the major references: Nicholas Goergescu-Roegen (1954) and John Chipman (1960); cf. José Encarnación, Jr. (1964).

2. Such uncertainty may result from the kind of consumer behavior described by Werner Hildenbrand (1971) and also from random differences in the timing of purchases by different households, fluctuations in income, etc.

3. Cf. Alan Kirman and Matthew Sobel (1974) who make such an assumption. The holding of inventories is an attractive feature of their model, but they assume price adjustment every period which seems contrary to observed behavior. We expect that if market conditions are stationary except for random elements in demand, the imperfectly competitive firm will quote a steady price; cf. Richard Levitan and Martin Shubik (1971). Jean-Pascal Benassy (1976) also assumes instantaneous production and price adjustment every period.

4. The effects of other firms' prices on these conditions could well be anticipated however, in which case each firm could change its price without waiting for the actual effects to take place. There could then be rapid adjustments towards a new equilibrium.

5. Condition (19) follows from the assumption that $0 < \partial N_i^0 / \partial N_i < 1$ for all i , but this would be unduly restrictive. It is N , not necessarily N_i , that creates a demand for i .

6. Since this fact holds also for the long run, there is no assurance of full employment even there; cf. Oliver Hart (1982) who gets a similar conclusion from his model.

7. It is interesting that a reinterpretation of Fig. 3 applies to the firm of Section III. Given the level of demand θ for its product let $N^E = \psi(p_0)$ be the number of workers required to produce its output if the price is p_0 . Since there is an optimal price given θ , this picks out a point on the N^E curve which determines the employment $N(\theta)$ corresponding to the given θ . Let $p_0 = \phi(N)$ be the locus of such points obtained by varying θ , expressing $\theta = N(\theta)$ in units of N . An increase in costs shifts the p_0 curve upwards, an increase in demand shifts the N^E curve rightwards, and by construction, the intersection of the two curves gives price and employment in the firm. One could thus speak of a representative firm.

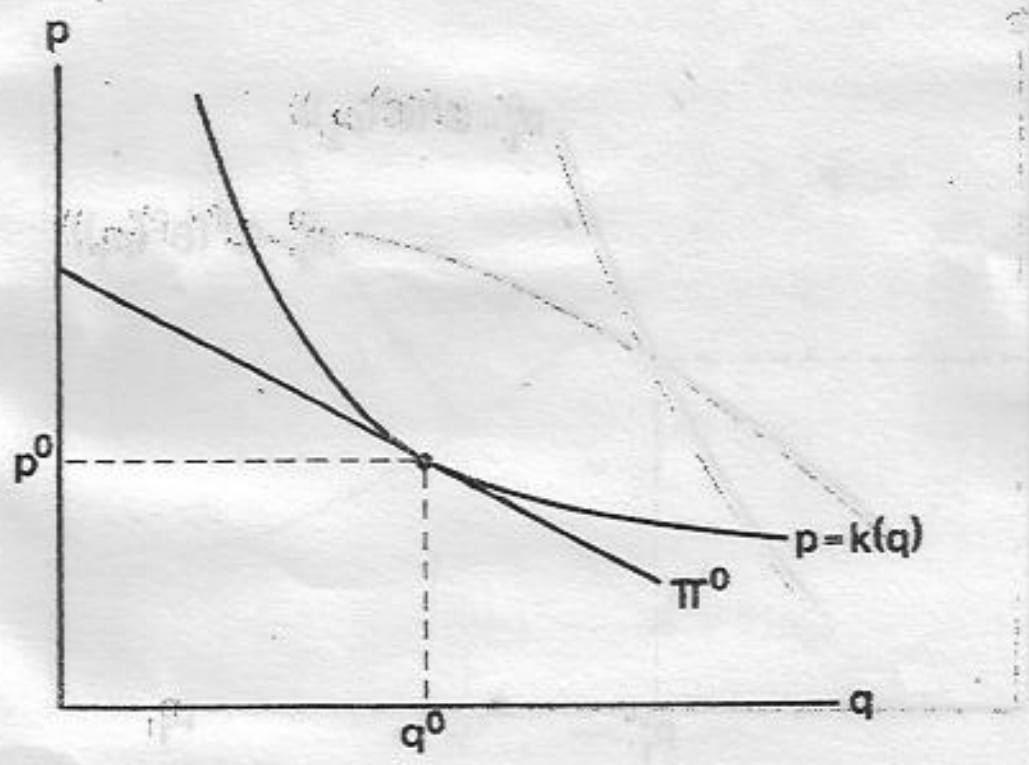


Fig. 1

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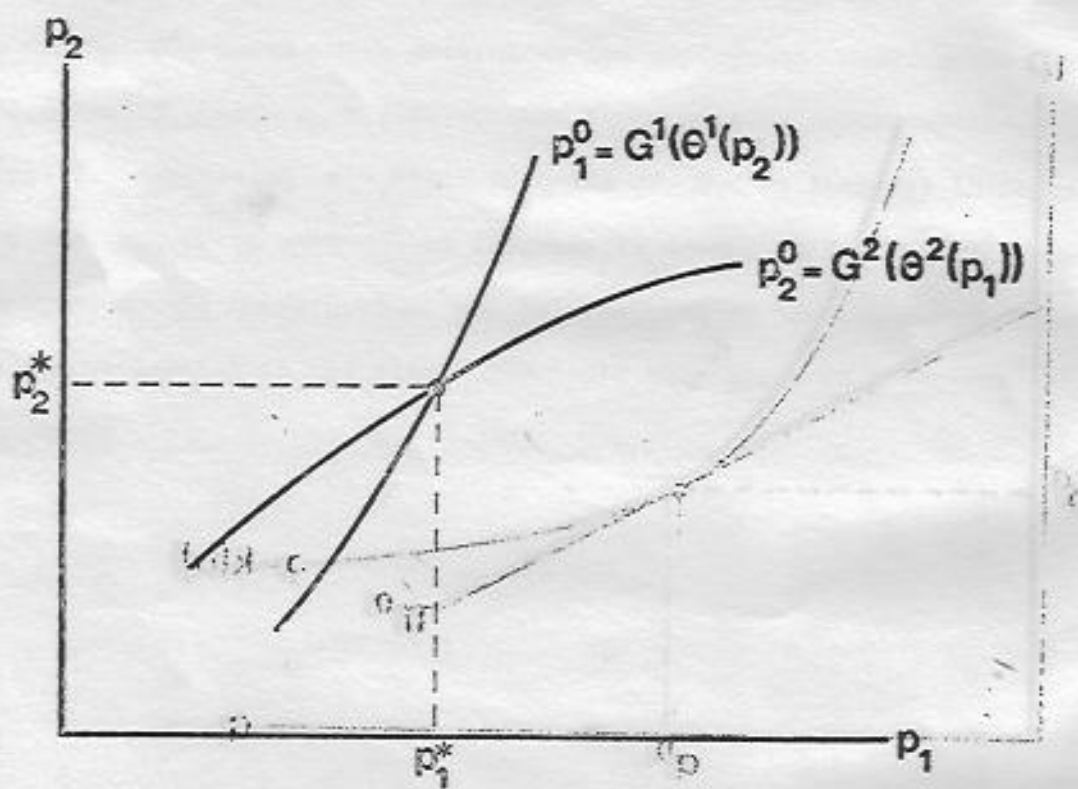


Fig. 2

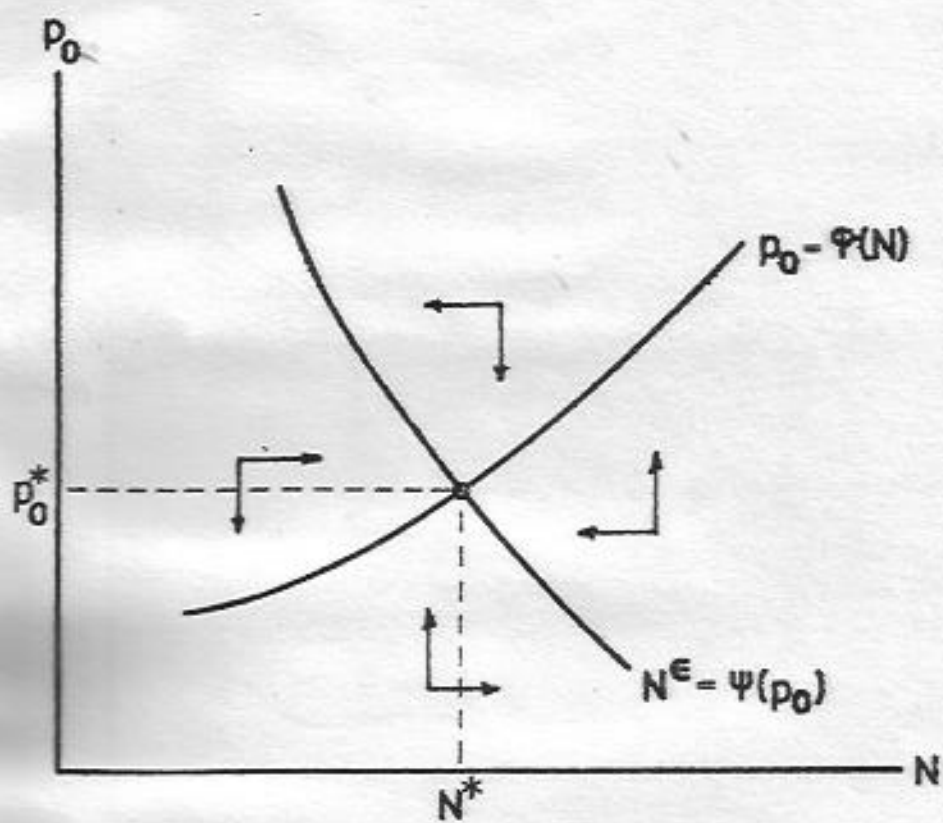


Fig. 3