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Incentives in Contracts for Public Sector Projects with Private Sector Participation

by

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Abstract

Optimal contracts are derived from a simple model where government guarantees two types of private investors participating in infrastructure projects. With asymmetric information, investors are offered a pair of incentive-compatible contracts covering production, tariff, and guarantee coverage. Both contracts offer identical production quantities, but the contract designed for high risk investors offers over-insurance and tariff below marginal cost, while the contract designed for low risk investors offers under-insurance with tariff above marginal cost. This benchmark outcome may motivate solutions to adverse selection, incentive and risk-sharing problems in contracts involving private sector participation in infrastructure development projects in the Philippines.

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This is a much-revised version of the earlier UPSE Discussion Paper 2002-05, "The Design of Asymmetric Information Contracts for Public Sector Projects with Private Sector Participation". The major changes from the earlier DP are:

a) a reversal in the earlier result that higher risks get partial guarantees;
b) new results on price/tariff-setting; and
c) new graphs depicting the symmetric and asymmetric information results.

I. Introduction

The introduction of private sector participation (PSP) in infrastructure development in the Philippines in the 1990's has led to palpable improvements in the quality and quantity of services in transportation and utilities. Initially developed to address crippling power shortages in the 1980's, templates for PSP in infrastructure involve a variety of modalities for private investor-contractors to build and/or own infrastructure facilities. In the Philippines, these templates are better known by the acronym, "BOT" (Build-Operate-Transfer, referring to the most common template: modalities for eventual transfer of ownership to government of an infrastructure facility built by private investor-contractors, usually after a period in which the investor has earned a sufficient return from operating it). Since their inception, BOT projects have posed great challenges to the Philippine government because they government to enter into complex contractual relationships with investors, contractors and creditors with often conflicting interests and greater familiarity with project finance.

Dealing with firms better-versed with structured finance for infrastructure projects has made it imperative for the Philippine government to enhance its capacity to screen applicant-investors and its capacity to structure BOT contracts that offer internationally competitive terms, investor protection and rewards, while at the same time giving proper incentives for optimal effort to be exerted. Unfortunately, this capacity has not developed at a pace that adequately addresses the need for good incentives and the increasingly complex requirements of investor-contractors, their shareholders and creditors.

Shortcomings in screening and contracting private firms are not benign. Standard government guarantee packages in BOT contracts are subject to the same moral hazard and adverse selection problems inherent in any insurance transaction. Improper structuring of incentives within contracts may lead to adverse selection, attracting a risky and/or inefficient mix of investors. Insufficient monitoring capacity and lax reporting requirements give rise to more unwanted incentive and moral hazard problems.

Unfortunately, the government has not found adequate mechanisms for properly "incentivizing" contracts. It has been suggested that the absence of adequate incentive mechanisms in contracts has aggravated the country's fiscal problems. Specifically, contingent liabilities, arising from generous provision of government guarantees in BOT contracts have been blamed for severely disrupting the timing of government cash flows. The Philippine experience suggests that the unintended selection of risky investor-contractors contributes to the frequency and severity of losses and therefore to the frequency and size of claims on the Treasury.

Thus, one of the central issues in the debate regarding government guarantee policy has been how to address the concomitant problems of adverse selection and moral hazard. Adverse selection in BOT projects may arise because the government may not be sufficiently informed about the capabilities of the private investors it contracts, resulting in sub-optimal risk-sharing. There are several explanations for this:

a) most of the investors are foreign; so government has encountered them for the first time;
b) government lacks familiarity with the market and industry altogether, so it may have insufficient basis for distinguishing efficient from inefficient contractors; and
c) government does not charge risk-adjusted guarantee premiums.

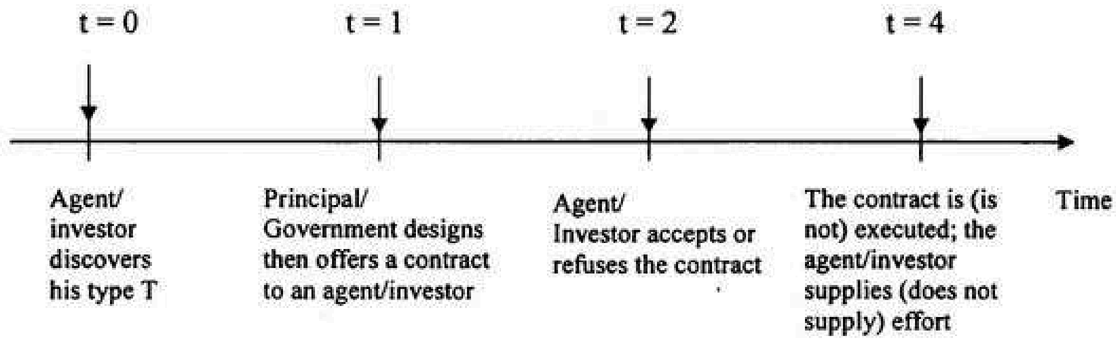
Although the government has considered charging risk-adjusted premiums in the past, doing so in practice has proven difficult, because of concerns it may deter investment. *Thus, the government has decided it will not levy risk-adjusted guarantee premiums, complicating the search for incentive-compatible BOT contracts. The government is presently more inclined to provide partial guarantees, but in an ad hoc manner, where the level of guarantee coverage is not the outcome of an optimization process.*

A related issue to designing contract incentives is determining how best to price infrastructure services delivered by private investor-contractors, as well as ensuring the correctness of the signals the allowed prices/tariffs convey to investor-contractors. Poorly designed price/tariff mechanisms may also create incentive problems in BOT contracts. However, tariff structures in BOT contracts are normally not used as tools to manipulate incentives in contracts, because they traditionally serve some other purpose. Prior to being privatized, prices of most infrastructure services are strictly regulated, usually pegged at below-market rates (to the detriment of the state-owned enterprises delivering them). Since the introduction of PSP in infrastructure development, however, the Philippine government has been compelled to create a more investor-friendly environment, implementing looser and more market-oriented pricing policies, emphasizing policies that make investor and bank credit to firms more sustainable. All this has come at high political cost, however, as customers otherwise accustomed to paying low, non-market tariffs have struggled to deal with higher (but more financially sustainable) tolls, water and electricity rates. At times, the Philippine government has had difficulty in justifying to the public the higher prices for services, and this struggle has been aggravated on several occasions where delivery by private contractors has been poor, or where quality has not discernibly improved over the pre-BOT quality. *An important issue then, is whether tariff structures exist which create better incentives for good performance of investor-contractors in projects. Can such tariff structures be combined with government guarantees to produce optimal incentives in contracts and the best investor mix given asymmetric information?*

In this study, we seek to find optimizing roles for production quantities, guarantees and tariffs to play in "incentivizing" project contracts. The previous paragraphs suggest that policies which balance interests among government and investors, should help create a better environment for infrastructure investment overall. For this to succeed, the approach should address the need for government to distinguish low from high risk investors, to allow them to exploit this information in order to minimize their risk and contingent liabilities, while at the same time giving proper incentives to low risk investors. Due to the demands for sustainable credit and sources of finance, BOT contract negotiations usually center around or emphasize the level of guarantee coverage by government. This invariably leads to government's assumption of an inordinate amount of risk. This study suggests that a better strategy for government during the contracting process is to utilize other variables within its control (prices), in order to create more incentive-compatible environments in projects.

Throughout the study, the following timing of events in the contracting process will be assumed to hold

Figure 1



Contracts are offered in the interim stage, so if there is asymmetric information between the contracting parties, it is already present when the government makes the offer. To further simplify this benchmark model, we have also assumed that the process above is a static, one-shot game. Extension into a dynamic setting is left for further study. Note that the events in Figure 1 preclude cases where applicant investors bid for project contracts, such as in the case of concessions. In the Philippines, therefore, the timing of events more closely resembles the process of awarding contracts in projects solicited by government.[1]

II. Adverse Selection: The Case of Symmetric Information

Consider the case of a risk-neutral government that offers contracts to risk-averse investors for the production and delivery of infrastructure services to a risk-averse public constituency. The contract specifies a project to be undertaken by the investor with payment based on the delivery of an observable output. The project to be undertaken, such as a toll road, a power generating plant, or an urban rail transit facility, creates important infrastructure services for the public, which pays for these services at tariffs regulated by the government. The project is inherently risky, since the value of the investor's revenue and/or expenditure flows are uncertain. However, the investor has a minimum threshold for income flows received, so that he will not participate in the project unless the flows he receives are higher than this threshold.

Assume that the project to be undertaken by the investor is so vital that the government offers to insure, or guarantee all or part of the flow of income received by the investor. If the government chooses to guarantee the entire flow of income, it offers the

[1] The majority of BOT projects in the Philippines are solicited by government.

investor a *full guarantee*. If the guarantee is only on a portion of the flow, it is a *partial guarantee*. Also assume that government does not charge guarantee premiums.

There are two states of nature, *good* and *bad*. Assume that these states are verifiable and project outcomes under each state are observable to the government. In the bad state, all or portion L of the value of the investor's flow of income is lost. But if a government guarantee is built into the contract, the investor can call on this guarantee and claim the amount q from the government, the value of the guarantee coverage. In the good state, the flow is not lost and the guarantee is never called. π is the probability that the bad state will occur. Of course, $0 < \pi < 1$. For a full guarantee, q = L : the entire loss is covered. For a partial guarantee, q < L : only a portion is covered. Note that q, the guarantee coverage of the investor, *is a contingent liability of the government* since it is to be paid by the Treasury to the investor contingent upon the realization of the state which the contract states can trigger a claim (and such a remedial action is so stipulated).

The outcome of the project for the investor can be described by a binomial distribution:

Table 1: Probability distribution of investor outcomes

State of Nature	Probability	Outcome
Good (No Loss)	$1 - \pi$	B [pQ – cQ – F]
Bad (Loss)	π	B [pQ – cQ – F – L + q]

The investor's utility is described by the function B [.]. Because the investor is assumed to be risk averse, his utility function is concave: B ' [.] > 0 and B '' [.] < 0. The argument within the utility function is the flow of income (net revenues, wealth or endowment) received by the investor across states. The quantity of goods produced by the investor is Q, sold to the public at the government-regulated per-unit tariff p.[2] Revenues equal pQ. In producing each unit of Q, the investor incurs a constant variable cost *c*. A fixed cost of production, *F*, is also incurred. Think of the variables c and F in terms of variable and fixed costs commonly incurred in the process of constructing and operating infrastructure facilities (see Table 2).

Table 2: Fixed and variable costs of construction and operation of infrastructure facilities

Type of Infrastructure Facility	Fixed cost of production, F	Variable cost of production, c
Toll roads	Cost of toll booths	Maintenance costs, cost of asphalt or concrete, labor
Power plants	Cost of constructing the power plant	Maintenance costs, cost of fuel inputs and wires, labor
Water supply	Cost of constructing	Maintenance costs, cost of

[2] In practice, most infrastructure projects with PSP do involve the production of goods and services at regulated tariffs. It is very common for road tolls, electricity and water tariffs to be sold at regulated tariff rates.

	pumping stations	water treatment and/or sewerage, pumping, pipes, electricity consumed in pumping water, labor
Urban rail (MRT-III)	Cost of rolling stock, cost of constructing train stations and laying train tracks	Maintenance costs, cost of fuel and/or electricity, labor

It is assumed that both parties make reasonable estimates of the project's return before the contract is signed, so that both *c* and *F* are planned costs. In the good state, the project proceeds with all costs being realized according to plan. In the bad state, the project loses the flow of income L due to unexpected increases in costs, due to delays or cost overruns. Calls or claims on contractual guarantees are assumed to be triggered by shortfalls of cash flow or income to the investor. This in turn triggers an outflow q from the national treasury to the investor. By restricting the loss L to a flow, the scope of work is restricted to the case where a flow of revenues or costs (or net revenues) accruing to the agent (investor) is at risk of being lost.

The Philippine experience with BOT projects suggests in fact that the loss of flows is most closely associated with delays, cost overruns, and other situations where unexpected reductions in revenues occur, leading to insufficient cash flow for amortizing a given stock of debt falling due. The loss of flows could also be broadly consistent with any situation where the government assumes the investor's (contractor's) payment for liabilities to sub-contractors (investor income or cash flow shortfall leads to default -- the trigger event -- which leads to outflows from the treasury). [3] The information in Table 2 suggests that cost overruns are most often associated with risky fixed costs (F).

The outcome for the public can also be described by a binomial distribution:

Table 3: Probability distribution of public outcomes

State of Nature	Probability	Outcome
Good (No Loss)	$1 - \pi$	$U(Q) - pQ$
Bad (Loss)	π	$U(Q) - pQ - q$

Since the public is risk averse, its (aggregate) utility function is concave: $U'[\,.\,] > 0$ and $U''[\,.\,] < 0$. It is assumed that in the bad state, the government ultimately passes onto the public the cost of claims made by investors, q through taxation. Thus, the public's utility is reduced by q in the bad state. We also assume for simplicity that all arguments outside the function U'( ) are expressed in (or converted into) utils of social welfare. That is, pQ in the good state and pQ - q in the bad state are multiplied by the variable v, where v is the social cost of public funds and $v = 1$.

[3] This is in fact, similar to what occurred in 2000 when the Philippines' MRT-3 project called on a government guarantee. Due to various cost overruns, the MRT-3 consortium of private investors required additional cash infusions to finance payments to their contractors (Reside, 2000 and Reside, 2001).

The contract between government and investor includes the terms of the guarantee and tasks to be carried out by the investor. It contains three parameters:

a) Q, the quantity of the infrastructure good or service produced and purchased by the government;
b) p, the per-unit tariff of good or service sold; and
c) q, the value of the guarantee coverage.

It is assumed that the government, when offering the contract to the investor, chooses the values of these parameters. *The question, therefore, is to find incentive-compatible contracts that maximize social welfare and mitigate agency problems, risk and contingent liabilities of government.*

Suppose there are two types of investors of varying competence, low risk (W) and high risk (H). They have the same planned variable and fixed costs for given project. The low risk investor is distinguished from the high risk investor by having a lower probability of failure in a project (or, a lower probability of calling on a guarantee, perhaps because its management is more competent and experience fewer delays and cost overruns). That is, $\pi^{H} > \pi^{W}$. We also assume that $\pi^{W} < \pi^{H} < \pi^{GOVERNMENT}$: government has a higher probability of failure than the riskiest private investor and government observes this, so it solicits private sector investors to undertake the project.

Under conditions of symmetric information, the government has complete information about the characteristics of the investor relevant to the task at hand. The government is able to observe investor type. If T is an index for investor type, the government's objective function is assumed to be the expected value of social welfare:

$$(1-\pi^{T})\left[U\left(Q^{T}\right)-p^{T}Q^{T}\right]+\pi^{T}\left[U\left(Q^{T}\right)-p^{T}Q^{T}-q^{T}\right] \qquad \text{for } T=H, W \tag{1}$$

For simplicity, social welfare is separable in all of its arguments. Because government has the ability to observe the type of each investor that applies for a contract and guarantee, it performs the maximization problem separately for each investor. *Also note that social welfare is reduced by two factors: total payments to the investor, pQ, as well as payment of claims on government guarantees, q.*

If the government were contracting with a type T investor (T = H, W) under symmetric information, the government would maximize social welfare subject to two inequality constraints:

$$(1-\pi^{T})\left[U\left(Q^{T}\right)-p^{T}Q^{T}\right]+\pi^{T}\left[U\left(Q^{T}\right)-p^{T}Q^{T}-q^{T}\right]\geq \underline{R}^{G} \tag{2}$$

$$(1-\pi^{T})B\left[p^{T}Q^{T}-cQ^{T}-F\right]+\pi^{T}B\left[p^{T}Q^{T}-cQ^{T}-F+L-q^{T}\right]\geq \underline{R} \tag{3}$$

The first constraint is government's participation constraint. The expected utility derived from purchasing the good or service must exceed the government's reservation level expected utility, $\underline{R}^G$. Think of $\underline{R}^G$ as the level of expected social welfare the public could achieve if government had undertaken the project itself (instead of contracting the services of a private investor). Thus, the government will only entertain offers from competent enough private contractors *(albeit differing in degree of risk)* who will provide *at least as high* a level of expected social welfare as the government could provide by itself. This is the rationale for privatizing the provision of infrastructure goods and services in the first place. The second constraint is the investor's participation constraint. For the investor to agree to undertake the project, his expected utility must exceed his reservation expected utility level ($\underline{R}$). Think about the reservation expected utility $\underline{R}$, for example, to be the level of utility associated with a level of income sufficient to satisfy the investor's required rate of return on the project.

Given these assumptions, the government faces the following problem of maximizing social welfare subject to the two constraints.

$$\max_{(Q^T, p^T, q^T)} \quad (1-\pi^T)\left[U(Q^T) - p^T Q^T\right] + \pi^T\left[U(Q^T) - p^T Q^T - q^T\right]$$

subject to

$$(1-\pi^T)\left[U(Q^T) - p^T Q^T\right] + \pi^T\left[U(Q^T) - p^T Q^T - q^T\right] \geq \underline{R}^G$$

$$(1-\pi^T)B\left[p^T Q^T - cQ^T - F\right] + \pi^T B\left[p^T Q^T - cQ^T - F - L + q^T\right] \geq \underline{R} \qquad (4)$$

The problem can be solved using conventional Kuhn-Tucker conditions. The first derivatives of the lagrangean with respect to Q^T, p^T, and q^T give rise to the following Kuhn-Tucker conditions:

$$\frac{\partial L}{\partial Q^T} = (1+\lambda)\left[U'(Q^T) - p^T\right] + (p^T - c)\gamma\begin{Bmatrix}(1-\pi^T)B'\left[p^T Q^T - cQ^T - F\right] \\ +\pi^T B'\left[p^T Q^T - cQ^T - F - L + q^T\right]\end{Bmatrix} = 0 \qquad (5)$$

$$\frac{\partial L}{\partial p^T} = -(1+\lambda) + \gamma\begin{Bmatrix}(1-\pi^T)B'\left[p^T Q^T - cQ^T - F\right] \\ +\pi^T B'\left[p^T Q^T - cQ^T - F - L + q^T\right]\end{Bmatrix} = 0 \qquad (6)$$

$$\frac{\partial L}{\partial q^T} = -(1+\lambda) + \gamma B'\left[p^T Q^T - cQ^T - F - L + q^T\right] = 0 \qquad (7)$$

also, $\gamma \geq 0$ and $\lambda \geq 0$.

Equations (6) and (7) give rise to the condition for efficient production:

$$U'(Q^{T}) = c \tag{8}$$

Marginal utility must equal the marginal cost of production.

Proposition 1a: In the case of symmetric information, the optimal guarantee is a full guarantee for each investor type ($q^T = q^H = q^W = L$).

Proposition 1b: In the case of symmetric information, the optimal price/tariff equals marginal cost, c for each investor type: $U'(Q^T) = c = p^T = p^H = p^W$.

Proof of Proposition 1a:

Equations (7) and (8) yield the condition that marginal utility in the bad state for each type of investor equals the expected value of marginal utility across states:

$$B'\left[p^{T}Q^{T} - cQ^{T} - F - L + q^{T}\right] =$$

$$\left\{(1-\pi^{T})B'\left[p^{T}Q^{T} - cQ^{T} - F\right] + \pi^{T}B'\left[p^{T}Q^{T} - cQ^{T} - F - L + q^{T}\right]\right\} \tag{9}$$

This condition will *only hold if* $L = q^T$. Q.E.D.

Proof of Proposition 1b:

Because B'( ) > 0 by assumption and the fact that $0 < \pi^T < 1$, then from equation (6), we know that the term $[(1-\pi^T)\,B'(p^T Q^T - cQ^T - F) + \pi^T B'(p^T Q^T - cQ^T - F - L - q^T)] > 0$. Since $\lambda \geq 0$ and $-(1+\lambda) < 0$, it follows that $\gamma > 0$. Now, from (5), $(1+\lambda) > 0$, $\gamma > 0$ and $[(1-\pi^T)\,B'(p^T Q^T - cQ^T - F) + \pi^T B'(p^T Q^T - cQ^T - F - L - q^T)] > 0$. Thus, the only way for the left hand side of equation (5) to be equal to zero is if p^T = marginal cost (= c) = $U'(Q^T)$. Q.E.D.

Thus, in the case of symmetric information, the optimal action for the government is to set output at a level that achieves efficiency in production. In this regard, it sets prices/tariffs at a competitive level, equal to marginal cost.

What are the characteristics of the set of optimal contracts $\{(Q^W, p^W, q^W), (Q^H, p^H, q^H)\}$ under symmetric information? Since marginal cost c is assumed to be the same for all investors, it turns out that $Q = Q^W = Q^H$: optimal quantity is the same across investor types. Because we assume that there is only one possible value of the loss L, we also have $q^W = q^H = q = L$. Each investor type gets the same level and class of guarantee (full guarantee to the extent of L).[4]

Because Q^T and q^T are the same, it follows that prices p^T are also the same for each investor type: $c = p = p^W = p^H$. Therefore, the *same* contract is offered to (and chosen by) both low *and* high risk investors. The solution is a *pooling equilibrium*. But this creates incentive problems when asymmetric information prevails. Although the low risk investors have a lower probability of failure, or calling on the guarantee, government offers identical contracts to both investor types. Because guarantees for both low and high risk investors are the same, high risk investors have a greater incentive to participate in the project than the low risk ones do. Low risk investors, aware of their own characteristics, know they are better than high risks, but they cannot obtain a better contract from the government. Because the symmetric information contract treats high and low risk investors exactly the same way, it follows that when the government is unable to observe investor type, the pooling equilibrium contract is sub-optimal.

III. Adverse Selection: The Case of Asymmetric Information

a. The Model

In case of asymmetric information, again assume that there are two types of investors: low risk and high risk. The latter have a higher probability π^H of incurring a given loss L than the former ($\pi^W < \pi^H$). Suppose the proportion of low risk investors among all investors is x, where $0 < x < 1$. The government can observe the proportion x and the loss probabilities (π^H and π^W), but cannot completely observe the exact type of each investor/applicant that approaches it and exhibits interest in undertaking a given project. Low and high risk investors differ only in their loss probabilities (both investor types have the same c and F). Assume that the functional form of the utility function is the same across investor types. The government's problem is to maximize social welfare:

[4] An alternative symmetric information solution for the government would be to ignore high risk applicants and wait to offer contracts only to select low risk investors, but that is not possible as we have assumed a static game.

$$\underset{\{Q^W,p^W,q^W\}\{Q^H,p^H,q^H\}}{\text{Max}} \quad (1-x)\{(1-\pi^W)[U(Q^W)-p^WQ^W]+\pi^W[U(Q^W)-p^WQ^W-q^W]\}+$$

$$x\{(1-\pi^H)[U(Q^H)-p^HQ^H]+\pi^H[U(Q^H)-p^HQ^H-q^H]\} \tag{10}$$

subject to

$$(1-\pi^W)[U(Q^W)-p^WQ^W]+\pi^W[U(Q^W)-p^WQ^W-q^W] \geq \underline{R}^G \tag{11}$$

$$(1-\pi^H)[U(Q^H)-p^HQ^H]+\pi^H[U(Q^H)-p^HQ^H-q^H] \geq \underline{R}^G \tag{12}$$

$$(1-\pi^W)B[p^WQ^W-cQ^W-F]+\pi^WB[p^WQ^W-cQ^W-F-L+q^W] \geq \underline{R} \tag{13}$$

$$(1-\pi^H)B[p^HQ^H-cQ^H-F]+\pi^HB[p^HQ^H-cQ^H-F-L+q^H] \geq \underline{R} \tag{14}$$

$$\begin{aligned}(1-\pi^W)B[p^WQ^W-cQ^W-F]+\pi^WB[p^WQ^W-cQ^W-F-L+q^W] \geq \\ (1-\pi^W)B[p^HQ^H-cQ^H-F]+\pi^WB[p^HQ^H-cQ^H-F-L+q^H]\end{aligned} \tag{15}$$

$$\begin{aligned}(1-\pi^H)B[p^HQ^H-cQ^H-F]+\pi^HB[p^HQ^H-cQ^H-F-L+q^H] \geq \\ (1-\pi^H)B[p^WQ^W-cQ^W-F]+\pi^HB[p^WQ^W-cQ^W-F-L+q^W]\end{aligned} \tag{16}$$

The first two constraints are the participation constraints of government. The public must accept the level of expected social welfare under each investor type (otherwise, the government will do the project, but it has the highest probability of failure). The third and fourth constraints are the participation constraints of each investor. Each investor should receive an expected utility greater than or equal to the reservation utility (assumed to be the same). The last two are the self-selection constraints. The low risk investor will never select the contract intended for the high risk investor because the expected utility from the contract intended for him will be greater than or equal to the expected utility from the contract intended for the high risk investor. Similarly, the high risk investor will never select the contract intended for the low risk investor because the expected utility from the contract intended for him will be greater than or equal to the expected utility from the contract intended for the low risk investor.

Note that the asymmetric information approach to the government's optimization problem is very different from the symmetric information approach. Instead of running separate maximization problems for each investor type, the government performs the maximization problem only once for both investor types. The objective function includes the expected utilities (for consumer surplus) for both investor types.

It turns out that in this model, only a subset of combinations of Q, p, and q will ensure that each investor type will have the incentive to participate and self-select in the project. In other words, the incentive-compatibility, or self-selection constraints for both types of agents (16) and (17) will only hold for a subset of combinations of Q, p, and q. Also, the participation constraints for both investor types will not be binding. For some range of values of Q, p, and q, one or both types of investors will not participate in the project.

The complete Lagrangean for the asymmetric information case is spelled out in Appendix A. The following is a summary of the first-order conditions of the lagrangean with respect to the contract variables Q^H, p^H, q^H, Q^W, p^W, and q^W

$$\frac{\partial L}{\partial Q^W} = (1-x+\lambda)\{[U'(Q^W)-p^W]\}+(p^W-c)\{(1-x+\alpha+\theta)A-\eta D\}=0 \tag{17}$$

$$\frac{\partial L}{\partial p^W} = -(1-x+\lambda)+\{(1-x+\alpha+\theta)A-\eta D\}=0 \tag{18}$$

$$\frac{\partial L}{\partial q^W} = -(1-x+\lambda)+\left\{(1-x+\alpha+\theta)C-\eta\left(\frac{\pi^H}{\pi^W}\right)C\right\}=0 \tag{19}$$

$$\frac{\partial L}{\partial Q^H} = (x+\delta)\{[U'(Q^H)-p^H]\}+(p^H-c)\{(x+\gamma+\eta)J-\alpha K\}=0 \tag{20}$$

$$\frac{\partial L}{\partial p^H} = -(x+\delta)+\{(x+\gamma+\eta)J-\alpha K\}=0 \tag{21}$$

$$\frac{\partial L}{\partial q^H} = -(x+\delta)+\left\{(x+\gamma+\eta)N-\alpha\left(\frac{\pi^W}{\pi^H}\right)N\right\}=0 \tag{22}$$

where

$$A=\{(1-\pi^W)B'[p^WQ^W-cQ^W-F]+\pi^W B'[p^WQ^W-cQ^W-F-L+q^W]\} \tag{23}$$

$$D=\{(1-\pi^H)B'[p^WQ^W-cQ^W-F]+\pi^H B'[p^WQ^W-cQ^W-F-L+q^W]\} \tag{24}$$

$$C=B'[p^WQ^W-cQ^W-F-L+q^W] \tag{25}$$

$$J = \{(1-\pi^H)B'[p^H Q^H - cQ^H - F] + \pi^H B'[p^H Q^H - cQ^H - F - L + q^H]\} \tag{26}$$

$$K = \{(1-\pi^W)B'[p^H Q^H - cQ^H - F] + \pi^W B'[p^H Q^H - cQ^H - F - L + q^H]\} \tag{27}$$

$$N = B'[p^H Q^H - cQ^H - F - L + q^H] \tag{28}$$

The final Kuhn-Tucker conditions are the inequality constraints, equations (11) to (16), along with $\lambda \geq 0$, $\delta \geq 0$, $\theta \geq 0$, $\gamma \geq 0$, $\alpha \geq 0$, and $\eta \geq 0$ for the lagrange multipliers.

We can now proceed to determine the optimal values for Q^W, p^W, q^W, Q^H, p^H, and q^H. The general plan for the succeeding parts of the paper are as follows:

1) Show that optimal Q is identical for both agent types;

2) Show that since $Q^W = Q^H$, the self-selection constraints imply that $q^W < L < q^H$ and $p^H < c < p^W$ in equilibrium; and

3) Further refine the result in (2) by qualifying the range of values q and p can take for each investor type.

4) Provide graphical intuition for the result.

b. Optimal Production

Proposition 2: The contracts offered by government to (and accepted by) both low and high risk investors specify the same quantity of the good or service to be produced.

Proof:

Combine (17) and (18) and we derive the condition for efficiency in production for the low risk agent:

$$U'(Q^W) = c \tag{29}$$

Combine (20) and (21) and we derive the condition for efficiency in production for the high risk agent:

$$U'(Q^H) = c \tag{30}$$

Thus, $Q^W = Q^H = Q$. Q.E.D.

c. Optimal Guarantee Coverage and Pricing of Infrastructure Services

Proposition 3a: The self-selection constraints will force the government to discriminate in equilibrium between low and high risk investors by offering differing guarantee coverage. It more than fully guarantees the high risk investor: $q^H > L$, while it less than fully guarantees the low risk investor: $q^W < L$. Thus, $q^W < L < q^H$.

Proposition 3b: With respect to tariffs, the regulated tariff offered to and accepted by the high risk investor is lower than the regulated tariff offered to and accepted by the low risk investor. $p^H < p^W$.

Proof:

Since the self-selection constraints must be satisfied in equilibrium, it follows that equilibrium will also be characterized by the outcomes in Proposition 3a and 3b. We can show that Proposition 3a and 3b hold by taking (15) and (16):

$$(1-\pi^W)B\left[p^W Q^W - cQ^W - F\right] + \pi^W B\left[p^W Q^W - cQ^W - F - L + q^W\right] \geq \\ (1-\pi^W)B\left[p^H Q^H - cQ^H - F\right] + \pi^W B\left[p^H Q^H - cQ^H - F - L + q^H\right] \qquad (15)$$

$$(1-\pi^H)B\left[p^H Q^H - cQ^H - F\right] + \pi^H B\left[p^H Q^H - cQ^H - F - L + q^H\right] \geq \\ (1-\pi^H)B\left[p^W Q^W - cQ^W - F\right] + \pi^H B\left[p^W Q^W - cQ^W - F - L + q^W\right] \qquad (16)$$

We can factor and rearrange the terms in (16) to get:

$$\left\{B\left[p^W Q^W - cQ^W - F\right] - B\left[p^H Q^H - cQ^H - F\right]\right\}\left\{\frac{1-\pi^W}{\pi^W}\right\} \geq \\ \left\{B\left[p^H Q^H - cQ^H - F - L + q^H\right] - B\left[p^W Q^W - cQ^W - F - L + q^W\right]\right\} \qquad (31)$$

We can factor and rearrange the terms in (17) to get:

$$\left\{B\left[p^H Q^H - cQ^H - F\right] - B\left[p^L Q^L - cQ^L - F\right]\right\}\left\{\frac{1-\pi^H}{\pi^H}\right\} \geq \\ \left\{B\left[p^W Q^W - cQ^W - F - L + q^W\right] - B\left[p^H Q^H - cQ^H - F - L + q^H\right]\right\} \qquad (32)$$

Noting that the RHS of (31) is merely the negative of the RHS of (32), we can combine (31) and (32) and impose $Q^W = Q^H = Q$ at the optimum to get:

$$\{B[p^H Q - cQ - F] - B[p^W Q - cQ - F]\}\left\{\frac{1-\pi^H}{\pi^H}\right\} \geq$$

$$\{B[p^W Q - cQ - F - L + q^W] - B[p^H Q - cQ - F - L + q^H]\} \geq$$

$$\{B[p^H Q - cQ - F] - B[p^W Q - cQ - F]\}\left\{\frac{1-\pi^W}{\pi^W}\right\}$$

Now, since $\left\{\frac{1-\pi^H}{\pi^H}\right\} < \left\{\frac{1-\pi^W}{\pi^W}\right\}$, the above condition can only hold if $p^W > p^H$ and $q^W < q^H$ in equilibrium. Q.E.D.

Proposition 4: At the optimum, the marginal rate of substitution of income across states for the low risk investor is greater than the marginal rate of substitution of income across states for the high risk investor. This holds in the symmetric, as well as in the asymmetric information solutions to the model.

Proof: See Appendix B.

This is a standard result in the insurance literature (Stiglitz, 1977 and Stiglitz and Rothschild, 1976) that describes the nature of the separating equilibrium.

Proposition 5: At the optimum, $p^H <$ marginal cost (= c) $< p^W$.

Proof: See Appendix C.

Thus, the high risk investor is offered a contract where it is to charge a price lower than marginal cost, but the low risk investor is allowed to charge a price greater than marginal cost. This outcome ensures that low risk investors have an incentive to participate in the project.

A summary of the key results that hold in equilibrium are:

1) $Q^W = Q^H = Q$;
2) $Q^W < L < q^H$; and
3) $p^H <$ marginal cost (= c) $< p^W$.

The intuition behind these results is the fact that government needs to be able to attract both types by keeping both of them interested in participating in the project. The only way to do this is by over-insuring high risks and under-insuring low risks, but *rewarding low risks with higher revenues. This ensures that the low risks will not want to mimic the high risks. In order to ensure that high risks do not mimic low risks, government gives high risks a sufficiently high level of over-insurance (or limits the amount of additional revenues earned by low risks). Thus, the government presents a menu of two contracts to each investor that applies. Depending on its type, the investor either accepts one or the other.*

This gives rise to one final issue. Just how much additional revenues are required by the low risks to dissuade them from mimicking high risks? And just how much over-insurance is required by the high risks to dissuade them from mimicking low risks with higher revenues? The self-selection constraints will be violated if the values of q, p and Q for both agent types are not properly chosen by the principal. This leads to Proposition 5.

Proposition 5: At the optimum, the set of contracts $\{(Q^W, q^W, p^W), (Q^H, q^H, p^H)\}$ are such that the following sequence of inequalities must hold:

$$L-(p^W-p^H)Q < q^W < L < q^H < L+(p^W-p^H)Q \tag{34}$$

Proof:

For the proof, we use the following arguments. We now know that $q^W < q^H$, $p^W > p^H$ and $Q^W = Q^H = Q$. Note that the self-selection constraint (16) in the maximization problem implies that

$$\left(\frac{1-\pi^H}{\pi^H}\right) \geq \frac{B[p^W Q-cQ-F-L+q^W]-B[p^H Q-cQ-F-L+q^H]}{B[p^H Q-cQ-F]-B[p^W Q-cQ-F]} \tag{35}$$

Since $p^W > p^H$, the denominator is negative. But we know the LHS must be positive, so that it must be the case that in the numerator:

$$p^W Q+q^W > p^H Q+q^H \tag{36}$$

or

$$q^W > q^H-(p^W-p^H)Q$$

$$q^W+(p^W-p^H)Q > q^H \tag{37}$$

Now, since $q^H > L$, we have

$$L < q^H < q^W+(p^W-p^H)Q \tag{38}$$

So that

$$L < q^W + (p^W - p^H)Q$$

$$L - (p^W - p^H)Q < q^W \tag{39}$$

This is the lower bound for the low risk agent's guarantee. The upper bound of the guarantee is established by taking equation (36):

$$q^W > q^H - (p^W - p^H)Q \tag{40}$$

But since $q^W < L$, we have

$$L > q^H - (p^W - p^H)Q$$

$$q^H < L + (p^W - p^H)Q \tag{41}$$

This is the upper bound for the high risk's guarantee.

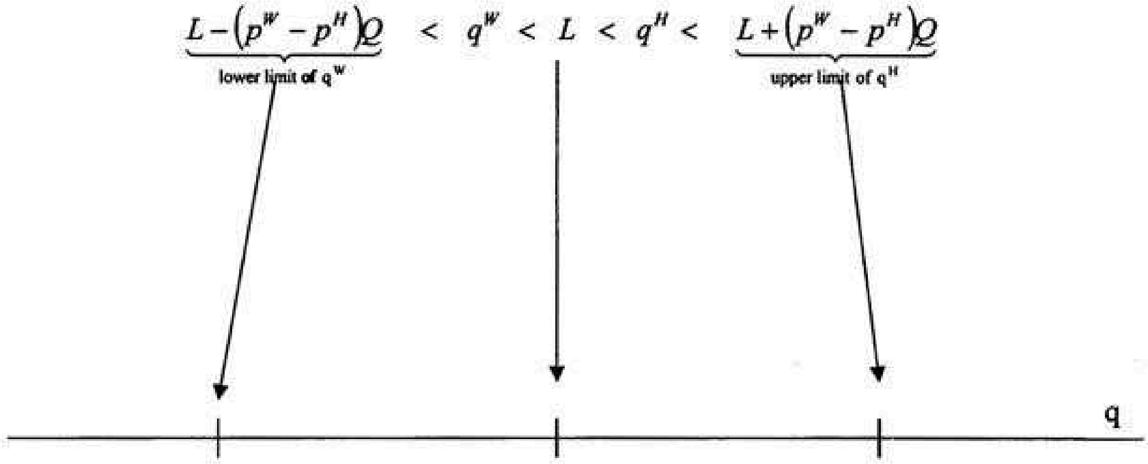
Under plausible assumptions, therefore, the amount of insurance or guarantee coverage is bounded from above and below

$$L - (p^W - p^H)Q < q^W < L < q^H < L + (p^W - p^H)Q \tag{42}$$

Q.E.D.

Note, therefore, that we end up with a pair of incentive-compatible contracts. Figure 3 illustrates the guarantee levels defined by the incentive-compatible contracts.

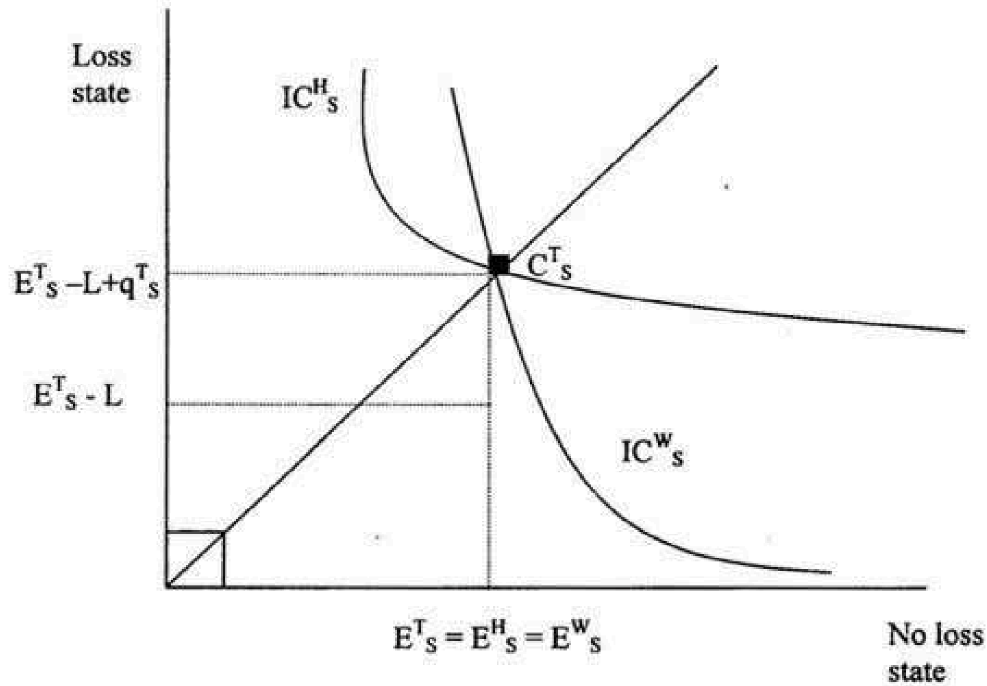
Figure 3



Region A:	Region B:	Region C:	Region D:
Guarantee and revenues are insufficient for low risk investor; adverse selection occurs	Guarantee and revenues are sufficient for low risk investor; no adverse selection occurs	Guarantee and revenues are sufficient for high risk investor; no adverse selection occurs	Guarantee and revenues are excessive for high risk investor; adverse selection occurs

The symmetric and asymmetric information solutions may be depicted graphically. The pooling equilibrium under symmetric information is depicted in Figure 4. The 45 degree line is the standard full-insurance line. Points below it mean that agents over-insure, while points under it mean agents under-insure. The indifference curve for the low risk agent is given by $IC^W{}_S$, which is steeper than the indifference curve for the high risk agent, $IC^H{}_S$ (Proposition 4). In equilibrium, the government can choose no loss (good state) endowments for both types, so on the horizontal axis, it gives the endowment $E^W{}_S$ to the low risks and $E^H{}_S$ to the high risks with $E^W{}_S = E^H{}_S = E^T{}_S = p^T{}_S Q^T{}_S - cQ^T{}_S - F$ and T = H,W. On the vertical (bad state) axis, the government fully compensates each type with $q^T{}_S$, so that the symmetric information pooling contracts $C^W{}_S$ and $C^H{}_S$ ($= C^T{}_S$) lie on the 45 degree line.

Figure 4: Pooling equilibrium with symmetric information



In order to depict the asymmetric information solution, its outcome must be analyzed in relation to the symmetric information outcome. Hence Proposition 6:

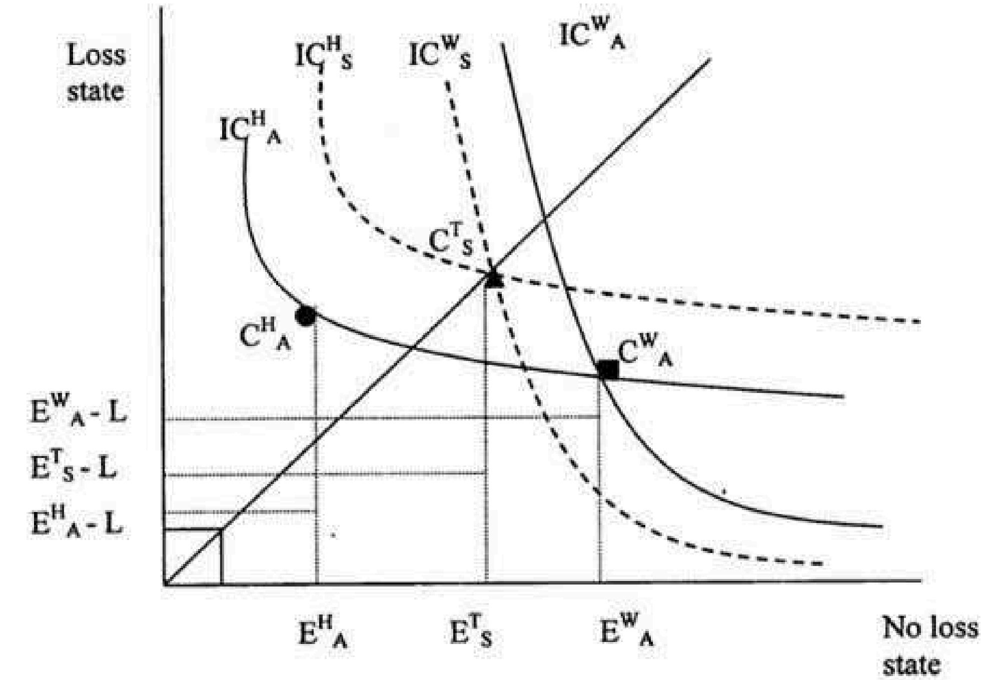
Proposition 6: The following inequalities hold in the good (no loss) state:

Endowment of the high risk investor under asymmetric information (= E^H_A) < endowment of each investor under symmetric information (= $E^H_S = E^W_S = E^T_S$) < endowment of the low risk investor under asymmetric information (= E^W_A)

The proof is straightforward. Since *c*, *F* and *Q* are equal across investor types under symmetric and asymmetric information, we only need to show that the price charged by the high risk investor under asymmetric information < price charged by each investor under symmetric information < price charged by the low risk investor under asymmetric information. But we have already proven this from Proposition 5. Q.E.D.

The asymmetric information solution can then be depicted graphically:

Figure 5: Separating equilibrium with asymmetric information



The pooling equilibrium described earlier is given by the broken curves IC^W_S and IC^H_S. The endowment of both agents is given by E^T_S on the horizontal axis. The full insurance contract is given by C^T_S on the 45 degree line where IC^W_S and IC^H_S intersect. Government implements the separating equilibrium under asymmetric information by giving endowment (E^H_A) to the high risk agent, lower than the endowment to the low risk agent (E^W_A). Proposition 6 is satisfied because $E^H_A < E^T_S < E^W_A$. Note that low risks under-insure (choosing contract C^W_A), so they are below the 45 degree line, while high risks over-insure (choosing contract C^H_A), so they are above the 45 degree line. MRS for low risks > MRS for high risks at the respective contracts, so that Proposition 4 is satisfied. Observe that the separating equilibrium solution dominates the pooling equilibrium when asymmetric information exists: government is better able to attract low risks (as they are at a higher indifference curve) and discourage high risks (as they are at a lower indifference curve). *The investor mix can improve.*

At the optimum, government chooses combinations of tariffs/price and levels of guarantee coverage as a mechanism for maximizing social welfare in the absence of a risk-adjusted guarantee premium. The government can afford to let the public pay higher tariffs to low risks, because they are partially insured, relieving the public from bearing high contingent liabilities (q is lower for low risks). Conversely, since the public must bear greater contingent liabilities in the case of high risks, government responds by compelling high risks to levy lower tariffs and earn lower revenues in the no-loss state.

The model above involves a principal that is a guarantor-insurer, agents who can only be of two types, and only two states of nature. Therefore, the structure of the simple maximization problem above is similar to that modeled in papers by Rothschild and Stiglitz (RS, 1976), and Stiglitz (S, 1977). Some of the properties of the solution in this study are similar to those found in these past studies. *In RS and S, low risks subsidize high risks by accepting less insurance. In this study, however, the subsidy is offset because low risks are rewarded with sufficiently higher no-loss endowments. This is how low risks can attain higher utility in the asymmetric information outcome.*

Furthermore:

1) In RS, and S, the principal is a risk-neutral monopolistic insurance company, while in this study, the principal is a risk-averse (a monopolistic guarantor) government;

2) In RS and S, the objective function is that of the agent (so it is the agent which performs the maximization of its expected utility), while in this study, the objective function is that of government (so the government performs the maximization of expected social welfare); and

3) In RS and S, the endowment or wealth of agents in each state of nature is given, while in this study, the government *has the capacity to determine* prices and quantities of the infrastructure good or service to be provided by the agent. Therefore, the government has the capacity to determine net revenues (endowment or wealth) for each type of agent under each state of nature (in addition to being able to determine q for each agent).

A more detailed comparison of the models is made below:

Table 4

Feature of the Model	Rothschild and Stiglitz (1976), Stiglitz (1977)	This study
Principal	Risk-neutral monopolist insurer (insurance firm)	Risk-neutral monopolist – insurer (government)
Whose objective function is optimized?	The agent's (profit function)	The principal's (social welfare function)
Agent's endowment in each state of nature	Given	Determined by principal
Outcome under symmetric information	Both agent types choose to fully-insure	Government chooses to offer a full guarantee to both agent types
Outcome for agent under asymmetric information	Agents self-select	Agents self-select

	High risk agent	Chooses a full insurance contract, signaling its type	High risk agent	Chooses the contract which over-insures him, but with lower price, signaling its type
	Low risk agent	Chooses a partial insurance contract, signaling its type	Low risk agent	Chooses the contract with a partial guarantee but higher tariff, signaling its type
Other implications of asymmetric information solution	Equilibria exist for which low risk agents choose not to insure at all. Riskier agent obtains full insurance from the principal (insurance company).		Government can eliminate contracts which induce low risks not to insure at all.	

IV. Conclusion

The solution to the problem in the paper is similar to those of standard problems where asymmetric information is present: a risk-sharing outcome eventually occurs between principal and agents. The contracts just derived are optimal from the point of view of maximizing social welfare: although the high risk investor knows it is more than fully insured, the per-unit output tariff it charges to the public is lower. Meanwhile, the low risk investor is only partially insured, so even though it sells output at a higher per-unit tariff, it must make an effort to reduce the likelihood of losses. The contracts are also optimal from the point of view of the agents. Low risk agents subsidize high risks by accepting lower guarantee coverage, but the subsidy is offset because high risks accept lower prices/tariffs. Thus, the incentives mechanism may be interpreted as one of offsetting cross-agent subsidies. In order to further insure that neither agent has an incentive to mimic the other's type, the levels of guarantee coverage are bounded from above (for the high risks) and below (for the low risks). Given the separating equilibrium and self-revelation by agents, the government can now implement measures to reduce exposure and risk to high risk agents, minimizing contingent liabilities.

The solution to the model suggests that it is not sufficient for government to merely pursue the present ad hoc strategy of offering partial guarantees as the sole *incentive mechanism in BOT projects.* The reason is that low levels of coverage may discourage all types from participating in projects (they may have no other incentive to participate). An additional variable – price, p – needs to be utilized as a tool to properly align incentives in contracts. The government usually treats issues of price independently of guarantees in existing contracts. *Tying price/tariff and guarantee coverage together could lead to the formulation of better incentives while at the same time addressing adverse selection and moral hazard problems.*

Optimal incentive contracts could help address adverse selection and moral hazard problems in contracts where government provides guarantees, but does not charge guarantee premia. An interesting aspect of the solution is that since combinations of tariffs and guarantee coverage are used to "incentivize" the contract, it does not necessarily reduce the contingent liabilities of government. High risks get over-insured while the low-risks get under-insured in equilibrium. Since the objective function is social welfare, the solution attempts to find the best combination of q and p to maximize it. *This does not necessarily mean that the principal will attempt to find the lowest q (i.e., the lowest contingent liability). The desirability of these contracts for the government does not hinge on the minimization of contingent liabilities, q (in fact, higher risks get higher q in the model). What is important is that government realizes that when maximizing social welfare, it has an alternative to minimizing q: it can offset the public's burden by mandating lower prices/tariffs for high risks.*

Also note that the existence of asymmetric information means that the government *will* have to settle for a certain mix of high and low risk contractors at the optimum. It cannot totally eliminate high risks because it cannot completely distinguish them from low risks. But at least, the government ensures that low risks are attracted, where their participation would not be certain under the current regime of ad hoc partial guarantees.

How can government implement policy recommendations in this study? In future contract negotiations, it may strengthen incentive structures by relating quantities, tariffs, and guarantee coverage during actual contract negotiations, the same way these variables are endogenized in this study. Increased recognition of their endogeneity should strengthen government's bargaining position. But this also requires an analysis of the relevant variables prior to the selection of bids from investors. This suggests that the applicability of the recommendations extends primarily to projects solicited by the government. By their nature, these are projects that are rationalized after some extensive studies are undertaken by government.

Without introducing any new mechanisms and variables for improved risk-sharing beyond those contained in existing contracts with PSP, and without charging risk-adjusted guarantee premia, this study demonstrates that it is possible optimal guarantee contracts to be crafted. This represents the initial study of its kind. More elaborate contract structures and variations on the benchmark model (e.g., different values for L, different cost structures, etc.) may be explored in future work.

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Appendix A

The complete Lagrangean is

$$
\begin{aligned}
L = {} & (1-x)\left\{(1-\pi^W)\left[U(Q^W)-p^W Q^W\right]+\pi^W\left[U(Q^W)-p^W Q^W-q^W\right]\right\} \\
& +x\left\{(1-\pi^H)\left[U(Q^H)-p^H Q^H\right]+\pi^H\left[U(Q^H)-p^H Q^H-q^H\right]\right\}+ \\
& \lambda\left\{(1-\pi^W)\left[U(Q^W)-p^W Q^W\right]+\pi^W\left[U(Q^W)-p^W Q^W-q^W\right]\right\}+ \\
& \delta\left\{(1-\pi^H)\left[U(Q^H)-p^H Q^H\right]+\pi^H\left[U(Q^H)-p^H Q^H-q^H\right]\right\}+ \\
& \theta\left\{(1-\pi^W)B\left[p^W Q^W-cQ^W-F\right]+\pi^W B\left[p^W Q^W-cQ^W-F-L+q^W\right]\right\}+ \\
& \gamma\left\{(1-\pi^H)B\left[p^H Q^H-cQ^H-F\right]+\pi^H B\left[p^H Q^H-cQ^H-F-L+q^H\right]\right\}+ \\
& \alpha\left\{\begin{array}{l}(1-\pi^W)B\left[p^W Q^W-cQ^W-F\right]+\pi^W B\left[p^W Q^W-cQ^W-F-L+q^W\right]- \\ \quad (1-\pi^W)B\left[p^H Q^H-cQ^H-F\right]+\pi^W B\left[p^H Q^H-cQ^H-F-L+q^H\right]\end{array}\right\}+ \\
& \eta\left\{\begin{array}{l}(1-\pi^H)B\left[p^H Q^H-cQ^H-F\right]+\pi^H B\left[p^H Q^H-cQ^H-F-L+q^H\right]- \\ \quad (1-\pi^H)B\left[p^W Q^W-cQ^W-F\right]+\pi^H B\left[p^W Q^W-cQ^W-F-L+q^W\right]\end{array}\right\}
\end{aligned}
\tag{A.1}
$$

where the variables λ, δ, θ, γ, α, and η are the lagrange multipliers for constraints (11) (16).

Appendix B

Proof of Proposition 4:

The proof under symmetric information is straightforward. Since Q, p and q are equal for both types under both states, the slopes of the indifference curves reduce to:

$\left(\frac{1-\pi^W}{\pi^W}\right)$ for low risks and $\left(\frac{1-\pi^H}{\pi^H}\right)$ for high risks. $\left(\frac{1-\pi^W}{\pi^W}\right) > \left(\frac{1-\pi^H}{\pi^H}\right)$.

Next, prove the asymmetric information case. Consider first-order condition (19), which can be rewritten as

$$\left\{(1-x+\alpha+\theta)-\eta\left(\frac{\pi^H}{\pi^W}\right)\right\}C=(1-x+\lambda) \quad \text{(B.1)}$$

Since we assume $0 < x < 1$ and $\lambda \geq 0$, it follows that the RHS of (B.1) is ≥ 0. Since $C > 0$, it must be that $\{1 - x + \alpha + \theta - \eta(\pi^H / \pi^W)\} > 0$. Since both $\eta \geq 0$ and $(\pi^H / \pi^W) > 0$, it must be true that $1 - x + \alpha + \theta > 0$.

Next, equate (18) and (19) and transpose terms:

$$\{(\underbrace{1-x+\alpha+\theta}_{>0})A-\eta D\}=\left\{(\underbrace{1-x+\alpha+\theta}_{>0})C-\eta\left(\frac{\pi^H}{\pi^W}\right)C\right\} \quad \text{(B.2a)}$$

$$(1-x+\alpha+\theta)(\mathrm{A}-\mathrm{C})=-\eta\left[\left(\frac{\pi^H}{\pi^W}\right)C-D\right] \quad \text{(B.2b)}$$

Since $1 - x + \alpha + \theta > 0$ and $\eta > 0$. Suppose we impose $q^W < L$. Then it follows that $A > C$ and $(\pi^H / \pi^W)\, C < D$.

When $A > C$,

$$\{(1-\pi^W)B'[p^W Q - cQ - F] + \pi^W B'[p^W Q - cQ - F - L + q^W]\}$$

$$> B'[p^W Q - cQ - F - L + q^W]$$

or

$$B'[p^W Q - cQ - F] > B'[p^W Q - cQ - F - L + q^W]$$

$$\frac{B'[p^W Q - cQ - F]}{B'[p^W Q - cQ - F - L + q^W]} > 1 \tag{B.3}$$

Next, we show that (π^H / π^W) C > D.

$$\left(\frac{\pi^H}{\pi^W}\right) B'[p^W Q - cQ - F - L + q^W]$$

$$< \{(1-\pi^H)B'[p^W Q - cQ - F] + \pi^H B'[p^W Q - cQ - F - L + q^W]\} \tag{B.4}$$

After manipulation, this becomes

$$\left(\frac{\frac{1-\pi^W}{\pi^W}}{\frac{1-\pi^H}{\pi^H}}\right) < \frac{B'[p^W Q - cQ - F]}{B'[p^W Q - cQ - F - L + q^W]} \tag{B.5}$$

Since the LHS is > 1, the only way for the RHS to be > 1 is if $q^W < L$ and this holds true in the model.

Next, show the implications of $q^H > L$. The proof is similar to the preceding analysis. Consider (22), which can be rewritten as

$$\left\{(x+\gamma+\eta) - \alpha\left(\frac{\pi^W}{\pi^H}\right)\right\} N = x + \delta \tag{B.6}$$

Since we assume $0 < x < 1$ and $\delta \geq 0$, it follows that the RHS of (B.6) is ≥ 0. Since $N > 0$, it must be that $\{ x + \gamma + \eta - \alpha(\pi^W / \pi^H)\} > 0$. Since both $\alpha \geq 0$ and $(\pi^W / \pi^H) > 0$, it must be true that $x + \gamma + \eta > 0$.

Next, equate (21) and (22) and transpose terms:

$$(x+\gamma+\eta)J - \alpha K = (x+\gamma+\eta)N - \alpha\left(\frac{\pi^W}{\pi^H}\right)N \tag{B.7a}$$

$$(x+\gamma+\eta)(J-N) = \alpha\left[K - \left(\frac{\pi^W}{\pi^H}\right)N\right] \tag{B.7b}$$

Since x + γ + η > 0 and α > 0, then if $q^H > L$, we have J < N and K < (π^W/π^H) N.

When J < N

$$\{(1-\pi^H)B'[p^HQ - cQ - F] + \pi^H B'[p^HQ - cQ - F - L + q^H]\}$$

$$< B'[p^HQ - cQ - F - L + q^H]$$

or

$$B'[p^HQ - cQ - F] < B'[p^HQ - cQ - F - L + q^H]$$

$$\frac{B'[p^HQ - cQ - F]}{B'[p^HQ - cQ - F - L + q^H]} < 1 \tag{B.8}$$

which will only hold if $q^H > L$.

Next, show that K < (π^W/π^H) N.

$$\left(\frac{\pi^W}{\pi^H}\right)B'[p^HQ - cQ - F - L + q^H]$$

$$> \{(1-\pi^W)B'[p^HQ - cQ - F] + \pi^W B'[p^HQ - cQ - F - L + q^H]\} \tag{B.9}$$

After some manipulation, this becomes

$$\left(\frac{\dfrac{1-\pi^W}{\pi^W}}{\dfrac{1-\pi^H}{\pi^H}}\right) < \frac{B'[p^HQ - cQ - F - L + q^H]}{B'[p^HQ - cQ - F]} \tag{B.10}$$

Since the LHS is > 1, the only way for the RHS to be > 1 is if $q^H > L$ but this is what we have in the model.

Note, however, that since $q^H > L$,

$$\left(\frac{\frac{1-\pi^W}{\pi^W}}{\frac{1-\pi^H}{\pi^H}}\right) > \frac{B'\left[p^H Q - cQ - F\right]}{B'\left[p^H Q - cQ - F - L + q^H\right]} \tag{B.11}$$

Thus,

$$\frac{B'\left[p^H Q - cQ - F\right]}{B'\left[p^H Q - cQ - F - L + q^H\right]} < 1 < \left(\frac{\frac{1-\pi^W}{\pi^W}}{\frac{1-\pi^H}{\pi^H}}\right) < \frac{B'\left[p^W Q - cQ - F\right]}{B'\left[p^W Q - cQ - F - L + q^W\right]} \tag{B.12}$$

Note that the slope of the indifference curve of the low risk is

$$\frac{B'\left[p^W Q - cQ - F\right]}{B'\left[p^W Q - cQ - F - L + q^W\right]}\left(\frac{1-\pi^W}{\pi^W}\right) \tag{B.13}$$

and the slope of the indifference curve of the high risk is

$$\frac{B'\left[p^H Q - cQ - F\right]}{B'\left[p^H Q - cQ - F - L + q^H\right]}\left(\frac{1-\pi^H}{\pi^H}\right) \tag{B.14}$$

Since $\pi^W < \pi^H$ and $q^W < L$ and $q^H > L$,

$$\frac{B'\left[p^W Q - cQ - F\right]}{B'\left[p^W Q - cQ - F - L + q^W\right]}\left(\frac{1-\pi^W}{\pi^W}\right) > \frac{B'\left[p^H Q - cQ - F\right]}{B'\left[p^H Q - cQ - F - L + q^H\right]}\left(\frac{1-\pi^H}{\pi^H}\right) \tag{B.15}$$

Under asymmetric information, the slope of the low risk's indifference curve is steeper than the slope of the high risk agent's indifference curve. Q.E.D.

Appendix C

Proof that p^H < marginal cost (= c) < p^W :

The proof comes in two parts. First, prove that $p^W > c$. From Appendix B,

$$(1-x+\alpha+\theta)A-\eta D>0 \quad \text{so that} \quad (1-x+\alpha+\theta)A>\eta D$$

Next, from (17) in the text,

$$\frac{\partial L}{\partial Q^W}=(1-x+\lambda)\left\{U'(Q^W)-p^W\right\}+(p^W-c)\left\{(1-x+\alpha+\theta)A-\eta D\right\}=0 \tag{17}$$

Since $0 < x < 1$ by assumption, and $\lambda \geq 0$, it follows that $1 - x + \lambda > 0$. Since $(1 - x + \alpha + \theta)A - \eta D > 0$ and $U'(Q^W) = c$, then either $p^W < c$ or $p^W > c$ holds. However, we can show that $p^W > c$ holds by showing that $p^H < c$ as it has already been established that $p^H < p^W$ from Proposition 3b.

Proof that $p^H < c$:

From Appendix B, we know that

$$(x+\gamma+\eta)J-\alpha K>0$$

Next, from equation (20):

$$\frac{\partial L}{\partial Q^H}=(x+\delta)\left\{U'(Q^H)-p^H\right\}+(p^H-c)\left\{(x+\gamma+\eta)J-\alpha K\right\}=0 \tag{20}$$

we know that $x + \delta > 0$ since $x > 0$ by assumption and $\delta \geq 0$. Also, $(x + \eta + \gamma)J - \alpha K > 0$ from the previous argument. Since $U'(Q^H) = c$, then either $p^H < c$ or $p^H > c$ holds.

Therefore:

1) either $p^W < c$ or $p^W > c$ holds; and
2) either $p^H < c$ or $p^H > c$ holds.

Given these options, the only possible solution consistent with the fact that $p^H < p^W$ from Proposition 3b, is that $p^H < c$ and $c < p^W$, so that $p^H < c < p^W$. Q.E.D.