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# SPECIFYING SEIGNIORAGE FOR TIME CONSISTENCY

by

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ABSTRACT

It is shown that the traditional specification of the revenue from money creation implies a retroactive inflation tax and leads to time inconsistency in monetary policy. The specification that leads to time consistency turns out to be Auernheimer's "honest government" seigniorage revenue.

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When what seems to be the best decision given the current situation would violate the optimal plan, we have a problem of time inconsistency.[1/] Calvo (1978) has called our attention to such a problem in monetary policy. He showed the government would repeatedly choose higher inflation rates than it would itself deem optimal from a different point in time even under perfect foresight. To this problem, he found no acceptable solution. We view the problem somewhat differently, however, and are therefore able to propose a solution.[2/]

We show time inconsistency arises from the use of the conventional specification of seigniorage. This specification, due most notably to Friedman (1971), is the rate of nominal money creation times the real money stock. We therefore propose an alternative specification, one derived by Auernheimer (1974) as "the honest government's revenue," which is the nominal interest rate times the real money stock.[3/] This alternative, however, has yet to gain wide acceptance in the literature, perhaps because it is thought to be based on a moral argument.[4/] The argument we provide--that it leads to time consistency--seems more compelling.

The conventional form of seigniorage implies government discretion over what is a retroactive tax, and this discretion is

at the heart of the problem.[5] It is not a problem, however, of the government misleading the public, for time inconsistency occurs with perfect foresight, or at least rational expectations. The nature of the problem is such that the harm the government inflicts, it inflicts on itself. The solution is cases like this, as Kydland and Prescott (1977) point out, is to institute a policy rule. Indeed the alternative form of seigniorage implies a simple money supply rule. But this is not Auernheimer's rule of preventing price jumps. We analyze, after all, a model in which all price increases are jumps.

First, we reconsider Auernheimer's analysis by introducing perfect foresight. Next, we present a discrete time model to show how the conventional form of seigniorage leads to time inconsistency. Then, we show how the alternative form solves the problem. Finally, we characterize the money supply rule that would implement the solution.

## 1. Auernheimer's honest government

Suppose, following Auernheimer, the real demand for money is a function of the nominal interest rate and can be represented by the curve LL in Fig. 1, with $r$ as the given real interest rate. At the outset the inflation rate is $\pi_0$ and is expected to stay there, so that the real money stock demanded is $m_0$. Suppose further there is no income growth so that the rate of nominal money creation

equals in inflation rate. In this case, the conventional measure of seigniorage is the inflation rate times the real money stock, or the area B + C in the figure.

Now suppose there is a sudden shift in money creation that raises in inflation rate to $\pi_1$. Assuming instantaneous adjustment, the real money stock will fall discretely to $m_1$. Seigniorage by conventional will then be A + B, and the policy change will bring the government an increment of A - C in seigniorage. But the adjustment here must be accommodated by a once-and-for-all rise in the price level. This price jump means a fall in the real value of money balances and therefore a capital loss to money holders. The amortized value of that loss Auernheimer shows to be the area E. In effect the government cheats money holders of that amount.

Auernheimer's honest government is obliged to prevent any jump in the price level by standing ready to redeem the excess real money stock. Because it effectively underwrites the capital loss, this government then gains from such a policy change only A - C - E in seigniorage. The point is that we obtain precisely the same result by simply measuring seigniorage as the nominal interest rate times the real money stock. By this alternative specification, seigniorage is B + C + D + E before the policy shift and A + B + D after the shift. Thus there is the same seigniorage gain of A - C - E.

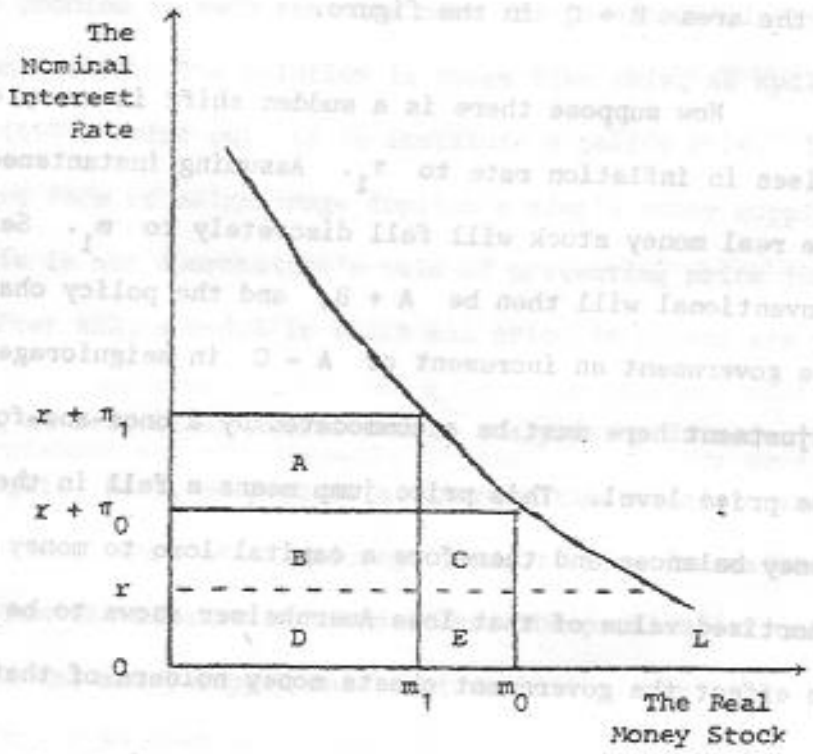


Fig. 1. The Real Demand for Money

But what if in place of such an honest government, we assumed perfect foresight? Sargent and Wallace (1973) have shown that money holders, in anticipation of the policy shift, would then try to divest themselves of real balances to forestall the capital loss. In the process, they would exert pressure on the price level, causing the inflation rate to exceed the yet unchanged rate of nominal money creation. This is where the rub comes. The ensuing decline in the real money stock must then yield a smaller seigniorage flow during this transition. The conventional specification of seigniorage, however, measures only the revenue flow from the time all adjustments have been completed. In neglecting the forerunning revenue effect, the government would therefore choose a higher inflation rate than is optimal.

Hence, once we assume perfect foresight, the problem of a dishonest government turns into a problem of time inconsistency. Instead of harming the public, the government ends up harming itself.

## 2. Time-inconsistent seigniorage

To enunciate the argument more fully, we now turn to a discrete-time model of money and perfect foresight. That this argument requires no assumption of morality can perhaps be dramatized by using as our example a government whose objective is the familiar one of maximizing revenue, that is, without regard for social welfare.[6/]

This government issues money only at the start of each period, exchanging it for output at the price level that prevails in the period. Thus if the government issues the nominal stock $M_t - M_{t-1}$ at the start of period $t$ and the price level is $P_t$, real seigniorage is $R_t = (M_t - M_{t-1})/P_t$. With $M_t$ as the resulting nominal stock, the price $P_t$ is determined by the condition $M_t/P_t = m_t$, where $m_t$ is the real demand for money for the period, which in turn is assumed to depend only on the inflation rate $\pi_t = 1 - (P_t/P_{t+1})$.[7/]

Substituting the real demands $m_t = M_t/P_t$ and $m_{t-1} = M_{t-1}/P_{t-1}$ and the inflation rate $\pi_{t-1} = 1 - (P_{t-1}/P_t)$ now allows us to write real seigniorage as

$$R_t = m_t - (1 - \pi_{t-1})m_{t-1}$$

$$= m_t - m_{t-1} + \pi_{t-1}m_{t-1} \qquad (1)$$

which can be seen simply as the discrete form of the conventional specification of seigniorage. This specification has two inflation rates, namely, $\pi_{t-1}$ and $\pi_t$. We assume as usual $m'_{t-1}(\pi_{t-1}) < 0$ and $m'_t(\pi_t) < 0$.

The government in this example views as its own optimal plan and structure of period inflation rates that maximizes the discounted sum

$$(2) \qquad z = \sum_{t-1}^{\infty} \frac{R_t}{(1 + r)^{t-1}}$$

where $r$ is a given discount rate. This is a simple dynamic programming problem, that is, we choose a particular inflation rate $\pi_t$ by taking all rates after period $t$ as optimally given. Hence, the optimal rates $\pi_{t-1}$ and $\pi_t$ must satisfy the conditions

$$(3) \qquad m_{t-1} + (r + \pi_{t-1})m'_{t-1} = 0$$

$$(4) \qquad m_t + (r + \pi_t)m'_t = 0$$

for an inferior solution.

Suppose now the government arrives at period $t$ and maximizes revenue from this period on. By choosing the size of money issue at the start of the period, this government can clearly control the variable $\pi_{t-1}$. In other words, it has discretion over a retroactive tax in that raising $\pi_{t-1}$ raises the inflation tax on real balances of the previous period. That this rate is indeed its control variable for the period conforms with the way this problem is typically formulated. Given that $R_t$ is seigniorage for the period, raising $\pi_{t-1}$ does serve to generate greater revenue. On the other hand, if we took $\pi_t$ as the control variable, we would get the bothersome result that raising the inflation rate serves to reduce seigniorage for the period.

Differentiating $R_t$ with respect to $\pi_{t-1}$ yields

$$m_{t-1} - (1 - \pi_{t-1})m'_{t-1} > 0 \tag{5}$$

which is clearly positive. Therefore, the government maximizes $R_t$ by setting $\pi_{t-1} = 1$, the upper bound. Note that this is consistent with maximizing the discounted sum of seigniorage from period $t$ on, since revenues after the period are constructed to be independent of $\pi_{t-1}$. However, this is clearly inconsistent with (3) and therefore violates the optimal plan. In period $t + 1$, the government will similarly set $\pi_t = 1$ to violate (4). Inflation rates would invariably be too high because of time inconsistency.

The problem arises because to specify seigniorage as $R_t$ is to take no account of the effect of $\pi_{t-1}$ on seigniorage in the previous period. Differentiating $\pi_{t-1}$ with respect to $\pi_{t-1}$ yields $m'_{t-1} < 0$. By neglecting this adverse forerunning effect, the government abuses the inflation tax, in this case, to its own detriment.

## 3. Time-consistent seigniorage

Writing out each $R_t$ in (2) and collecting terms in each $m_t$, we obtain

$$Z = \sum_{t=1}^{\infty} \frac{S_t}{(1 + r)^{t-1}} \tag{6}$$

where this maximand is now written in terms of

$$S_t = \frac{(r + \pi_t)m_t}{1 + r} \tag{7}$$

except for period 1, for which we have

$$S_1 = \frac{(r + \pi_1)m_1}{1 + r} - (1 - \pi_0)m_0$$

The extra term in $S_1$ is there only because we arbitrarily specified $Z$ in terms of $R_t$ first. We can exercise our one degree of freedom in specifying initial conditions to assume the term away. In any case, the term will not matter for the choice of inflation rates from period 1 on.

Notice that $S_t$ is simply the discrete form of Auernheimer's honest-government revenue. But here we have not changed the maximand for the optimal plan. The same structure of inflation rates would yield the same seigniorage stream whether we specify $Z$ in terms of $R_t$ or $S_t$. Moreover, $S_t$ does not depend on transfers that would prevent price jumps, since all price increases here are jumps.

Specifying seigniorage as $S_t$ does make a profound difference in monetary policy. The use of $S_t$ now implies the use of $\pi_t$ as the control variable for the period. There is, after all, no other inflation rate in $S_t$. This time, maximizing $S_t$ then yields

(8) $$m_t + (r + \pi_t)m'_t = 0$$

which is precisely the condition (4) characterizing the optimal plan. No longer is there a problem of time inconsistency.

Time consistency is achieved because the use of $S_t$ takes away from the government discretion over $\pi_{t-1}$, thus preventing the abuse of a retroactive tax. This solution is clearly not based on a moral argument, since its implementation is in the government's own interest. It remains now to provide a policy rule to implement such a solution.

## 4. A money supply rule

In the spirit of Kydland and Prescott, we propose a simple money supply rule to assure time consistency. At the start of period $t$, the central bank must allocate resources between two users: (i) $(1 - \pi_t)m_t/(1 + r)$ to a sinking fund to earn the real rate of return $r$; and (ii) $(r + \pi_t)m_t/(1 + r)$ to the appropriate branch of government to use as seigniorage revenue for the period. This means, of course, theat while the actual issue of money yields only $m_t - (1 - \pi_{t-1})m_{t-1}$ in seigniorage, the central bank must have on hand $m_t$ with which to allocate. But it would indeed have that amount, since it would acquire $(1 - \pi_{t-1})m_{t-1}$ from the previous period's sinking fund, and it would in turn pass on $(1 - \pi_t)m_t$ to the next period.

The main problem with adopting such a rule, apart from the obvious practical problems of engineering money supply, seems to be the initial sacrifice of funds to establish the sinking fund in the first period the rule is implemted.[8/] But the reward is time consistency in monetary policy.

FOOTNOTES

1/ The first to identify this problem was Strotz (1955-56). He found that an individual is bound to disobey his own plans even if his expectations are confirmed. This problem arises because in a sense Strotz allows the indivudual's tastes to change over time.

2/ Calvo's analysis is difficult but he concludes by attributing the problem to "(a) the nature of the demand for money and, (b) the fact that its creation implies in general the use of distortionary taxation" (p. 1420). In the present analysis, however, we use the same money demand function, implying also a distortionary inflation tax, to produce both time inconsistency and consistency.

3/ The same specification is implicit in Johnson's (1969) analysis of inside and outside money, although he assumes a zero rate of money creation after the initial issue. Since then, Auernheimer, Phelps (1973), and Marty (1978) seem to be the only ones to use this specification in spite of the plethora of papers on inflationary finance.

4/ Calvo himself dismisses this alternative by arguing, "...its moral appeal is greatly diminished inthe present context because, as pointed out above, we encounter time inconsistency even when the government attempts to maximize the welfare of the representative individual, that is to say, in a context where there is not a shade of malevolence or dishonesty at play." (p. 1422)

5/ This retroactivity in the inflation tax is emphasized by Brennan and Buchanan (1980). Their main concern, hoever, is not with time inconsistency, but that the government can try to impose higher inflation rates than people expect.

6/ Calvo assumes a government whose objective is to maximize social welfare. It should be clear that the solution we propose would apply also in that case.

7/ The base period for this inflation rate is the next one. We use this specification because it is convenient.

8/ The sacrifice is only to the government. There is no necessary loss to society since these funds are, after all, placed in the capital market.

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