University of the Philippines
SCHOOL OF ECONOMICS

Discussion Paper 8204 February 1982

Social Values and Individual Choices

by

José Encarnacion, Jr.

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Abstract

This paper sketches a theory of social choice based on a lexicographic ordering of multiple social values that can be pursued to greater or lesser degrees. Distinctions are made among the following concepts: (a) the vector-valued social decision, which results from individual choices regarding its components; (b) the social choice, which calls for compliance by the members of society with the social decision; and (c) social welfare judgements, which apply to situations where individual choices violate the social choice. A resolution of the Sen-Gibbard libertarian paradox follows from the theory.

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University of the Philippines

Introduction

The purpose of this paper is to sketch a theory of social choice based on social values. Under the theory, social welfare judgements apply particularly to situations where individual choices violate the social choice. We use the term "theory" in the usual sense of a system of concepts and relationships that aims to put more order in the subject under study.

In his classic Social Choice and Individual Values, Arrow (1963, p. 87) noted that "the alternatives, among which social preference is to be defined, may be interpreted in (at least) two ways: (1) each alternative is a vector whose components are values of the various particular decisions actually made by the government, such as tax rates, expenditures, ...; (2) each alternative is a complete description of the state of every individual." It is the second interpretation that has become standard in the literature, but we will have occasion to use both representations.

Gärdenfors (1973) and Hansson (1973) have distinguished two main approaches to the determination of social preference. The positionalist makes social preference on a set of objects depend on their absolute positions in the individuals' preference orderings of all possible objects, as in the Borda rule. The non-positionalist makes it depend only on their relative positions, as in Arrow's "independence of irrelevant alternatives" condition. Both approaches assume that social preference is a function of individual preferences regarding the objects. A third way which we will follow is to drop this assumption and instead make social preference a function of certain parameters (to be called the social parameters) determined by the members of society.

The following sections discuss social values and decisions, the social choice, welfare judgements and the social preference ordering. The penultimate section is a comment on the Sen-Gibbard libertarian paradox (Sen 1970, Gibbard 1974), and the final section is a concluding remark.

I. Social Values and Social Decisions

Our starting point is the fact of multiple social values: respect for human life, personal freedom over one's private affairs, civil liberties, property rights are among the more prominent ones in most societies. Social values--using this term to include the political and the economic--may be ranked in importance or priority differently in different societies: a decent livelihood for all, hence a right to employment, might be considered more important than property rights in one society, but the reverse might hold in another. At the same time, a particular social value (e.g. civil liberties) could be broader or narrower in scope--it could be preserved or promoted to a higher or lesser degree.

Though we use the singular, we view a social decision as a bundle of descriptive and normative propositions about the society and about relationships among its members (corresponding to Arrow's first interpretation of an alternative). It states facts, e.g. a new public highway connecting two cities or a new artesian well in a village; it also states the rules governing the society, using "rules" broadly to include tax legislation and all statutory laws as well as traffic rules and regulations.

The various components of a social decision have generally different implications for different social values, some of which may be advanced farther than others. A new public highway might mean more employment for urban workers but also more traffic deaths. A new artesian well might mean better health for villagers but also less employment for water carriers. One has of course to look at a social decision as a whole to see its overall net effects. The important point is that a social decision is made to promote or preserve social values and not for any other reason. Even so-called special legislation that would seem to favor only one individual finds a rationale, at least in principle, by an appeal to some social value that it promotes; otherwise it could not be justified.

We propose to formalize the relationships between social values and social decisions in the following way. For a given society, let u_i be a real-valued function (like the standard "ordinal" utility function, unique up to a positive monotonic transformation) such that if $u_i(x) > u_i(x')$, the social decision x promotes the ith social value more than does x' $(i = 1, 2, \ldots)$. Each x can accordingly be considered as a point $(u_1(x), u_2(x), \ldots)$ in $(u_1, u_2, \ldots)$-space. For convenience on occasion we will also refer to the ith social value as u_i. Corresponding to each u_i we postulate a particular number u_i^* such that if $u_i(x) \geqq u_i^*$, society considers x acceptable or sufficient for the preservation of the ith value. Writing $q_i(x) = \min\{u_i(x), u_i^*\}$, to each x thus corresponds the vector $q(x) = (q_1(x), q_2(x), \ldots)$. Let $q(x) > q(x')$ mean that the first nonvanishing component of $q(x) - q(x')$ is positive, i.e. let the $q(x)$'s be lexicographically ordered.[1/] We can then define a total ordering relation Q on the set of conceivable social decisions X:

xQx' if and only if $q(x) \geq q(x')$.

Suppose a unanimous agreement among its members that society is to adopt an ordering relation Q on X. As Arrow (1963, p. 83) has remarked, "it must be demanded that there be some consensus on the ends of society, or no social welfare function can be formed." Q would clearly state the ends of society in a structured way. To define Q it is necessary to determine (i) the social ranking $u_1, u_2, \ldots,$ regarding which there could be different views among the members of society, and then (ii) the corresponding acceptable levels $u_1^*, u_2^*, \ldots,$ which we will call social parameters.

We assume that each person has a view as to what the social ranking should be, in which case the following variant of the Borda rule may be applied. To the social value that a person considers most important a score of 1 is assigned, to the second most important a score of 2, and to the mth a score of m. The social value that gets the lowest total score then gives u_1 for society, the one with the second lowest gives u_2, and so on. Having fixed (i), the parameters have to be chosen seriatim. We assume that each person has single-peaked preferences as to what u_1^* should be, given the social ranking already determined.2/ Each person would thus have a "candidate" for u_1^*; anything less would be inadequate in his view, while anything higher would be excessive. Accordingly, Black's (1958) theorem on single-peaked preferences is applicable, and only the median candidate for u_1^* could win by simple majority rule over any other possible choice for u_1^* in pairwise comparisons. We therefore say that the social u_1^* is the median of individual candidates (or the median choice) for u_1^*. A repetition of the procedure fixes u_2^*, and so on. Q is thus defined in terms of individual choices for the social ranking and the parameters, and could then be used by society to order the x's without having to refer to individual preferences on X.

II. The Social Decision Function

Let $A \subset X$ be the set of feasible social decisions. A nonnull subset $f(a)$ needs to be defined as the social decision set, one of whose elements would then be the social decision. From Section I it is natural to say that[3/]

$$f(a) = \{x \in A \mid \forall\, x' \in A:\ xQx'\}$$
$$= A \cap A_1 \cap A_2 \cap \ldots$$

where

$$A_i = \{x \in A_{i-1} \mid u_i(x) \geqq \theta_i = \max_x \{q_i(x) \mid x \in A_{i-1}\}\},\quad i = 1, 2, \ldots,$$

and $A_0 = A$.

Suppose that $\theta_1 < u_1^*$. Then A_1 is a one-element set unless the solution to the problem of maximizing $u_1(x)$ subject to $x \in A$ is not unique. In general it would be unlikely for A_i to have more than one element if $\theta_i < u_i^*$. Suppose then that j is the smallest integer in $\{\mu \mid A_\mu$ is a one-element set$\}$, and consider the "normal" case where $\theta_i = u_i^*$, $i = 1, \ldots, j - 1$. In this case society maximizes the social value u_j subject to the condition that for all $i < j$, $u_i \geqq u_i^*$. If the feasible set gets larger so that $u_1, \ldots, u_j$ can all be greater, u_j is increased further until u_j^* is reached, at which point u_{j+1} becomes the maximand and $u_j \geqq u_j^*$ is added to the list of constraints. The social decision accordingly preserves as many of the social values (at acceptable levels) as possible, starting with the most important, and the next social value is advanced to the extent feasible. With larger feasible sets, more social values can be preserved at acceptable levels. (A richer society could be concerned with environmental pollution and prohibit the setting up of a new factory, while a poorer one which must first attend to problems of livelihood and employment might take the pollution with the factory.)

Whether the case is "normal" or not, if the (possibly infinite) number of components is sufficiently large, we may expect $f(A)$ to be a one-element set since maximization at each stage i should in general make A_i a proper subset of A_{i-1}. If, nonetheless, $f(A)$ still turns out to have multiple elements, they would all belong to the same indifference class from the viewpoint of social decision. In what follows we will denote by a an element of $f(A)$. (In any event, the social decision must be one particular x.)

The social decision a being a bundle of facts and rules, it cannot be a subject for majority decision by the members of society. What can be subjects for majority decision are the components of a, and indeed we find that majority decision is a common method for choosing which facts are to materialize for society and which statutory laws are to hold. In effect, such majority decisions on the components of a can be considered as the operational means for society's determination of the u_i^*'s. If, for example, the laws restrict the use of private property for certain purposes, that simply shows that property rights are not absolute in that society.

We observe that the process for yielding the social decision is highly economical. One need only fix the u_i^*'s (only as many as are required to get a one-element set A_j) and the social decision is fully determined given A. Representing social decisions by lexicographically ordered vectors thus results in a much simpler decision process than would the alternative assumption of a real-valued representation along standard lines. In the case of the latter, tradeoffs would generally vary at different points in $(u_1, u_2, \ldots)$-space.

III. The Social Choice

Suppose there are n persons in the society indexed by $k = 1, \ldots, n$, and let $z = (z^1, \ldots, z^n)$ where z^k is k's choice in the set of personal decisions Z^k that he could conceivably make as an individual (and not as a member of society expressing his choices for social parameters). We have $z \in Z$, where Z is the cartesian product $Z^1 \times \ldots \times Z^n$, and we denote a state of society by $\sigma = (x, z) \in \Sigma = X \times Z$.

As stated earlier, the social decision a consists of facts and rules. Thus a determines for the members of society what is possible given the facts in a and what is permissible given the rules in a. (The facts may make it possible to drive faster between two cities; the rules may prohibit driving faster than 55 miles an hour.)

Let $\phi(a) = \{z \mid z$ is possible given $a\}$ and $\Psi(a) = \{z \mid z$ is permissible given $a\}$. Denoting the social choice by $F(A)$, we propose to say that

$$F(A) = \{a\} \times Z \cap \phi(a) \cap \Psi(a).$$

Writing $Z^+(a) = Z \cap \phi(a) \cap \Psi(a)$, we can also write

ϕ = possible

$$Z^+(a) = Z^{1+}(a) \times \ldots \times Z^{n+}(a)$$

ψ = permissible

where

$$Z^{k+}(a) = Z^k \cap \phi^k(a) \cap \Psi^k(a),$$

$\phi^k(a)$ being the kth component of $\phi(a)$ and similarly for $\Psi^k(a)$. The choice of k is constrained by the possibilities to belong to $Z^k(a) = Z^k \cap \phi^k(a)$, so k could possibly choose $z^k \in Z^{k-}(a) = Z^k(a) - \Psi^k(a)$. If the rules are to be followed, however, we must have $z^k \in Z^{k+}(a)$, in which case we will say that z^k is admissible, and if z^k is admissible for all k, z is admissible.

The distinction between social decision and social choice that we wish to make is that while society can decide a, which is within its control, society can only choose $F(A)$ and hope that some element of $F(A)$ will turn out to be the case, considering that z is individually decided. (One may choose a horse to win in a race, but it depends on the horse how fast it will run. The only decision one has in a horse race is which horse to bet on.) From society's viewpoint it does not matter which element of $\Gamma(A)$ is realized since all elements of $Z^+(a)$ are within the rules. If the actual z is admissible, society is getting its choice.

IV. Social Welfare Judgements

Matters are different if $z \in Z^-(a) = Z \cap \phi(a) - \Psi(a)$. If the actual z is in $Z^-(a)$, at least one rule governing society is being violated by one or more persons. We suggest that given a, the domain of "operational" (in contrast to "hypothetical," see below) social welfare judgements is $Z(a) = Z^+(a) \cup Z^-(a) = Z \cap \phi(a)$. Social welfare would be "greater" with any admissible z--all admissible z's being equal in this regard--than with any element of $Z^-(a)$. A social welfare ordering on $Z^-(a)$ has now to be defined.

Associated with a is the vector $(q_1(a), q_2(a), \ldots)$, and associated with each $q_i(a)$ is a constraint set $\Psi_i(a)$ such that $\Psi(a) = \Psi_1(a) \cap \Psi_2(a) \cap \ldots$. We interpret $\Psi_i(a)$ as the set of z's that do not violate the rules designed to preserve the ith social value at the decided level $q_i(a)$. (It might be noted that $\Psi_i(a) = \Psi_i^1(a) \times \ldots \times \Psi_i^n(a)$ and $\Psi^k(a) = \Psi_1^k(a) \cap \Psi_2^k(a) \cap \ldots$.)

It seems quite reasonable to say that a violation of the rules defining $\Psi_i(a)$ is worse than a violation of those defining $\Psi_j(a)$ if $i < j$. Murder is worse than burglary if respect for human life is more important than property rights, and the corresponding penalties are more severe in the one case than in the other. At the same time, violations can be more extensive or less frequent, and less is better than more. A social welfare ordering on $Z^-(a)$ needs to reflect these aspects of the matter.

Given a, let c_{ai} be a real-valued function such that if $c_{ai}(z) > c_{ai}(z')$, then z complies more with (involves less violations of) $\Psi_i(a)$ than does z' ($i = 1, 2, \ldots$). The c_{ai}'s may be standardized so that $c_{ai} = 1$ means full compliance and $c_{ai} = 0$ means zero compliance. Suppose there exist acceptable degrees of compliance or tolerable violation levels c^*_{ai} and define $b_{ai}(z) = \min\{c_{ai}(z), c^*_{ai}\}$. Writing

$$w_a(z) = (b_{a1}(z), b_{a2}(z), \ldots ; c_{a1}(z), c_{a2}(z), \ldots),$$

we propose to say that given a, social welfare is at least as great with z as with z' (or $zW_a z'$) if and only if $w_a(z) \gtrsim w_a(z')$. Since $w_a(z) = (c^*_{a1}, c^*_{a2}, \ldots ; 1, 1, \ldots)$ for all $z \in Z^+(a)$, social welfare is the same for all of them and greater than with any element of $Z^-(a)$.

In order to determine the c^*_{ai}'s, the same procedure that fixed the u^*_i's could be used, assuming that each person has single-peaked preferences as to what each c^*_{ai} should be. Black's theorem yields the social c^*_{a1}, then c^*_{a2}, etc., and the social welfare ordering on $Z(a)$ is determined. It may be that $c^*_{ai} = 1$ for some i, but it would seem likely that for most i, $c^*_{ai} < 1$. If all $c^*_{ai} = 1$, one would expect law enforcement agencies to allocate all their resources to the prevention of murder and none to the protection of property rights, which is not the case.

The social parameters, i.e. the u^*_i's and the c^*_{ai}'s, are functions of individual choices regarding them. Each individual k also has a choice on $Z^k(a)$, hence on $Z^{k+}(a)$, which is realizable if z^h is admissible for all $h \neq k$, on the assumption that if everyone else follows the rules, a person's choice would be constrained only by what is possible given the facts. But an inadmissible z^h may prevent k's choice from being realized. (If h purloins k's wallet, k cannot buy the basket of goods that he wants.) The ability of k to get his choice on $Z^k(a)$ is thus conditional on what others do. (Formally this means that a realizable z is subject to a "compatibility" constraint $z \varepsilon \zeta(a)$ where $\zeta(a)$ is the set of z's whose components are realizable simultaneously given the facts of a. An actual z must belong to $\zeta(a) \subset Z(a)$.) A person's choice might itself be conditional on other people's choices, as in the example in Section VI. But in any case, it is only when k chooses $z^k \varepsilon Z^{k-}(a)$ that his choice becomes subject to social judgement.

In addition to "operational" welfare judgements that take a as given, there are what might be called hypothetical judgements that consider a different social decision a' and the resulting $Z^+(a')$. One could compare $Z^+(a)$ and $Z^+(a')$ and argue that the latter is "better" and that the social decision should be a' instead. The difference between a and a' may be relatively minor, involving only a few components corresponding to lower ranked social values (i.e. those u_j with higher indices j). In order to get a' as the social decision from the same feasible set A, only a few lower ranked parameters $u_j^\ddagger$ would then need to be revised. The difference between a and a' would be a major one if a revision of a higher ranked parameter $u_i^\ddagger$ is required, which is likely to entail changes in the components of the social decision corresponding to social values u_j, $j > i$, even if the parameters $u_j^\ddagger$ were unchanged. In any case, at any given time there is a social decision that forms the basis for making operational social welfare judgements.

V. The Social Preference Ordering

Given x, $w_x(z)$ is well defined for any $z \varepsilon Z$. It is thus possible to define the social preference ordering relation R on $\Sigma = X \times Z$ by writing

$$r(\sigma) = r(x, z) = (q(x), w_x(z))$$

and having $\sigma R \sigma'$ if and only if $r(\sigma) \overset{L}{\geq} r(\sigma')$. Society then prefers σ to σ' (or $\sigma P \sigma'$) if $\sim \sigma' R \sigma$.

It needs to be emphasized that R depends only on the social parameters and not on individual preferences regarding the σ's. We have not made any particular assumptions about individual preferences on X or Σ because they are not necessary for the purpose of determining the social choice and the welfare ordering. R is a superfluity for this purpose but has theoretical interest in relation to the Arrow (1963) conditions for a social preference ordering, to which we turn.

The usual format of social choice theory makes social preference on a set of objects a function of individual preferences on that set. Although the theory of this paper does not require it, consider a model where each k has a preference ordering relation R^k on Σ, so that k prefers σ to σ' (or $\sigma P^k \sigma'$) if $\sim \sigma' R^k \sigma$. Murakami's (1961) formulation of the Arrow impossibility theorem shows that R cannot satisfy all of the following conditions (R_S is society's preference ordering on $S \subset \Sigma$ and R_S^k is k's):

(i) there exists a 3-element set $T \subset \Sigma$ on which each k could have any logically possible R_T^k (the free triple condition);

(ii) there is no k such that for all $\sigma, \sigma' \in T$, $\sigma P^k \sigma' \rightarrow \sigma P \sigma'$ (non-dictatorship);

(iii) R_S is invariant with respect to any changes in $\{R^k\} = (R^1, \ldots, R^n)$ that leave $\{R_S^k\} = (R_S^1, \ldots, R_S^n)$ unchanged (independence of irrelevant alternatives);

(iv) if $\sigma P^k \sigma'$ for all k, then $\sigma P \sigma'$ (Pareto principle).

Conditions (i) and (ii) are easily satisfied. The case of (iii) is different. In our theory R, hence R_S, remains the same no matter how $\{R^k\}$ changes provided only that individual choices for the social parameters remain the same. Indeed, individual choices could change without affecting R provided the median choices for the parameters remain the same. The determination of R_S is thus divorced from $\{R^k\}$, hence from $\{R_S^k\}$, in the model we are considering. Since (iii) requires R_S to be determined uniquely by $\{R_S^k\}$, a violation of (iii) is possible. We do not consider this a defect of the model, however, much less of the theory which does not require that $\{R^k\}$ exist. As mentioned in the Introduction, there are three ways of determining R_S. One way is to require (iii) or something similar. Another, the positionalist approach, is to make R_S a function of $\{R^k\}$. The third, which we have followed, makes R_S a function of social parameters determined by individual choices. For this last, which is internally consistent, (iii) is simply irrelevant.

In the case of (iv), the same fact that R does not depend on $\{R^k\}$ makes a violation possible. Since Arrow's impossibility theorem is a logical necessity, R must violate one or more of (i)-(iv). That R should fail (iii) is to us quite acceptable but failure of (iv) would seem intolerable. The Pareto principle has strong traditional appeal as a normative proposition, but here we should like to argue that it should not be considered to have universal application.

Suppose $\sigma = (a, z)$, $\sigma' = (a', z')$, $a = a'$, and $z, z' \in Z^*(a)$. It may be that $\sigma P^k \sigma'$ for all k, in which case $\sigma P \sigma'$ by the Pareto principle. But in our theory, σ and σ' belong to the same social indifference class and therefore $\sim \sigma P \sigma'$. In our view, it is none of society's business to express a

Society decides only on means

preference either for σ or for σ' if z and z' are both admissible. (Such a view also permits a resolution of the Sen-Gibbard paradox, discussed in the next section.)

Consider also a case with $z, z' \in Z^{-}(a)$ where z and z' are identical except in one respect: in z a little boy, hungry, steals a loaf of bread and gets caught; in z' he goes free. The rules may say $\sigma P \sigma'$ but conceivably, $\sigma' P^k \sigma$ for all k. It will not do to say that the rules should be changed in such a case, since whatever the rules might be, one can always think of some circumstances where rigid application of the rules is hard for everyone. The conclusion we draw about the Pareto principle is that occasional violations are tolerable in the context of social judgements, though it may be mandatory for small group (committee) decisions where more flexibility about the group's own "rules" is possible.

VI. The Libertarian Paradox

A considerable literature has been generated by Sen's (1970) demonstration that the libertarian principle (in his sense) and the Pareto principle are incompatible with the existence of a social preference ordering (see Sen 1976 and more recently Gaertner and Krüger 1981 for reviews of this literature). Gibbard (1974) has since shown that the Pareto principle is not needed to show an inconsistency.

Write $\lambda^k(z)$ for the vector resulting from the replacement of the kth component of z by λ^k, i.e. $\lambda^k(z) = (z^1, \ldots, z^{k-1}, \lambda^k, z^{k+1}, \ldots, z^n)$. What might be called the Sen-Gibbard interpretation of the libertarian principle is that

(1) $\lambda^k(z)P^kz \rightarrow \lambda^k(z)Pz$ for all k, z, $\lambda^k(z)$,

where we have suppressed a in (a, z) and $(a, \lambda^k(z))$ since a remains the same throughout the discussion. The argument is that z and $\lambda^k(z)$ are the same except in their kth components, so (1) would seem reasonable. But suppose that

$$(\alpha,\beta)P^1(\alpha',\beta) \quad \& \quad (\alpha',\beta')P^1(\alpha,\beta')$$

(2)

$$(\alpha',\beta)P^2(\alpha',\beta') \quad \& \quad (\alpha,\beta')P^2(\alpha,\beta)$$

where we have suppressed $z^3, \ldots, z^n$ which remain the same throughout. Then by (1) and the transitivity of P, one has $(\alpha,\beta)P(\alpha,\beta)$ which contradicts $-(\alpha,\beta)P(\alpha,\beta)$.

In our view the source of the problem lies with (1), which assumes that in order to reflect libertarian values, one needs to say that $\lambda^k(z)Pz$ if $\lambda^k(z)P^kz$ because, the only difference between $\lambda^k(z)$ and z being k's choice, he should therefore decide the matter. But surely the correct thing to say is that the matter is k's concern and none of society's if z and $\lambda^k(z)$ are both admissible. In the theory of this paper, the libertarian principle is simply the statement that each k is free to choose any element of $Z^{k+}(a)$. The whole thrust of personal liberty as a social value is that one is free to do what one pleases provided no rules of society are violated, and society is not to pass judgement one way or the other regarding private choices that satisfy the rules. All the situations in (2) are presumably admissible, in which case they would all belong to the same social indifference class. Our resolution to the paradox is thus to drop (1), which is not necessary for a formalization of the libertarian principle.

The question remains as to what the outcome of example (2) might be. Since each person's choice is conditional on what the other person decides to do, it is up to them to arrive at some agreement. One possibility is that each person gets his way half of the time. In any event, it is a private matter for them to settle, not society's, unless some situation in (2) is inadmissible. In the latter case, the welfare ordering of Section IV would apply.

Among the various proposals for resolving the paradox, our view is closest to that of Nozick (1974) who would have social preference hold only over alternatives that already satisfy libertarian values. In the example of (2), Nozick would have all four situations eliminated as possible subjects for social judgement. But his solution is incomplete, as Sen (1976) has pointed out, for it leaves unanswered the question of social judgement when libertarian values are violated. In our theory, $w_a(z) > w_a(z')$ if z and z' are otherwise identical but z' involves a violation of the rules protecting libertarian values and z does not.

VII. Concluding Remark

The social choice literature is now replete with impossibility theorems (Kelly 1978), and perhaps it is time to allocate more effort to look into social choice models even if they violate one or another condition in some impossibility theorem.

The theory of this paper finds a place for multiple social values that can be pursued to greater or lesser degrees, a social decision that results from majority rule applied to its components, and social welfare judgements based on the prevailing norms. The social decision is determined by individual choices regarding not the feasible decisions themselves but the parameters of social evaluation, i.e. the social decision process.

Arrow remarked in a neglected passage that "unanimous agreement on the decision process may resolve the conflicts as to the decisions themselves," in which case "our social welfare problem may be regarded as solved" provided there is also "a widespread agreement on the desirability of everyday decisions" resulting from the decision process (Arrow 1963, pp. 90-91). This proviso would be satisfied in a stable ongoing society, which is the context of the present paper.

Notes

1. For a review of the literature on lexicographic orders and decision rules, see Fishburn 1974. The major references are Georgescu-Roegen 1954 and Chipman 1960; see also Encarnación 1964, 1964a, 1964b, 1965, 1969, and Day and Robinson 1973 which discuss various applications of lexicographic utility functions involving acceptable u_i^* levels. The skepticism of some economists about the analytical usefulness of such utility functions is actually well directed only when $u_i^* = \infty$ for all i; cf. Encarnación 1964.

2. It is hard to imagine how a person's preferences for u_1^* could have, say, a bimodal distribution, considering the meaning of u_1^*. Something is acceptable or it is not.

3. Since Q is a total ordering, the decision function f satisfies various conditions for "rational" choice functions, including the Weak and Strong Axioms of Revealed Preference and Arrow's Definition C4; see Arrow 1959, Theorem 2. Arrow uses the term "weak ordering" for total ordering (the latter follows Chipman 1960).

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