

University of the Philippines
SCHOOL OF ECONOMICS

Discussion Paper 8002

February 1980

A SIMPLE MODEL OF ECONOMIC GROWTH AND FLUCTUATIONS

by

José Encarnación, Jr.

NOTE: UPSE Discussion Papers are preliminary versions circulated privately to elicit critical comment. They are protected by the Copyright Law (PD No. 49) and not for quotation or reprinting without prior approval.

UPSEDP '97
11-26-97

Abstract

This paper considers a simple Keynesian macroeconomic model that allows for growth and fluctuations. Expansion and contraction are assumed to be due to temporarily systematic shocks that propel the economy away from a long-run equilibrium growth path. When such shocks have dissipated their effects, the economy returns to equilibrium growth. The model is compatible with unemployment as the usual state of affairs and price inflation in the presence of growing unemployment.

A SIMPLE MODEL OF ECONOMIC GROWTH AND FLUCTUATIONS

By José Encarnación, Jr.

I. Introduction

There appear to be essentially two ways of modeling economic growth and fluctuations. The more usual way is to postulate an inherently cyclical growth path, possibly with floors and ceilings (Allen 1967, ch. 20). An alternative approach involves a long-run growth path that is eventually stable (in a sense to be defined below) under certain conditions, but those conditions are not likely to be fulfilled always so that the actual sequence of events fluctuates around the long-run growth path. This paper presents a simple model of the latter type which is compatible with the following observations: full employment is not the usual state of affairs; employment and the real wage both rise during the recovery phase, and the unemployment rate increases while the real wage falls during a recession (Bodkin 1969); price inflation with growing unemployment is possible. Briefly, the model is Keynesian in the long-run (in having a long-run unemployment equilibrium) as well as in the short-period.

II. The Model

There are 12 variables: population H , gross investment I , capital stock K , money supply M , employment (demand for labor) N , labor supply N^S , price level P , interest rate r , saving S ,

money wage W , output and income Y , and unemployment rate u .

Consider the following relationships pertaining to a short-period (the subscript t is omitted):

$$(1) \quad Y = F(K, N)$$

$$(2) \quad N = g(W/P)K$$

$$(3) \quad N^S = h(W/P)H$$

$$(4) \quad u = (N^S - N)/N^S$$

$$(5) \quad S = (-\alpha + \beta W/P)N + \gamma(Y - NW/P)$$

$$(6) \quad I = (n + \delta + j(r))K + \theta(vY - K)$$

$$(7) \quad M/P = L(K, Y, r)$$

$$(8) \quad I = S$$

In eq. (1) we are assuming a smooth aggregate production function homogeneous of degree one, with output the same good as the capital stock. Thus in (2) the demand for labor corresponding to any particular real wage is proportional to $K^{1/\epsilon}$. Labor supplied is assumed to be proportional to H in (3). Equation (4) is definitional, and we require $u \geq 0$.

It is assumed in (5) that savings propensities out of wage and profit incomes are different, so that aggregate saving depends on the income distribution. Saving out of wages is $(-\alpha + \beta W/P)N$ and that out of profits is $\gamma(Y - NW/P)$, where we expect the parameters α, β, γ to be all positive with $\gamma > \beta$.

The investment function (6) is based on the following considerations. Suppose that the expected demand for output in the next period is $(1+n)Y$, n being the population growth rate, and suppose that there is a normally desired capital-output ratio v , a parameter. Then the capital stock desired at the end of the period (the same as the beginning of the next), K^d , equals $(1+n)vY$, and the investment needed to give K^d would be

$$K^d - (1-\delta)K = (n+\delta)K + (1+n)(vY - K)$$

where δ is the depreciation rate. In (6) the parameter θ , $0 < \theta \leq 1+n$, allows for the possibility that the capital deficiency $vY - K$ is not required to be fully made up in the current period. The interest rate affects investment through $j(r)$, with $j'(r) < 0$ and $j(r) = 0$ for some value of r , say r^* .

As usual, in (7) we assume that the real-balances demand function L is such that $L_1 > 0$, $L_2 > 0$, $L_3 < 0$; further, we assume that L is homogeneous of degree one in K and Y , so that in particular, $\lambda M/P = L(\lambda K, \lambda Y, r^*)$ would hold. Equations (7) and (8) are equilibrium conditions.

With H , K , M , and W given for the period, eqs. (1)-(8) should yield a solution for the remaining 3 variables. In general the solution is one with $u > 0$ rather than $u = 0$ (which would be fortuitous) so that we have an unemployment equilibrium which we assume to be stable; i.e., $\partial S/\partial Y > \partial I/\partial Y$, which is equivalent to $\partial S/\partial N > \partial I/\partial N$ in the short-period.

From (5),

$$(5') \quad S/N = -\alpha + \gamma Y/N - (\gamma - \beta)W/P = G(K/N)$$

This equation can be graphed in Fig. 1 with the general shape as shown, since Y/N and W/P are both positive functions of K/N . (If the elasticity of factor substitution is less than unity, the S/N function would have a maximum). In each short-period, the economy is at some point on the S/N curve, since (5') must be satisfied.

The economy will move to a new equilibrium from one period to the next depending on the new values of H , K , M and W . For H and K we simply have

$$(9) \quad H = (1 + n)H_{-1}$$

$$(10) \quad K = (1 - \delta)K_{-1} + I_{-1}$$

Now (6), putting

$$(i) \quad j(r) = 0$$

$$(ii) \quad vY - K = 0$$

gives

$$(6') \quad I/N = (n + \delta)K/N$$

which is graphed in Fig. 1 and can be thought of as a long-run relationship. We assume that (5') and (6') intersect at two points, ruling out the case of no intersection (i.e. no long-run equilibrium) and the singular case of one tangency point. Suppose, then, that the intersection point c is determined by (1)-(10) and (i)-(ii).

These 12 equations determine all the current variables (including

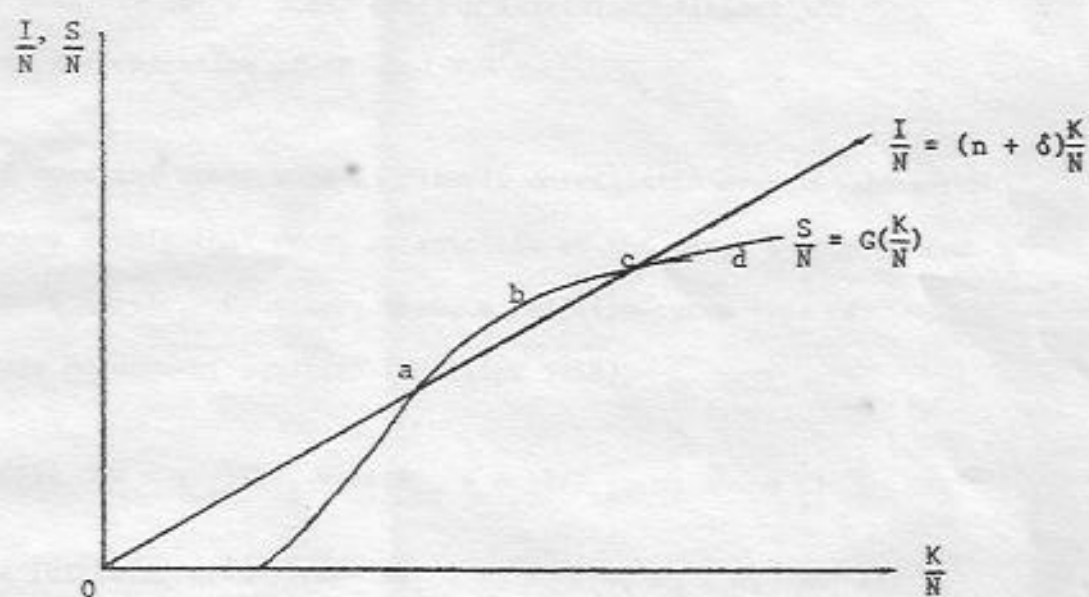


Fig. 1

M and W in particular) in, say, period t . At $t + 1$, given appropriate values of M_{+1} and W_{+1} , eqs. (1)-(10) would then maintain the state at c . The simplest case would be to have $M_{+1} = (1 + n)M$ and $W_{+1} = W$. It is quickly verified that the values of the variables are then either the same as or else $(1 + n)$ times their previous values. A long-run equilibrium is thus possible at c under certain initial conditions and "correct" determination of M and W .^{2/}

A constant money wage is clearly unrealistic over the long-run, and a money supply that grows exogenously at the rate n per period is not much better. Consider, then, a Phillips-curve type of money-wage adjustment equation (Phillips 1958)

$$(11) \quad (W - W_{-1})/W_{-1} = f((P_{-1} - P_{-2})/P_{-2}, u, u - u_{-1})$$

with the following properties: $f_1 > 0$, $f_2 < 0$, $f_3 < 0$, and for any u , $f(x, u, 0) = x$. We also suppose that

$$(12) \quad (M - M_{-1})/M_{-1} = (Y - Y_{-1})/Y_{-1}$$

Given the initial conditions, the system (1)-(12) determines all current variables and subsequently the future path of the system. It is apparent that under the right initial conditions, a long-run equilibrium still exists at c , which in general one may expect to have $u > 0$.

We now wish to show that the equilibrium point c is eventually stable in the following sense: If for some reason K/N has been falling and has reached a point between a and c where K/N does not fall any further,^{3/} say b , then the system (1)-(12) will move K/N from b to c ; similarly, if K/N has been rising and has reached a point to the right of c where K/N does not rise any further, say d , then the system will move K/N from d to c . Fluctuations around the long-run growth path would then be explained by temporarily systematic shocks, e.g. by movements in money supply not in accordance with (12) or investment spending not in accordance with (6), which propel the economy during contraction (from c to b) and expansion (from c to d). Depending on such external shocks, fluctuations would vary in amplitude and duration.

Suppose, then, that the initial state is b . At b , $S = I > (n + \delta)K$ so that in the next period $t + 1$, $K_{+1} = \lambda K$, say, where $\lambda > 1 + n$. If the state remains at b , $N_{+1} = \lambda N$ and

$$\lambda Y = F(\lambda K, \lambda N)$$

$$\lambda N = g(W/P)\lambda K$$

$$S_{+1} = (-\alpha + \beta W/P)\lambda N + \gamma(\lambda Y - \lambda NW/P) = \lambda S$$

$$I_{+1} = (n + \delta + j(r_{+1}))\lambda K + \theta(v\lambda Y - \lambda K)$$

$$\lambda M/P_{+1} = L(\lambda K, \lambda Y, r_{+1})$$

The real wage thus remains the same, $W_{+1}/P_{+1} = W/P$, but the unemployment rate is less and, referring to (11), the money wage increases (if not at $t+1$, then at some later period,^{4/} which could be labeled $t+1$). Hence the price level is higher and therefore also the interest rate, so that $I_{+1} < \lambda I$. But $\lambda I = \lambda S$, so $I_{+1} < S_{+1}$. Therefore we cannot have $N_{+1} = \lambda N$, and the capital-labor ratio must be higher at $t+1$. A repetition of the argument shows that if $K_{+2} = \mu K_{+1}$ and also $N_{+2} = \mu N_{+1}$, $\lambda > \mu > 1+n$, then $I_{+2} < S_{+2}$. For equilibrium we must have $N_{+2} < \mu N_{+1}$, for $N_{+2} > \mu N_{+1}$ implies that investment increases more than saving at a higher level of employment, which is the opposite of what is required for stability of short-period equilibrium. In short, the movement is towards c . If, on the other hand, d is the initial state, the reasoning proceeds in exactly the same way except that inequalities are reversed, so that again the movement is towards c .

Having established what we have called the eventual stability of c under the system (1)-(12), it is clear that the real wage rises during the recovery phase (from b to c) and falls during recession (from d to c). Moreover, employment rises during recovery, for an increasing unemployment rate is incompatible with the money wage rising faster than the price level in (11). Similarly, the rate of unemployment increases during recession.

This raises the twin possibilities of rising employment and falling prices during recovery and of growing unemployment and rising prices during recession. At b it is possible that the interest rate is unusually high (i.e. greater than r^*) and there is a capital deficiency. The course from b to c would then be one of falling r and increasing K/Y . This implies from (7) and (12) that P falls through time while, as we have already seen, employment increases. An opposite sequence is thus similarly possible from d to c .

III. Concluding Remarks

Harrod-neutral technical progress that is steady through time could be easily incorporated in the model but would add little by way of new possibilities. What seems important about technical progress for economic fluctuations is its non-steady nature--it occurs in spurts. Within the confines of the model, this could be represented by a shift of $F(K, N)$ in (1) to $\eta F(K, N)$, $\eta > 1$, and the value of η would be different with different technical advances. The effect, *ceteris paribus*, is to increase the real wage, the return to capital, investment (through θ), and therefore growth and employment. External shocks like oil price increases have opposite effects: the real wage falls, as also the return to

capital, investment, growth and employment because less output can now be had for the same inputs of capital and labor ($0 < \eta < 1$).

Finally, in view of the recent interest in disequilibrium models (Schwödiauer 1978), it is of some interest to see that the short-period Keynesian equilibrium model, suitably extended, can be used to describe both growth and fluctuations phenomena.

NOTES

1/ This demand for labor (which equates its marginal product to the real wage) is "latent" (Edwards 1959) and becomes actual, given the money wage, when the output produced can be sold at the corresponding price level.

2/ Although we have taken v as a parameter, one could also say that it is a function of long-run equilibrium values; specifically, $v = v((W/P)^*, r^*, \delta)$ where $(W/P)^*$ and r^* are the long-run equilibrium values of the corresponding variables.

3/ A lower limit to K/N would be imposed by some subsistence working wage (since K/N and W/P are positively related), and an upper limit by some minimum acceptable rate of return on capital. It is not necessary, however, that these limits should have to be reached in any particular fluctuation.

4/ The money wage may not increase right away if the price level has been falling, but with the real wage constant at b , the decreasing unemployment rate must sooner or later increase the money wage according to (11).

REFERENCES

- Allen, R. G. D. Macro-Economic Theory, London: Macmillan, 1967.
- Bodkin, R. G. "Real Wages and Cyclical Variations in Employment:
A Re-Examination of the Evidence," Canadian Journal of
Economics, August 1969, pp. 353-74.
- Edwards, E. O. "Classical and Keynesian Employment Theories: A
Reconciliation," Quarterly Journal of Economics, August 1959,
pp. 407-28.
- Phillips, A. W. "The Relation Between Unemployment and the Rate of
Change of Money Wage Rates in the United Kingdom, 1861-1957,"
Economica, November 1958, pp. 283-99.
- Schwödiauer, G. (ed.) Equilibrium and Disequilibrium in Economic
Theory, Dordrecht: Reidel, 1978.