

Discussion Paper No. 0205

August 2002

**The Design of Asymmetric Information  
Contracts for Public Sector Projects  
with Private Sector Participation**

*by*

*Renato E. Reside, Jr.\**

\*Assistant Professor, School of Economics,  
University of the Philippines.

Note: UPSE Discussion Papers are preliminary versions circulated privately to elicit critical comment. They are protected by the Copyright Law (PD No. 49) and not for quotation or reprinting without prior approval.

### **Abstract**

Over the last few years, multilateral lending institutions and governments have expressed increasing concern over the accumulation of contingent liabilities and their role in aggravating fiscal and financial fragility in developing economies. The accumulation of these contingent liabilities, brought about mostly by the provision of government guarantees, has prompted researchers to more closely scrutinize government contracts, specifically contracts for the provision of infrastructure goods and services in projects where there is private sector participation. The problem with these contracts seems to stem from adverse selection. Because of the adverse selection problem, contracts negotiated by government with private investors do not seem to be designed in a manner that achieves an optimal sharing of risk. As a result, the government is saddled with financial claims: investors seeking compensation for risks assumed by government. This study uses a simple model to solve for the optimal contracts under a situation where the government has asymmetric information about the quality of investors in essential infrastructure goods and services. The analysis concludes that it is possible for the government to offer a set of incentive-compatible contracts to investors. The terms in these contracts cover three variables: quantity, price/tariff and the level of guarantee coverage. In the set of optimal contracts, these three variables are endogenous variables.

## **I. Introduction**

Over the last two decades, governments in developing countries around the world have tried to cope with the complex tasks of financing and undertaking public infrastructure projects. The ever-increasing need to modernize roads, transport and utilities to maintain international competitiveness have led such countries to find innovative ways for building and financing the provision of such critical public goods. Because there usually are large differences in the structure of markets characterized by state provision as opposed to markets characterized by private provision of public goods, the transition from the former to the latter often entails a long drawn out process of bargaining between governments and private investors. The investment decision usually hinges on the extent to which governments allow disincentives and barriers to investment in markets for infrastructure products and services to be relaxed.

At present, many infrastructure services, mostly utilities in power, water and telecommunications, or in civil works, such as roads, bridges and ports, are provided in markets that are:

- 1) dominated by a large and inefficient state provider;
- 2) heavily regulated;
- 3) subject to uncertainties in demand; and/or
- 4) burdened by tariff distortions.

In the absence of additional incentives, private investors are discouraged by these conditions. Consistent with economic theory on externalities, there will be underinvestment in essential infrastructure if there is insufficient support from institutions that internalize these externalities. Underinvestment leads to a general decline in a country's international competitiveness and quality of life. A government facing budget constraints and a desire to maximize social welfare and to exploit potentially tremendous social externalities through the provision of greater and better infrastructure and other public services is tempted to implement investor-friendly policies in infrastructure, such as generous provision of guarantees. But, in so doing, the government faces a trade-off. On the one hand, if it generously provides guarantees for private investors, it increases the likelihood that any given investment in crucial or essential sectors will be pursued. On the other hand, it is already well known that guarantees are potential fiscal time bombs. They are contingent liabilities, liabilities whose values are random variables: stochastic and uncertain. The very timing and magnitude of claims against government is a random variable. Even the exact form of their probability distribution functions may be uncertain.

Furthermore, guarantees are off-balance sheet in nature, so even if they accumulate without claims ever being filed, they would not necessarily be captured by the cash accounting system adopted by most governments. Table 1 lists the various risks typically assumed (e.g., guaranteed) by the Philippine government in infrastructure projects with PSP. In addition to guarantees provided by sponsoring state-owned enterprises (SOE's) on individual risks, the Philippine government routinely signs



Performance Undertakings (PU's) in each project, a PU is normally provided by the national government in order to guarantee that the lead or sponsoring government agency or SOE its duties and obligations under the main investment contract.

Given the large number of risks the Philippine government has already had to assume in existing contracts, it is not surprising to note that large and unexpected financial claims by private investors on government guarantees provided in such projects have already occurred. These claims have occasionally disrupted the budgetary programming process, undermining fiscal stability. These claims have usually led to unexpected requirements for additional (mostly foreign) borrowing, usually in short-term notes, potentially leading to liquidity problems.

Thus, while it is an innovative and initially successful way of enhancing the efficiency of production of public goods, the introduction of PSP in the building of critical infrastructure facilities has also given rise to concomitant new challenges in fiscal management.

There are several approaches to addressing the problem of contingent liabilities. One approach involves estimating the size of the current stock of contingent liabilities. Several studies have attempted to measure the value of this stock in various countries, including the Philippines. In the Philippines, a study has concluded that the value is of a magnitude sufficient to threaten fiscal stability (Reside, 2001). After reviewing several contracts with PSP line-by-line, then identifying conditions which trigger claims against government, and then finally simulating the values of these claims, the study concludes that the magnitude of contingent liabilities is substantial relative to the budget deficit. The study then proposes charging risk-adjusted premiums for guarantees. This proposed solution has yet to be implemented.

Another approach to addressing the contingent liabilities problem involves identifying flaws in the structure and design of contracts between the government and private investors. A study by Sebastian and Silva (2001)<sup>1</sup>, utilizes a scoring method to rate the riskiness of a number of recent infrastructure contracts with PSP. This study finds that while there is large variance in the riskiness of contracts across sectors (e.g., urban rail contracts are found to be riskier than road contracts, which in turn are riskier than water concession contracts), *within sectors, there is little change in the level of risk assumed by government over time in contracts*. Specifically, the level of risk being assumed by government in contracts with IPP's have tended to remain the same since governments started contracting with them. The same is true for the level of risks assumed by government in contracts with toll road contractors. So, in spite of the fact that the accumulation of contingent liabilities has been a long-acknowledged problem, the Philippine government has not made significant strides in improving the design of contracts with PSP – the very basis and source of contingent liabilities.

---

<sup>1</sup> Their study won second place in the University of the Philippines School of Economics Sicat award competition for best undergraduate thesis in 2001.

Table 1: Common Risks in Infrastructure Projects in the Philippines

Type of Project-Specific Risks	Examples
<p>Project performance risks</p> <ul style="list-style-type: none"> <li>High cost of service</li> <li>Bad/inefficient service</li> </ul>	<p>Power – Power purchase agreements (PPAs) mandate minimum power plant performance level which the proponent has to satisfy.</p> <p>Water – The Metropolitan Waterworks and Sewerage System (MWSS) concession agreement states the minimum level of project performance to be satisfied by the proponent. The concessionaires would bear the risk of poor project performance if they are penalized by the MWSS Regulatory Office.</p> <p>Transport – Most toll road concession agreements state the minimum level of project performance to be satisfied by the proponent.</p>
<p>Project completion risks</p> <ul style="list-style-type: none"> <li>Delays</li> <li>Cost overruns</li> <li>Site availability</li> </ul>	<p>Power – The National Power Corporation (NPC) normally guarantees right-of-way and site availability for power projects.</p> <p>Water – The MWSS concession agreement stipulates that cost overruns in projects may be passed onto consumers provided these are covered in grounds for extraordinary price adjustments (EPA). Otherwise, such costs are borne by the concessionaires.</p> <p>Transport – Responsibility for constructing access and feeder roads necessary for ensuring the viability of many toll roads are assumed by the government.</p>
<p>Fuel and other inputs risk</p> <ul style="list-style-type: none"> <li>Fuel availability</li> <li>Skilled labor</li> </ul>	<p>Power – In many instances, PPAs include commitments by the NPC (also the off-taker) to guarantee the supply of fuel inputs to independent power producers (IPPs).</p> <p>Water – The MWSS concession agreement transfers input risk to the concessionaire, unless there are grounds for extraordinary price adjustments.</p> <p>Transport – Inputs for road and bridge construction are usually carried by the contractor.</p>
<p>Market risk</p> <ul style="list-style-type: none"> <li>User demand for services</li> </ul>	<p>Power – At the height of the power crisis, the NPC agreed to bear significant market risks by adopting minimum off-take contracts with IPPs.</p> <p>Water – The MWSS concession agreement transfers market risk to the concessionaire. However, a number of bulk water service contracts with pending approvals have minimum off-take provisions with government-owned off-takers.</p>



Type of Project-Specific Risks	Examples
	Transport – The MRT-3 contract includes a stipulation of minimum ridership levels below which government must compensate the contractor.
Payment Risk <ul style="list-style-type: none"> <li>• Creditworthiness of buyers of output</li> </ul>	Power – All PPAs stipulate that the NPC's commitments carry a full government guarantee for minimum offtake amounts. Thus, the relevant credit risk is that of the NPC and government.  Water – Many proposed service contracts between bulk water providers and offtakers, usually municipal water districts, carry guarantees of payment from the latter. Thus, the relevant credit risk is of the municipal water districts or the municipal government.  Transport – There are no off-takers in most transport projects.

Source: Reside (2000)

## II. The Nature of Information Problems in the Design of Contracts in Projects with PSP

An important step in addressing the contract design problem is to acknowledge that government guarantees are subject to the same information asymmetry problems present in any insurance transaction. After all, guarantees are a form of insurance. In any insurance, or guarantee transaction, the principal (the guarantor) faces an adverse selection problem. With incomplete information about the agent (the person buying the insurance or the guarantee), the possibility exists that the guarantor will select the agent with a high likelihood of filing a claim or calling on the insurance or guarantee. In this way, the adverse selection problem contributes to the accumulation of the government's contingent liabilities.

In the case of projects with PSP, the government is principal, while the investor is the agent. The government's problem is how to discriminate between investors that are more likely to fail in one or a number of tasks within a project (and therefore be more likely to call on a guarantee or to file a financial claim against government) – “bad” investors, and investors with a low likelihood of failure and thus a low likelihood of filing claims – “good” investors. Any solution to the optimal contract design problem should be able to address this problem of asymmetric information.

The solution to asymmetric information problems is fundamentally different from solutions to symmetric information problems. In a symmetric information problem, the principal has complete information about the characteristics of agents that are relevant to the activity at hand. Therefore, it can design contracts with the knowledge that it will be able to screen “bad” investors later.

In contrast, the principal has incomplete or no information about the agent in the asymmetric information case. Thus, if it offers only the symmetric information contract,

it has no ability to screen “bad” investors. Therefore, the symmetric information contract is sub-optimal when a situation of asymmetric information exists. In this case, the optimal response of the principal is to design a set of contracts that will force agents to self-select. That is, each of the contracts in the set offered to the investor set contains incentives that are designed to maximize the utility of only one particular type of agent. Thus, agents of a different type will not accept contracts not designed for them. In selecting the only contract designed for (i.e., that maximizes utility for) his particular agent type alone, the investor in essence reveals information about itself.

This study addresses the contract problem directly and explores the design of incentives built-into contracts for government projects with PSP. Using conventional tools of contract theory, this study uses a model of the contracting process to solve for a set of optimal contracts between government and private investors in a situation where the government does not have complete information about the investor.

To be effective, the model in this study is built to closely resemble the apparatus under scrutiny: public sector contracts involving PSP. In the model in this study, the contracted variables are:

- a) the quantity of output to be produced;
- b) the per-unit tariff to be charged when selling the output; and
- c) the (peso) level of insurance or guarantee coverage.

Typically, these three variables are the object of the bargaining process in any conventional PSP investment contract. Note that no premia are charged for the provision of guarantees. But that is consistent with the prevailing practices in the Philippines and in other countries. No other variables are introduced beyond those present in conventional contracts. The objective is to show that the government can achieve an optimal contract design solution through a mix of incentives using variables present in existing contracts. Table 2 lists quantities and tariffs that are the subject of bargaining between government and various types of agents.

**Table 2: Quantities and Tariffs Across Sectors**

Type of Agent	Quantity	Tariff
Independent power producer (IPP)	Kilowatt-hours generated	Pesos per kilowatt-hour
Toll road	Length of toll road	Toll
Water supply	Cubic meters of water	Water tariff
Urban Rail	Length of rail line	Fare



Typically, solutions to asymmetric information problems yield interesting propositions. This study is no different. The methodology adopted is as follows:

- 1) Solve for the optimal contract under symmetric information;
- 2) Show that the setup of constraints under an insurance problem with asymmetric information differs from the conventional setup of constraints in non-insurance-related asymmetric information problems;
- 3) Solve for the optimal contract under asymmetric information;
- 4) Prove that the optimal guarantee for a good investor is for the government to provide insurance coverage in excess of the value of the loss (i.e., the coverage exceeds the level of a full guarantee);
- 5) Prove that the optimal guarantee for a bad investor is a partial guarantee (i.e., coverage is less than the value of the loss);
- 6) Prove that the optimal contract for the good investor is to produce the same quantity of output as the bad investor, but sell this output at lower per-unit tariffs yet with higher insurance or guarantee coverage; and
- 7) Prove that the contracts designed for the good and bad investors are incentive-compatible.

### III. The Case of Symmetric Information

Consider the case of a risk-averse government that enters into a contract with a risk-averse investor. The contract specifies a project to be undertaken by the investor for which he is to be paid. The project to be undertaken, such as a toll road, a power generating plant, or an urban rail transit facility, creates important infrastructure services for the public, which pays for these services at tariffs regulated by the government. The project is inherently risky, so that the value of the investor's revenue and/or expenditure flows are uncertain. However, the investor has a minimum threshold for income flows received, so that he will not participate in the project unless the flows he receives are higher than this threshold.

Although the government is risk-averse, the project to be undertaken by the investor is so vital that the government offers to insure, or guarantee all or part of the flow of income received by the investor. If the government chooses to guarantee the entire flow of income, it offers the investor a *full guarantee*. If the guarantee is only on a portion of the flow, what is offered is a *partial guarantee*. Because it may discourage investors from contracting with the government, the government does not charge a premium to cover the cost of the guarantee.

Assume that there are only two states of nature, *good* and *bad*. In the bad state, all or portion  $L$  of the value of the investor's flow of income is lost. But if a government guarantee is built into the contract, the investor is able to call on this guarantee and claim the amount  $q$  from the government, the value of the insurance or guarantee coverage. In the good state, the flow is not lost and the project proceeds without the insurance or guarantee ever being called.  $\pi$  is the probability that the bad state will occur. Of course, 0



$< \pi < 1$ . In the case of a full guarantee,  $q = L$  : the entire loss is covered. In case of a partial guarantee,  $q < L$  : only portion of the loss is covered.

Note that  $q$ , the value of the insurance or guarantee coverage of the investor, is a *contingent liability of the government*. This is because it is to be paid to the investor contingent upon the occurrence of the bad state.

The outcome of the project for the investor can be described by a binomial distribution:

State of Nature	Probability	Outcome
Good	$1 - \pi$	$B [pQ - cQ - F]$
Bad	$\pi$	$B [pQ - cQ - F - L + q]$

The investor's utility is described by the function  $B [ \cdot ]$ . The argument within the utility function is the flow of income (net revenues) received by the investor across states. The quantity of goods produced by investor is  $Q$ , sold to the buying public at the government-regulated per-unit output tariff  $p$ .<sup>2</sup> Revenues therefore equal  $pQ$ . In producing each unit of  $Q$ , the investor incurs a variable cost of  $c$ . The investor also incurs a fixed cost of production,  $F$ . In the bad state (the state where a loss,  $L$  occurs), the investor loses the flow  $L$ , but is compensated with the flow  $q$  from the government.

Because the investor is assumed to be risk averse, his utility function is concave:  $B' [ \cdot ] > 0$  and  $B'' [ \cdot ] < 0$ .

How closely does this mimic what happens in reality? For simplicity, this study abstracts from cases where a variety of risks are assumed by government in a single contract. By restricting the loss  $L$  to a flow, we restrict our scope of work to the case where a flow of revenues or costs (or net revenues) accruing to the agent (investor) is at risk of being lost. We can think of  $L$  as a flow of revenues that is lost (perhaps due to adverse fluctuations in market demand) or an unexpected increase in costs. The principal (government) guarantees a certain level of such net revenues by allowing claims to be filed by the investor to offset loss of net revenues or to finance increased costs incurred. In terms of Table 1, we may associate the loss of flows most closely with project completion risk (costly delays, cost overruns) where unexpected costs are incurred, and with market risk, and payment risk, where unexpected reductions in revenues are incurred. We may also think of  $L$  as a cost overrun, that leads to a level of cashflow that is not sufficient to amortize a given stock of debt falling due.<sup>3</sup>

<sup>2</sup> In practice, most infrastructure projects with PSP do involve the production of goods and services at regulated tariffs. It is very common for road tolls, electricity and water tariffs to be sold regulated tariffs.

<sup>3</sup> This is in fact, similar to what occurred in 2000 when the Philippines' MRT-3 project called on a government guarantee. Due to various cost overruns, the MRT-3 consortium of private investors required additional cash infusions to finance payments to their contractors.

The outcome for the public can also be described by a binomial distribution:

State of Nature	Probability	Outcome
Good	$1 - \pi$	$U(Q) - pQ$
Bad	$\pi$	$U(Q) - pQ - q$

Since the public is risk averse, this means that its (aggregate) utility function is concave:  $U'[\cdot] > 0$  and  $U''[\cdot] < 0$ . The public's utility is reduced by the flow  $q$  when the bad state occurs. It is assumed that in the bad state, the government ultimately can pass onto the public the cost of any claims made by the investors,  $q$ . This can be achieved through taxation.

The contract between the government and the investor, therefore, covers both the terms of the guarantee and the tasks to be carried out by the investor. It contains three key parameters:

- a)  $Q$ , the quantity of the good or service produced by the investor and purchased by the government;
- b)  $p$ , the per-unit tariff of good or service sold by the investor; and
- c)  $q$ , the value of the insurance or guarantee coverage.

It is assumed that the government, when offering the contract to the investor, can choose the values of these parameters.

Under conditions of symmetric information, both government and investor have full information about each other. The government's objective function is the expected value of social welfare:

$$(1 - \pi)[U(Q) - pQ] + \pi[U(Q) - pQ - q] + (1 - \pi)B[pQ - cQ - F] + \pi B[pQ - cQ - F - q] \quad (1)$$

Social welfare in this case includes the welfare of the public, as well as the welfare of the private investor. It is assumed, for simplicity, that social welfare is separable in all of its arguments. Maximization of social welfare is subject to two inequality constraints:

$$(1 - \pi)[U(Q) - pQ] + \pi[U(Q) - pQ - q] \geq 0 \quad (2)$$

$$(1 - \pi)B[pQ - cQ - F] + \pi B[pQ - cQ - F - q] \geq \underline{R} \quad (3)$$

The first constraint is the public's participation constraint. The expected utility derived from purchasing the good or service must exceed zero for the public to find the project acceptable. The second constraint is the investor's participation constraint. For the investor to agree to undertake the investment, his expected utility must exceed his reservation expected utility level ( $\underline{R}$ ). We can think about the reservation expected

utility  $R$ , for example, to be the level of utility associated with a level of income deemed sufficient to service a given stock of debt, borrowed to finance the project.

The government thus faces the following problem of maximizing social welfare subject to the three constraints.

$$\text{Max}_{(Q, p, q)} (1 - \pi)[U(Q) - pQ] + \pi[U(Q) - pQ - q] + (1 - \pi)B[pQ - cQ - F] + \pi B[pQ - cQ - F - L + q]$$

subject to

$$(1 - \pi)[U(Q) - pQ] + \pi[U(Q) - pQ - q] \geq 0$$

$$(1 - \pi)B[pQ - cQ - F] + \pi B[pQ - cQ - F - L + q] \geq R \quad (4)$$

The problem can be solved using conventional Kuhn-Tucker complementary slackness conditions. The lagrangean is:

$$\begin{aligned} L = & (1 - \pi)[U(Q) - pQ] + \pi[U(Q) - pQ - q] + (1 - \pi)B[pQ - cQ - F] + \pi B[pQ - cQ - F - L + q] \\ & + \lambda[(1 - \pi)[U(Q) - pQ] + \pi[U(Q) - pQ - q]] + \gamma[(1 - \pi)B[pQ - cQ - F] + \pi B[pQ - cQ - F - L + q]] \end{aligned} \quad (5)$$

The first derivative of the lagrangean with respect to  $Q$ ,  $p$ , and  $q$  give rise to the following Kuhn-Tucker conditions:

$$\frac{\partial L}{\partial Q} = (1 + \gamma)[U'(Q) - p] + (1 - \pi)(1 + \lambda)B'[pQ - cQ - F] + \pi B'[pQ - cQ - F - L + q] = 0 \quad (6)$$

$$\frac{\partial L}{\partial p} = (1 + \gamma) + (1 + \lambda)[(1 - \pi)B'[pQ - cQ - F] + \pi B'[pQ - cQ - F - L + q]] = 0 \quad (7)$$

$$\frac{\partial L}{\partial q} = -(1 + \gamma) + (1 + \lambda)B'[pQ - cQ - F - L + q] = 0 \quad (8)$$

also,  $\gamma \geq 0$  and  $\lambda \geq 0$ .

Equations (6) and (7) give rise to the condition for efficient production:

$$U'(Q) = c \quad (9)$$



Marginal utility must equal the marginal cost of production.

**Proposition 1:** In the case of symmetric information, the optimal guarantee is a full guarantee.

**Proof:**

Equations (7) and (8) yield:

$$B'[pQ - cQ - F - L + q] = \{(1 - \pi)B'[pQ - cQ - F] + \pi B'[pQ - cQ - F - L + q]\} \quad (10)$$

Which will hold *only if*  $L = q^*$ . Thus, in the case of symmetric information, the optimal action for the government is set output at a level that achieves efficiency and to fully insure or fully guarantee the investor. This solution is made possible by the fact that the government, possessing full information, is able to distinguish good investors (e.g., investors with a small likelihood of failure) from bad ones (e.g., investors with a large likelihood of failure), thereby allowing it to select only good investors to participate in projects and fully insure them. Q.E.D.

#### IV. The Case of Asymmetric Information

In the case of asymmetric information, the government does not have enough information about the investor to enable it to distinguish between good and bad investors. Thus, if it selects an investor with a high probability of failure to participate in a project, it is not optimal to provide a full guarantee. However, since the government has no way of telling good from bad investors, it may design a set of contracts that allows each investor type to select the contract best suited for it, and in so doing, reveal their own characteristics (e.g., designing a set of contracts which forces each type of investor to self-select).

In the case of asymmetric information, we can assume that there are two types of investors: good and bad. The latter have a higher probability  $\pi^B$  of incurring a given loss  $L$  than the former ( $\pi^B > \pi^G$ ). Suppose the proportion of good investors among all investors is  $x$ . It therefore follows that the proportion of bad investors is  $(1 - x)$ . Also, good and bad investors only differ in their loss probabilities (both investor types have the same  $c$  and  $F$ ). We assume that the functional form of the utility function is the same across investor types. The government's problem is thus to maximize social welfare:

$$\begin{aligned}
& \text{Max} \\
& \{Q^G, p^G, q^G\}, \{Q^B, p^B, q^B\} \\
& (1-x) \left\{ (1-\pi^G)[U(Q^G) - p^G Q^G] + \pi^G[U(Q^G) - p^G Q^G - q^G] + \right. \\
& \quad \left. (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \right\} \\
& + x \left\{ (1-\pi^B)[U(Q^B) - p^B Q^B] + \pi^B[U(Q^B) - p^B Q^B - q^B] + \right. \\
& \quad \left. (1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B] \right\} \quad (11)
\end{aligned}$$

subject to

$$(1-\pi^G)[U(Q^G) - p^G Q^G] + \pi^G[U(Q^G) - p^G Q^G - q^G] \geq 0 \quad (12)$$

$$(1-\pi^B)[U(Q^B) - p^B Q^B] + \pi^B[U(Q^B) - p^B Q^B - q^B] \geq 0 \quad (13)$$

$$(1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \geq R \quad (14)$$

$$(1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B] \geq R \quad (15)$$

$$\begin{aligned}
& (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \geq \\
& (1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B] \quad (16)
\end{aligned}$$

$$\begin{aligned}
& (1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B] \geq \\
& (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \quad (17)
\end{aligned}$$

The first two constraints are the participation constraints of the public. The public must accept the output of each investor type. The third and fourth constraints are the participation constraints of each investor. Each investor should receive an expected utility greater than or equal to the reservation utility (assumed to be the same). The last two are the self-selection constraints. The good investor will never select the contract intended for the bad investor because the expected utility from the contract intended for him will be greater than or equal to the expected utility from the contract intended for the bad investor. Similarly, the bad investor will never select the contract intended for the good investor because the expected utility from the contract intended for him will be greater than or equal to the expected utility from the contract intended for the good investor.

In conventional asymmetric information problems, at least one constraint in the above problem becomes redundant. However, that is not the case here.

**Proposition 2:** Since no *a priori* constraints are imposed on the amount of insurance or guarantee provided by the government, *all* of the constraints are relevant in the problem above.

**Proof:**

In more conventional asymmetric information models, the process of eliminating the participation constraint of one type of agent (investor) involves combining one self-selection constraint and one participation constraint. We try to do this as follows.

Take the self-selection constraint (17). Suppose we try to relate it to the participation constraint (12) as such:

$$\begin{aligned} (1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B] &\geq \\ (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] &\geq \\ (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] &\geq R \end{aligned}$$

Whether or not the second term is greater than or equal to the third term is ambiguous. We know that  $\pi^B > \pi^G$ , so that if  $q^G > L$ , the inequality will hold. However, if  $L > q^G$ , then for very small values of  $q^G$ , it is possible for the inequality between the second and third terms is violated.

Next, take self-selection constraint (16). Suppose we try to relate it to participation constraint (13) as such:

$$\begin{aligned} (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] &\geq \\ (1-\pi^G)B[p^B Q^B - cQ^B - F] + \pi^G B[p^B Q^B - cQ^B - F - L + q^B] &\geq \\ (1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B] &\geq R \end{aligned}$$

Whether or not the second term is greater than or equal to the third term is ambiguous. We know that  $\pi^B > \pi^G$ , so that if  $L < q^B$ , then the inequality between the second and third terms is violated. However, if  $q^B > L$  (the amount of insurance or guarantee is substantial), it is possible for the inequality to hold.



If participation constraint (12) is related to self-selection constraint (16), no conclusion is possible. The same is true if participation constraint (13) is related to self-selection constraint (17). Q.E.D.

It therefore follows that all of the constraints in the problem are necessary. The complete Lagrangean is

$$\begin{aligned}
L = & (1-x) \left\{ (1-\pi^G)[U(Q^G) - p^G Q^G] + \pi^G[U(Q^G) - p^G Q^G - q^G] + \right. \\
& \left. (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \right\} \\
& + x \left\{ (1-\pi^B)[U(Q^B) - p^B Q^B] + \pi^B[U(Q^B) - p^B Q^B - q^B] + \right. \\
& \left. (1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B] \right\} + \\
& \lambda \{ (1-\pi^G)[U(Q^G) - p^G Q^G] + \pi^G[U(Q^G) - p^G Q^G - q^G] \} + \\
& \delta \{ (1-\pi^B)[U(Q^B) - p^B Q^B] + \pi^B[U(Q^B) - p^B Q^B - q^B] \} + \\
& \theta \{ (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \} + \\
& \gamma \{ (1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B] \} + \\
& \alpha \left\{ \begin{aligned} & (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] - \\ & (1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B] \end{aligned} \right\} + \\
& \eta \left\{ \begin{aligned} & (1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B] - \\ & (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \end{aligned} \right\} \quad (18)
\end{aligned}$$

where the variables  $\lambda$ ,  $\delta$ ,  $\theta$ ,  $\gamma$ ,  $\alpha$ , and  $\eta$  are the lagrange multipliers for constraints (12), (13), (14), (15), (16), and (17), respectively. We can now form the Kuhn-Tucker conditions. The first Kuhn-Tucker condition is:

$$\begin{aligned}
\frac{\partial L}{\partial Q^G} = & (1-x) \left\{ (1-\pi^G)[U'(Q^G) - p^G] + \pi^G[U'(Q^G) - p^G] + \right. \\
& \left. [(p^G - c)\{(1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G]\}] \right\} \\
& + \lambda \{ (1-\pi^G)[U'(Q^G) - p^G] + \pi^G[U'(Q^G) - p^G] \} \\
& + \theta (p^G - c) \{ (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \} \\
& + \alpha (p^G - c) \{ (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \} \\
& - \eta (p^G - c) \{ (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \} \\
= & 0
\end{aligned} \tag{19}$$

This can be simplified into

$$\frac{\partial L}{\partial Q^G} = (1-x+\lambda) \{ [U'(Q^G) - p^G] \} + (p^G - c) \{ (1-x+\alpha+\theta)A - \eta D \} = 0 \tag{20}$$

where

$$A = \{ (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \} \tag{21}$$

$$D = \{ (1-\pi^G)B[p^G Q^G - cQ^G - F] + \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \} \tag{22}$$

The second Kuhn-Tucker condition is:

$$\begin{aligned}
\frac{\partial L}{\partial p^G} = & (1-x) \left\{ -(1-\pi^G)Q^G - \pi^G Q^G + \right. \\
& \left. \left\{ (1-\pi^G)B'[p^G Q^G - cQ^G - F]Q^G + \pi^G B'[p^G Q^G - cQ^G - F - L + q^G]Q^G \right\} \right\} \\
& + \lambda \{ (1-\pi^G)Q^G + \pi^G Q^G \} \\
& + \theta \{ (1-\pi^G)B'[p^G Q^G - cQ^G - F]Q^G + \pi^G B'[p^G Q^G - cQ^G - F - L + q^G]Q^G \} \\
& + \alpha \{ (1-\pi^G)B[p^G Q^G - cQ^G - F]Q^G + \pi^G B[p^G Q^G - cQ^G - F - L + q^G]Q^G \} \\
& - \eta \{ (1-\pi^G)B[p^G Q^G - cQ^G - F]Q^G + \pi^G B[p^G Q^G - cQ^G - F - L + q^G]Q^G \} \\
= & 0
\end{aligned} \tag{23}$$

This can be simplified into

$$\frac{\partial L}{\partial p^G} = (1-x+\lambda) + \{(1-x+\alpha+\theta)A - \eta D\} = 0 \tag{24}$$

The third Kuhn-Tucker condition is:

$$\begin{aligned}
\frac{\partial L}{\partial q^G} = & (1-x) \left\{ -\pi^G + \pi^G B'[p^G Q^G - cQ^G - F - L + q^G] \right\} \\
& - \lambda \pi^G + \theta \{ \pi^G B'[p^G Q^G - cQ^G - F - L + q^G] \} \\
& + \alpha \{ \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \} - \eta \{ \pi^G B[p^G Q^G - cQ^G - F - L + q^G] \} \\
= & 0
\end{aligned} \tag{25}$$

This can be simplified into

$$\frac{\partial L}{\partial q^G} = -(1-x+\lambda) + \left\{ (1-x+\alpha+\theta)C - \eta \left( \frac{\pi^G}{\pi^G} \right) C \right\} = 0 \tag{26}$$

where

$$C = B'[p^G Q^G - cQ^G - F - L + q^G] \tag{27}$$



We can now proceed to determining the optimal values for  $Q^G$ ,  $p^G$  and  $q^G$ . Combine (20) and (24) and we derive the condition for efficiency in production:

$$U'(Q^G) = c \quad (28)$$

Combine (24) and (26) and we get:

$$(1-x+\lambda) + \{(1-x+\alpha+\theta)A - \eta D\} = (1-x+\lambda) + (1-x+\alpha+\theta)C - \eta \left( \frac{\pi^B}{\pi^G} \right) C$$

$$(1-x+\alpha+\theta)A - \eta D = (1-x+\alpha+\theta)C - \eta \left( \frac{\pi^B}{\pi^G} \right) C$$

or

$$\begin{aligned} & (1-x+\alpha+\theta) \left\{ (1-\pi^G)B'[p^G Q^G - cQ^G - F] + \pi^G B'[p^G Q^G - cQ^G - F - L + q^G] \right\} - \\ & \eta \left\{ (1-\pi^B)B[p^G Q^G - cQ^G - F] + \pi^B B[p^G Q^G - cQ^G - F - L + q^G] \right\} \\ & = (1-x+\alpha+\theta)B'[p^G Q^G - cQ^G - F - L + q^G] - \eta \left( \frac{\pi^B}{\pi^G} \right) B'[p^G Q^G - cQ^G - F - L + q^G] \quad (29) \end{aligned}$$

**Proposition 3:** In the case where there are two investors and asymmetric information, the optimal action of government is to more than fully insure or fully guarantee the good investor:  $q^G > L$ .

**Proof:**

From an inspection of (29),  $q^G = L$  is not possible because  $\pi^B > \pi^G$ .

Next, suppose  $q^G > L$ . This means that the first term on the LHS of (29) is  $>$  the first term on the RHS of (29). For the equality to continue to hold, it must be true that the second term on the LHS of (29) is  $>$  the second term on the RHS of (29). Eliminating terms:

$$\left( \frac{1-\pi^G}{\pi^G} \right) B'[p^G Q^G - cQ^G - F - L + q^G] > \left( \frac{1-\pi^B}{\pi^B} \right) B'[p^G Q^G - cQ^G - F] \quad (30)$$

This must be true, because  $\pi^B > \pi^G$  and  $q^G > L$ .

Now, suppose  $q^G < L$ . Eliminating terms:

$$\left(\frac{1-\pi^G}{\pi^G}\right)B[p^G Q^G - cQ^G - F - L + q^G] < \left(\frac{1-\pi^B}{\pi^B}\right)B[p^G Q^G - cQ^G - F] \quad (31)$$

Manipulating, we get:

$$\frac{\left(\frac{1-\pi^G}{\pi^G}\right)}{\left(\frac{1-\pi^B}{\pi^B}\right)} > \frac{B[p^G Q^G - cQ^G - F]}{B[p^G Q^G - cQ^G - F - L + q^G]} \quad (32)$$

If  $q^G < L$ , then it must be that the RHS is  $< 1$ . But the LHS cannot be  $< 1$ , because  $\pi^B > \pi^G$ .

Thus, the only possible case is  $q^G > L$ , the government more than fully insures or fully guarantees the good investor.

The fourth Kuhn-Tucker condition is

$$\begin{aligned} \frac{\partial L}{\partial Q^B} = & \left\{ (1-\pi^B)[U'(Q^B) - p^B] + \pi^B[U'(Q^B) - p^B] + \right. \\ & \left. (p^B - c)\{(1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B]\} \right\} \\ & + \delta \{(1-\pi^B)[U'(Q^B) - p^B] + \pi^B[U'(Q^B) - p^B]\} \\ & + \gamma(p^B - c)\{(1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B]\} \\ & + \eta(p^B - c)\{(1-\pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B]\} \\ & - \alpha(p^B - c)\{(1-\pi^G)B[p^B Q^B - cQ^B - F] + \pi^G B[p^B Q^B - cQ^B - F - L + q^B]\} \\ & = 0 \end{aligned} \quad (33)$$

This can be simplified into

$$\frac{\partial L}{\partial Q^B} = (x + \delta) \{ [U'(Q^B) - p^B] \} + (p^B - c) \{ (x + \gamma + \eta)J - \alpha K \} = 0 \quad (34)$$

where

$$J = \{ (1 - \pi^B)B[p^B Q^B - cQ^B - F] + \pi^B B[p^B Q^B - cQ^B - F - L + q^B] \} \quad (35)$$

$$K = \{ (1 - \pi^G)B[p^B Q^B - cQ^B - F] + \pi^G B[p^B Q^B - cQ^B - F - L + q^B] \} \quad (36)$$

The fifth Kuhn-Tucker condition is:

$$\begin{aligned} \frac{\partial L}{\partial p^B} = & x \left\{ -(1 - \pi^B)Q^B - \pi^B Q^B + \right. \\ & \left. \{ [(1 - \pi^B)B[p^B Q^B - cQ^B - F]Q^B + \pi^B B[p^B Q^B - cQ^B - F - L + q^B]Q^B] \} \right\} \\ & + \delta \{ (1 - \pi^B)Q^B + \pi^B Q^B \} \\ & + \gamma \{ (1 - \pi^B)B[p^B Q^B - cQ^B - F]Q^B + \pi^B B[p^B Q^B - cQ^B - F - L + q^B]Q^B \} \\ & + \eta \{ (1 - \pi^B)B[p^B Q^B - cQ^B - F]Q^B + \pi^B B[p^B Q^B - cQ^B - F - L + q^B]Q^B \} \\ & - \alpha \{ (1 - \pi^G)B[p^B Q^B - cQ^B - F]Q^B + \pi^G B[p^B Q^B - cQ^B - F - L + q^B]Q^B \} \\ & = 0 \end{aligned} \quad (37)$$

This can be simplified into

$$\frac{\partial L}{\partial p^B} = -(x + \delta) + \{ (x + \gamma + \eta)J - \alpha K \} = 0 \quad (38)$$



The sixth Kuhn-Tucker condition is:

$$\begin{aligned}
\frac{\partial L}{\partial q^B} &= x \left\{ -\pi^B + \pi^B B' [p^B Q^B - cQ^B - F - L + q^B] \right\} \\
&\quad - \delta \pi^B + \gamma \left\{ \pi^B B' [p^B Q^B - cQ^B - F - L + q^B] \right\} \\
&\quad - \alpha \left\{ \pi^G B [p^B Q^B - cQ^B - F - L + q^B] \right\} + \eta \left\{ \pi^B B [p^B Q^B - cQ^B - F - L + q^B] \right\} \\
&= 0
\end{aligned} \tag{39}$$

This can be simplified into

$$\frac{\partial L}{\partial q^B} = -(x + \delta) + \left\{ (x + \gamma + \eta)N - \left( \frac{\pi^G}{\pi^B} \right)N \right\} = 0 \tag{40}$$

where

$$N = B' [p^B Q^B - cQ^B - F - L + q^B] \tag{41}$$

The final Kuhn-Tucker conditions are the inequality constraints, equations (12) to (17), along with  $\lambda \geq 0$ ,  $\delta \geq 0$ ,  $\theta \geq 0$ ,  $\gamma \geq 0$ ,  $\alpha \geq 0$ , and  $\eta \geq 0$  for the lagrange multipliers.

Drawing implications from the first order conditions for the bad investor is straightforward, because the Kuhn-Tucker conditions are similar. The G superscripts are replaced by B, and the appropriate lagrange multipliers are used.

For the optimal values for  $Q^B$ ,  $p^B$  and  $q^B$ , combine (34) and (38) and we derive the condition for efficiency in production:

$$U'(Q^B) = c \tag{42}$$

Combine (38) and (40) and we get:

$$-(x + \delta) + \{(x + \gamma + \eta)J - \alpha K\} = -(x + \delta) + (x + \gamma + \eta)N - \left( \frac{\pi^G}{\pi^B} \right)N$$

$$(x + \gamma + \eta)J - \alpha K = (x + \gamma + \eta)N - \left( \frac{\pi^G}{\pi^B} \right)N$$

or

$$\begin{aligned}
& (x + \gamma + \eta) \{ (1 - \pi^B) B' [p^B Q^B - cQ^B - F] + \pi^B B' [p^B Q^B - cQ^B - F - L + q^B] \} \\
& - \alpha \{ (1 - \pi^G) B' [p^B Q^B - cQ^B - F] + \pi^G B' [p^B Q^B - cQ^B - F - L + q^B] \} \\
& = (x + \gamma + \eta) \{ B' [p^B Q^B - cQ^B - F - L + q^B] \} - \left( \frac{\pi^G}{\pi^B} \right) \{ B' [p^B Q^B - cQ^B - F - L + q^B] \} \quad (43)
\end{aligned}$$

**Proposition 4:** In the case where there are two investors and asymmetric information, the optimal action of government is to less than fully insure or fully guarantee the bad investor:  $q^B < L$ .

**Proof:**

From an inspection of (43),  $q^B = L$  is not possible because  $\pi^B > \pi^G$ .

Next, suppose  $q^B < L$ . This means that the first term on the LHS of (43) is  $<$  the first term on the RHS of (43). For the equality to continue to hold, it must be true that the second term on the LHS of (43) is  $<$  the second term on the RHS of (43). Eliminating terms:

$$\left( \frac{1 - \pi^B}{\pi^B} \right) B' [p^B Q^B - cQ^B - F - L + q^B] < \left( \frac{1 - \pi^G}{\pi^G} \right) B' [p^B Q^B - cQ^B - F] \quad (44)$$

This must be true, because  $\pi^B > \pi^G$  and  $q^B < L$ .

Now, suppose  $q^B > L$ . Eliminating terms:

$$\left( \frac{1 - \pi^B}{\pi^B} \right) B' [p^B Q^B - cQ^B - F - L + q^B] > \left( \frac{1 - \pi^G}{\pi^G} \right) B' [p^B Q^B - cQ^B - F] \quad (45)$$

Manipulating, we get:

$$\frac{\left( \frac{1 - \pi^B}{\pi^B} \right)}{\left( \frac{1 - \pi^G}{\pi^G} \right)} > \frac{B' [p^B Q^B - cQ^B - F]}{B' [p^B Q^B - cQ^B - F - L + q^B]} \quad (46)$$

If  $q^B > L$ , then it must be that the RHS is  $> 1$ . But the LHS cannot be  $> 1$ , because  $\pi^B > \pi^G$ .

Thus, the only possible case is  $q^B < L$ , the government partially insures or partially guarantees the bad investor.

**Proposition 5:** The government's optimal action is to offer two different contracts. One contract, designed to be selected by the good investor, has the terms  $\{Q^G, p^G, q^G\}$ . The other contract, designed to be selected by the bad investor, has the terms  $\{Q^B, p^B, q^B\}$ . In addition,

- 1)  $Q^G = Q^B = Q$ . The contract is designed so that each type of investor has to produce the same quantity of the good or service;
- 2)  $q^B < L < q^G$ ;
- 3) The regulated tariff offered to the bad investor is higher than the regulated tariff offered to the good investor.  $p^G < p^B$ ; and finally

**Proof:**

- 1) The first result follows because both  $Q^G = Q^B = c$  from equations (28) and (42).
- 2) The second result follows because  $q^G > L$  from equation (30) and  $q^B < L$  from equation (44).
- 3) The proof of the third result is based on the following arguments. Take the final self-selection constraint in the maximization problem (17) and impose the first two results:

$$\begin{aligned} (1 - \pi^B)B[p^B Q - cQ - F] + \pi^B B[p^B Q - cQ - F - L + q^B] \geq \\ (1 - \pi^B)B[p^G Q - cQ - F] + \pi^B B[p^G Q - cQ - F - L + q^G] \end{aligned} \quad (47)$$

Since  $q^G > q^B$ , it follows that the only way for the inequality to hold is if  $p^G < p^B$ .

One final issue to be resolved. We now know that  $q^G > q^B$ ,  $p^G < p^B$  and  $Q^G = Q^B = Q$ . But a glance back at the maximization problem confirms that these results are still not sufficient to satisfy the two self-selection constraints (16) and (17). For example, in (16), while  $p^G < p^B$ , it may be that  $q^G$  is set so high so as to violate the constraint. Similarly, in (17), while  $q^G > q^B$ , it may be that  $p^B$  is set so high as to violate the constraint.

There must be some guide to setting insurance coverage or tariffs, so that the self-selection constraints are not violated. This leads to the penultimate proposition.

**Proposition 6:** Under plausible assumptions, (1), (2) and (3) in Proposition 5 imply that the following sequence of inequalities must hold:



$$L - (p^B - p^G)Q < q^B < L < q^G < L + (p^B - p^G)Q \quad (48)$$

**Proof:**

Note that the self-selection constraint (16) in the maximization problem implies that

$$\left( \frac{1 - \pi^G}{\pi^G} \right) \geq \frac{B[p^B Q - cQ - F - L + q^B] - B[p^G Q - cQ - F - L + q^G]}{B[p^G Q - cQ - F] - B[p^B Q - cQ - F]}$$

or

$$\left( \frac{1 - \pi^G}{\pi^G} \right) \leq \frac{B[p^G Q - cQ - F - L + q^G] - B[p^B Q - cQ - F - L + q^B]}{B[p^G Q - cQ - F] - B[p^B Q - cQ - F]}$$

Which can be simplified into

$$1 \leq \pi^G \left\{ \left[ \frac{B[p^G Q - cQ - F - L + q^G] - B[p^B Q - cQ - F - L + q^B]}{B[p^G Q - cQ - F] - B[p^B Q - cQ - F]} \right] + 1 \right\} \quad (49)$$

If  $0 < \pi^G < 1$ , then it must be the case that the bracketed term must be  $> 1$ . This implies that

$$\left\{ \frac{B[p^G Q - cQ - F - L + q^G] - B[p^B Q - cQ - F - L + q^B]}{B[p^G Q - cQ - F] - B[p^B Q - cQ - F]} \right\} > 0 \quad (50)$$

The denominator is negative, since  $p^G < p^B$ . Thus, for the entire term to be  $> 0$ , the numerator must be less than zero. This will only be the case if

$$q^B > q^G - (p^B - p^G)Q \quad (51)$$

Transposing terms, we get

$$q^G < q^B - (p^G - p^B)Q$$

$$q^G < q^B + (p^B - p^G)Q \quad (52)$$

Equation (52), the fact that  $p^B > p^G$  and the fact that  $q^B < L$  imply that

$$\begin{aligned} L &> q^G - (p^B - p^G)Q \\ q^G &< L + (p^B - p^G)Q \end{aligned} \quad (53)$$

Note that we now have an upper limit for the value of  $q^G$ .

Equation (52) and the fact that  $q^G > L$  imply that

$$\begin{aligned} L &< q^B - (p^G - p^B)Q \\ q^B &> L - (p^B - p^G)Q \end{aligned} \quad (54)$$

Note that we now have an upper limit for the value of  $q^B$ .

Under plausible assumptions, therefore, the amount of insurance or guarantee coverage is bounded from above and below

$$L - (p^B - p^G)Q < q^B < L < q^G < L + (p^B - p^G)Q \quad (55)$$

Q.E.D.

All of the preceding results lead to the final proposition:

**Proposition 7:** The pair of optimal contracts  $\{Q^G, p^G, q^G\}$  and  $\{Q^B, p^B, q^B\}$  are incentive-compatible.

**Proof:**

Note that because  $p^B > p^G$ , the term  $(p^B - p^G)Q (> 0)$  is the revenue advantage of the bad investor. Conversely, the term  $(p^G - p^B)Q (< 0)$  is the revenue disadvantage of the good investor.

The bad investor will never accept the contract intended for the good investor because:

- 1) The good investor has a revenue disadvantage and, from (53),  $q^G < L + (p^B - p^G)Q$ . The guarantee coverage intended for the good investor,  $q^G$ , is less than the loss plus the revenue advantage of the bad investor. Put in another way,  $q^G - L < (p^B - p^G)Q$ : the level of over-insurance of the good investor (left hand side of the inequality) is less than the revenue advantage of the bad investor (right hand side); and

- 2)  $q^B + (p^B - p^G) Q > L$  : the guarantee coverage intended for the bad investor,  $q^B$ , plus his revenue advantage, is greater than the loss, so that the bad investor is more than sufficiently compensated for the partial guarantee;

On the other hand, the good investor will never accept the contract intended for the bad investor because:

- 1)  $q^G > L + (p^G - p^B) Q$  : The guarantee coverage intended for the good investor exceeds the sum of his revenue disadvantage and loss (so he ends up with a net gain in utility). This can be seen from that portion of the inequality in (48) where  $L - (p^B - p^G) Q < q^B < L < q^G$ . It follows by transitivity that  $q^G > L - (p^B - p^G) Q$ , so  $q^G > L + (p^G - p^B) Q$ . Put in another way,  $q^G - (p^G - p^B) Q > L$ , so the good investor's guarantee coverage less his revenue disadvantage still exceeds the loss; and
- 2)  $q^G > q^B$ . The guarantee coverage of the good investor exceeds the guarantee coverage of the bad investor. The good investor is more than sufficiently compensated for his revenue disadvantage by receiving generous guarantee coverage. We know this to be true from propositions 3 and 4, as well as the inequality in (48).

**Note, therefore, that we end up with a pair of incentive-compatible contracts. Q.E.D.**

What accounts for these results? The government best manages its contingent liabilities by over-insuring the investor with the smaller likelihood of calling on the guarantee and under-insuring the investor with the higher likelihood of a call. This way, the government can minimize the expected (financial) value of calls on guarantees. The good investor receives a premium (over-insurance).

The contingent liability,  $q$ , only becomes a real, actual liability when the bad state occurs. But this is more likely to occur when the bad investor undertakes the project, so the government offers it only a partial guarantee. The bad investor, however, is compensated by receiving a higher tariff for his output, which consumers pay regardless of the state of nature that occurs.

To ensure incentive compatibility given tariffs, however, the pair of optimal contracts limit the level of over-insurance for the good investor and provide a sufficient partial cover of losses for the bad investor.

Also note the importance of the differential between tariffs allowed to be charged by the two investors. A given differential in tariffs also determines the levels of  $q^G$  and  $q^B$  to be provided. For example, the closer is  $p^G$  to  $p^B$ , the lower  $q^G$  needs to be set to ensure that the incentive compatibility condition  $q^G + (p^G - p^B) Q - L > 0$  holds. But conversely, this also means that a higher  $q^B$  needs to be set to ensure the condition  $q^B + (p^B - p^G) Q > L$  holds.



**Another key property of the solution is that quantities, tariffs and guarantee coverage are endogenous variables. The efficacy of the incentive mechanisms in the set of optimal contracts in this study rests squarely on the endogeneity of these variables during contract negotiations.** Incentive compatibility rests on the recognition that these variables need to be interdependent. In past contracts, perhaps, treating these variables as exogenous has been a contributing factor to the contingent liabilities problem.

## **V. Conclusion**

The solution to the problem of guarantees is similar to those of problems where asymmetric information is present: risk-sharing eventually occurs between the principal and the agent. The risk-sharing contracts we have just derived are optimal and are incentive compatible: although the good investor knows it is more than fully insured, it must keep its losses to a minimum because its per-unit output tariff is lower, so that it will be less capable of using its revenues to cushion the impact of large losses. Meanwhile, the bad investor is aware that it is only partially insured, so even though it sells output at a higher per-unit tariff, it must make an effort to reduce the likelihood of losses.

How can government implement the policy recommendations set forth in this study? In future contract negotiations, it may pay closer attention to the need to endogenize the relationship between quantities, tariffs, and guarantee coverage during actual contract negotiations, the same way these variables are endogenized in this study. Increased recognition of the need for endogeneity should strengthen the government's bargaining position. But this also requires an analysis of the relevant variables prior to the selection of bids from investors. This suggests that the applicability of the recommendations extends primarily to projects solicited by the government. By their nature, these are projects that are rationalized after some extensive studies are undertaken by government.

If the variables in optimal contracts are known to the government, they should be offered to investors *ex ante* – before they are actually selected. This will lead to improved incentives and risk management.

Without introducing any new mechanisms and variables for improved risk-sharing beyond those contained in existing contracts with PSP, this study has demonstrated that it is possible for the government to offer a set of incentive-compatible contracts. The government has the capability of solving for a set of optimal contracts in a scenario where the government contracts with private investors, and at the same time, provides guarantees. This represents the initial study of its kind. More elaborate contract structures may be explored in future work.

## **Bibliography**

Reside, Renato, Jr. E., 2000. "Developing a Risk Management Framework for the Philippine Government" University of the Philippines Discussion Paper No. 0007, July.

Reside, Renato, Jr. E., 2001. "Estimation of the Contingent Liabilities of the Philippine Government." United States Agency for International Development. May.

Sebastian, Iva R. and Maria Teresa P. Silva, 2001. "Evaluating the Risk-Efficiency of Build-Operate-Transfer (BOT) Contracts Using Risk-Based Indicators". Undergraduate thesis, University of the Philippines School of Economics.