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SHARECROPPING, PRODUCTION EXTERNALITIES AND THE THEORY OF CONTRACTS

by

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A B S T R A C T

A corollary of the Debreu-Scarf theorem is that, under ideal conditions, contracts serve as perfect substitutes for markets. Applying this proposition to sharecropping provides a rigorous foundation for the competitive theory of share tenancy. In addition, it is shown that the competitive theory serves as a good approximation even for a small number of landowners.

Applying the proposition to production externalities provides a precise and non-trivial version of the Coase theorem, i.e., the contracting equilibrium is equivalent to a Walrasian equilibrium with universal markets, including a market for the "externality."

Thus under ideal conditions the contract solution, the market solution, and the government intervention solution (e.g. Pigouvian taxes) are identical. Their relative efficiency cannot be assessed without incorporating transactions costs into the analysis. Even without such complication, however, the abstract model developed here appears to be useful for positive purposes, such as explaining certain patterns in agricultural contracts.

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JAMES ROUMASSET*

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In his most celebrated article, Coase (1960) showed that efficiency is not a necessary consequence of externalities; under well-defined property rights, voluntary arrangements are possible which can internalize externalities. In a similar spirit, Cheung (1969) has shown that share tenancy does not necessarily cause allocative inefficiency; the terms of a sharing arrangement may in fact pay factors their marginal products.

Both contributions can be viewed as different applications of the same point, namely that, under certain conditions, contracts are perfect substitutes for competitive markets. The purpose of the present paper is to illuminate what those conditions are and to show that a related theorem which has been developed and proven by Debreu and Scarf (1963) in another context serves as a solid cornerstone of the theories of sharecropping, production externalities, and contracts in general.

A modified version of the Coase/Cheung proposition is stated in part I as a corollary of the Debreu-Scarf theorem. Since proofs of the theorem generally rely on advanced mathematics,[1] part II presents a demonstration of the proposition using an example of a simple sharecropping economy. Part III

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extends the proposition to production externalities. The last section discusses how the proposition can be used to explain some patterns in agricultural contracts and how other patterns can only be explained by incorporating enforcement costs, information costs, and bargaining costs.

I. A FIRST-BEST THEORY OF CONTRACTS

Perhaps the most fundamental concept in the theory of resource allocation is the Walrasian or "competitive" equilibrium.[2] The relevance of the Walrasian equilibrium has been much assailed on the grounds that real economic agents do not act as price takers, and, even if they did, there is no strong reason for believing that an equilibrium would be attained. Indeed, the relevance of the Walrasian equilibrium seems to rest on the *ad hoc* defense of a mythical Walrasian auctioneer.[3]

Roughly a century ago, Edgeworth (1881) defended the relevance of the Walrasian equilibrium using the concept of the core of an economy. The Walrasian equilibrium is a non-cooperative concept based on the notion that agents respond to parametric prices without interagent negotiation. In contrast the core is a solution concept for cooperative games wherein interagent bargaining is costless. Specifically, the core is the set of *unblocked* allocations. (An allocation is said to be *blocked* if some subset of agents can form a new coalition whereby it can guarantee that all of its members will be better off.) Edgeworth showed that as the number of consumers in a two-commodity exchange economy is increased, the core "shrinks" until, in the limit it contains only the Walrasian equilibrium.[4]

Edgeworth's "limit theorem" was resurrected by Shubik (1959) and extended to many commodities and production by Debreu and Scarf (1963). It is now commonly referred to as the Debreu-Scarf theorem.[5] The entire literature on the relationship of Walrasian equilibria and the core, including further extensions, is reviewed by Hildenbrand (1974).[6]

While the primary interest in the limit theorem has been to establish the relevance of the Walrasian equilibrium, it also serves as the basis for a theory of contracts. Just as the Walrasian equilibrium is a market solution, the core, with its assumption of unrestricted bargaining, can be regarded as a contract solution. Thus, the limit theorem implies that, with many traders, the contract solution is the same as the market solution.

More precisely, we can now state the following proposition:

> *If property rights are well-defined and contracting costs are zero, then as the number of agents of each type becomes large, all viable contractual arrangements approximate the Walrasian equilibrium with universal markets, regardless of whether markets exist or not.*

This is a corollary of the Debreu-Scarf theorem. How close the approximation is depends on how large is the number of agents. "Universal markets" is used, following Arrow (1969), to mean that markets exist for all commodities and activities which enter into either utility functions or production functions. The above proposition is a "first-best" theory of contracts in the sense that "second-best" considerations, i.e., transactions costs and other constraints, are ignored.[7]

When the corollary is applied to share contracts, we get Cheung's (1969) proposition that share tenancy allocates resources as if a profit-maximizing farmer hired land and labor from competitive factor markets. When it is applied to production externalities, we get a precise and non-trivial version of the Coase theorem. These points are elaborated in parts II and III below.

II. THE CORE OF A SHARECROPPING ECONOMY

Cheung (1969) conjectured that aside from the allocative effects of uncertainty and contracting costs, the equilibrium terms of share contracts will be such that factors are allocated and compensated as if they were hired on competitive factor markets. His reasoning was that if the terms of the share contract were such as to induce inefficiency, then recontracting would occur to exploit that inefficiency. Furthermore, if a landowner or tenant were receiving less than his marginal product, he would take advantage of competition and recontract where he could receive his marginal product.

Cheung's attempt to provide a mathematical proof of his conjecture was not successful, but Reid (1976) later proved that if landowners and tenants regard the terms of the contract (labor intensity and the sharing rate) as fixed, then the equilibrium contract terms are those which pay factors their marginal products. Given such terms, the allocation of land and labor will be identical to the competitive market solution.[8] The assumption of contract-term-taking behavior, however, appears to be inappropriate in a one-to-one contractual arrangement such as share contracting, where the contracting parties are frequently in contact. Since the costs of negotiation are relatively low, a bargaining model seems more appropriate.[9]

~~However,~~ Applying the corollary from part I, we see immediately that in a bargaining context, sufficient competition guarantees that the Walrasian equilibrium or Cheung solution is attained even if the landowner and tenant regard the terms of the contract as negotiable. We now turn to a demonstration of this proposition. At the same time we investigate how "fast" the core shrinks to the Walrasian allocation.

Consider a simple one-good (rice) economy consisting of one landowner with land, H, and one tenant with labor, L. Production technology for rice is given by: $Q = H^{\frac{1}{2}}L^{\frac{1}{2}}$. Utility of each is assumed to be an increasing function of his own share of the rice production. Since there are no other uses for land and labor, the core solution requires that all land and labor be employed in rice production. The only remaining variable to be established is the landowner's percentage share, r, of the gross harvest. But since neither the landowner or tenant can guarantee himself any output whatsoever -- land or labor must be jointly employed -- there is no value of r which can be "blocked" by either tenant or landowner. That is, the core consists of all values from

0 to 1. In set notation, we denote the core for one landowner and one tenant, C_{11}, as

$$C_{11} = \{r | 0 \leq r \leq 1\},$$

i.e., all values of r such that r is not less than 0 nor greater than 1.

Now consider an economy with two (identical) landowners and two (identical) tenants. Without loss of generality, we further assume that each landowner has one unit of land and each tenant has one unit of labor. A landowner with one tenant working his land therefore receives income equal to r. The extent to which landowners can "exploit" labor, i.e., r_{max}, is limited by the potential of one landowner breaking the coalition with the other and forming a new coalition with the two tenants. r_{max} is that rental share such that if r were to rise any higher, then a coalition of one of the landowners and the two tenants could guarantee themselves a higher payment than in the two-landowners two-tenants coalition. After paying the tenants what they were receiving in the old coalition, the landowner's share in the present coalition is $1^{\frac{1}{2}}2^{\frac{1}{2}} - (1 - r)2^{\frac{1}{2}}2^{\frac{1}{2}}$. To compute r_{max}, set this residual equal to the income received by each landowner assuming that landowners received equal share in the old coalition, i.e., r. (If they received different shares, then the landowner with the lower share could unite with the tenants and block the old coalition at some level less than r_{max}.) Therefore, r_{max} is the solution of the equation,

$$1^{\frac{1}{2}}2^{\frac{1}{2}} - 2(1 - r) = r,$$

$$\text{i.e., } r_{max} = 2 - 2^{\frac{1}{2}} = .586.$$

To solve for r_{min}, the lowest undominated landowner share, simply reverse the roles of land and labor and investigate how much the "tenants can exploit their landowners," i.e., how much income they can extract before it becomes possible for two landowners and one laborer to form a blocking coalition. Thus, $r_{min} = 2^{\frac{1}{2}} - 1 = .414.$

The limits of the core for the general case can be derived by induction. For n landowners and n tenants, the relevant blocking coalition (RBC) is simply the coalition of all agents excluding one agent with the highest share.[10] In order to prevent the RBC from becoming effective, the group with the higher share (landowners or tenants) will divide their joint income equally. If landowners have the advantage, the RBC will exclude one of the landowners with an equal share of total landowners' income.

Thus the RBC which limits landowner exploitation is the coalition of n tenants and n-1 landowners, i.e., $RBC_{(n-1),n}$. r_{max} is therefore the solution of the equation,

$$(n - 1)^{\frac{1}{2}} n^{\frac{1}{2}} - n(1 - r) = (n - 1)r$$

$$\text{or } r_{max} = n - (n - 1)^{\frac{1}{2}} n^{\frac{1}{2}}.$$

Similarly, r_{min} is computed with reference to $RBC_{n,(n-1)}$, i.e., r_{min} is the solution of:

$$n^{\frac{1}{2}}(n - 1)^{\frac{1}{2}} - nr = (n - 1)(1 - r)$$

$$\text{or } r_{min} = n^{\frac{1}{2}}(n - 1)^{\frac{1}{2}} - (n - 1) = 1 - r_{max}.$$

This formula was used to simulate the effects of increasing the number of agents on the core. Table 1 shows the limit of the core, r_{max} and r_{min}, for economies with increasing number of agents. It is readily seen that the core shrinks to the competitive equilibrium solution, i.e., $r = .5$. Morever, the shrinking is "fast." With only three pairs of landowners and tenants, the core consists of those solutions which are within approximately ten percent of $r = .5$. More generally, for a production function of the form

$$Q = H^{\alpha} L^{1-\alpha}$$

$$r_{max} = n - (n - 1)^{\alpha} n^{(1-\alpha)},$$

$$r_{min} = 1 - r_{max}$$

and r_{max} and r_{min} converge to α as n becomes large.[11]

Table 1: Core Shrinking for Equal Number of Landowners and Tenants

n (number of landowners)	r_{max}	r_{min}
1	$1 - 0\sqrt{1} = 1$	0
2	$2 - \sqrt{1}\sqrt{2} = .586$	.414
3	$3 - \sqrt{2}\sqrt{3} = .551$	.449
4	$4 - \sqrt{3}\sqrt{4} = .536$	.464
5	$5 - \sqrt{4}\sqrt{5} = .528$	.472
10	$10 - \sqrt{9}\sqrt{10} = .513$	.487
25	$25 - \sqrt{24}\sqrt{25} = .506$	.494
100	$100 - \sqrt{99}\sqrt{100} = .502$	.498
500	$500 - \sqrt{499}\sqrt{500} = .500$	.500

Table 2: Core Shrinking with Twice as Many Tenants as Landowners

n (number of landowners)	r_{max}	r_{min}
1	1	.414
2	.586	.445
5	.528	.486
10	.513	.492
50	.503	.497
500	.500	.500

Behavior of the core with more tenants than landowners

Assume now that there are always twice as many tenants as landowners. In order to focus on the effect of this assumption on the bargaining solution, we assume that each tenant is endowed with ½ of a unit of labor. This suffices to keep the land-labor ratio equal to 1 for all core solutions, just as it was for the economy investigated above.

Beginning with an economy with 1 landowner and 2 tenants, $r_{max} = 1$ since $RBC_{0,2}$ has no land. r_{min} is determined relative to $RBC_{1,1}$ and satisfies

$$1^{\frac{1}{2}}(\tfrac{1}{2}) - r = \frac{(1 - r)}{2},$$

$$\text{i.e., } r_{min} = \frac{2}{\sqrt{2}} - 1 = .415.$$

Again proceeding by induction, the limits of the core for the general case of n landowners and 2n tenants are:

$$r_{max} = n - (n - 1)^{\frac{1}{2}} n^{\frac{1}{2}}$$

$$\text{and } r_{min} = 2n^{\frac{1}{2}}(n - \tfrac{1}{2})^{\frac{1}{2}} - 2n + 1.$$[12]

The limits of the core for various values of n are given in Table 2.

Tables 1 and 2 about here

Notice that the r_{max} column in Table 2 is identical to that in Table 1. This is because the RBC with n-1 landowners and 2n tenants, each with ½ unit of labor, is indistinguishable from the RBC (from case one) of n-1 landowners and n tenants, each with 1 unit of labor. On the other hand, r_{min} is no longer symmetrical to r_{max}. The larger number of tenants makes exploitation of landowners more difficult because leaving one tenant behind and forming a new coalition with landowners now becomes less costly in terms of the output foregone.

Limiting our attention to economies where the number of landowners does not exceed the number of tenants, we can now draw the following conclusions from these and similar exercises. First, the core shrinks rapidly to the Walrasian equilibrium regardless of the ratio of tenants to landowners. Second, for production function of the form $Q = H^{\alpha}L^{(1-\alpha)}$, r_{max} converges to α[13], the rate of convergence being independent of the tenant-landowner ratio. Third, $r_{max} \geq r_{min} \geq 1 - r_{max}$, i.e., r_{min} converges to α (from below) at least as fast as r_{max} approaches α (from above).

These results provide a justification for the assumption of contract-term-taking behavior on the part of the landowners and tenants. Despite the assumption that landowners and tenants are free to bargain with one another about possible deviations from the equilibrium set of contracts, no such changes are viable in the sense of making all of the recontracting parties better-off. Thus, Cheung's (1969) intuition, that the power of individual landowners to establish inefficient exploitative contracts is limited by competition among landowners, can be shown to be formally correct. Furthermore, it has been established that for the competition to be effective, the requisite number of landowners (with at least as many tenants) is relatively small.

III. APPROPRIABLE PRODUCTION EXTERNALITIES AND THE COASE THEOREM

Coase (1960) has been credited with the insight that externalities, i.e., non-market interdependencies among agents, are not a sufficient condition for inefficiency. Private negotiation may suffice to "internalize" the externality. More specifically, Coase asserted that if transactions costs are zero, private negotiation *will* remove the externality. This simple idea has been extracted from his article -- a profound and pioneering treatise on law and economics -- and labeled "the Coase theorem."

There has been an extensive debate in the economics literature regarding the proper statement of the Coase theorem and whether or not various versions are correct. The following is one of the better stated versions.

> *If costless negotiation is possible, rights are well-specified, and redistribution does not affect marginal values, then*
>
> 1. *The allocation of resources will be identical, whatever the allocation of legal rights.*
> 2. *The allocation will be efficient, so there is no problem of externality.*[14]

As Arrow (1969) notes, the second part of the theorem can only be proven by assuming that the only admissible solution concepts for cooperative games are those which are Pareto optimal by definition. That is, since the conclusion must be stated in the premise, the proposition is trivial.

However, by applying the limit theorem to the case of production externalities, we get a precise, non-trivial, and meaningful version of the Coase theorem, to wit:

> *The core of an economy with appropriable spillover effects in production shrinks to the Walrasian equilibrium with universal markets as the numbers of generators and recipients of the spillover grow large.*

Thus the proposition states that the contracting equilibrium in an economy with spillover effects is the solution that would exist if there were markets for all commodities, including the spillover effect. A hypothetical institution of a market for a spillover effect is Dales' (1968) market in "polluter's rights."

The limitation to "appropriable" spillover effects is critical here. An externality is "appropriable" in the sense that a particular unit of the external effect "spills over" onto one particular recipient. In contrast, a "joint" externality is one where the same unit of externality, e.g., smoke, spills over onto more than one recipient. Most of Coase's (1960) examples of externalities were appropriable production externalities. The rancher's cattle trample or eat a particular farmer's crop and the confectioner's noise bothered a particular dentist. Even the railroad's sparks are appropriable since each spark falls on the land of a specific farmer.

For appropriable spillover effects, our corollary applies immediately. In a model without transactions costs, there is nothing to analytically distinguish, for example, the pollinating services of bees from an ordinary commodity. That is, the Walrasian equilibrium is well-defined and includes a market clearing price for pollination. With large number of agents, including many beekeepers and apple orchard owners, the core also shrinks to the Walrasian equilibrium. Thus, the contracting equilibrium is just the Walrasian equilibrium with universal markets.

For a joint externality such as air pollution, however, the revised Coase proposition does not apply. The universal market solution, also known as the Lindahl equilibrium, still exists but involves different agents paying different amounts for the externality according to their individual marginal benefits.[15] But the core no longer shrinks in this case.

To see intuitively why the core does not shrink where externalities are not appropriable, consider the case of a public good. A pure public good can be analyzed as a positive production externality which spills over onto all the members of the community. Now since all the members of the community are needed as members of the coalition which consumes the public good in order to minimize its average cost, there are no viable competing coalitions which can provide the competition required to make the core "shrink." Thus, the core of an economy with joint externalities or public goods contains a large number of solutions, each with its own prescription for sharing the surplus generated by internalizing the externality or by group provision of the public good.[16]

It is instructive to apply the revised Coase theorem to the case of water pollution. Following Starrett and Zeckhauser (1974), consider the case of Upton's paper mill which dumps waste into a river which is used by Downley Baths. The universal market solution for this example is depicted in Figure 1. The graph shows the value of the marginal product, VMP, (to Upton) of the right to dump G units of "garbage" in the river and the minimum marginal cost, MC, to Downley

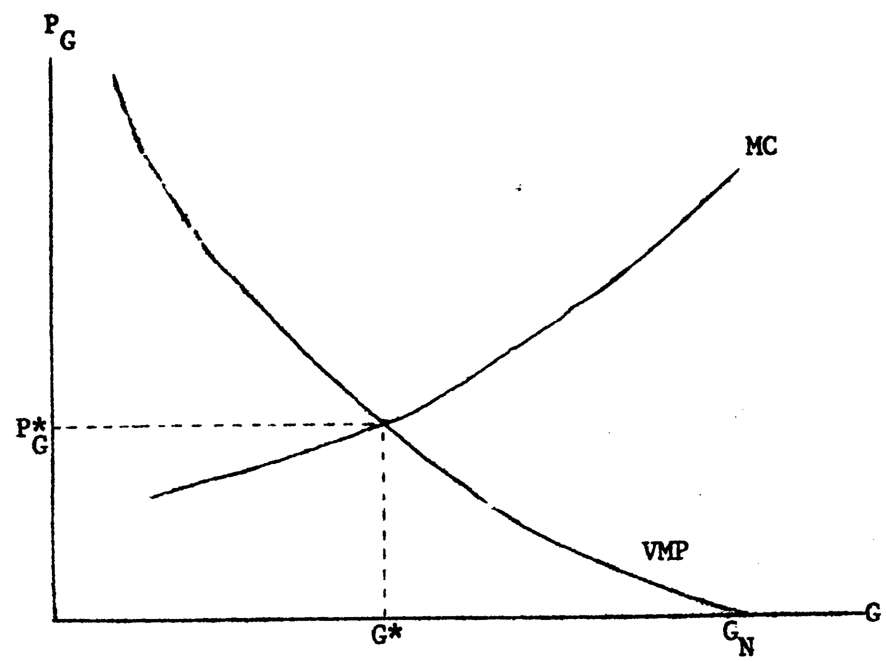

Figure 1: The Lindahl Pollution Solution

Figure 1 about here

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of successive units dumped. In the case of pollutee rights, the universal market solution involves Upton buying G^* "pollution rights" (Dales, 1968) from Downley at the equilibrium price, P_G^*. This is equivalent to imposing a Pigouvian tax, $t = P_G^*$, on Upton per unit of G discharged and giving the proceeds to Downley on the basis of the minimum marginal cost.[17] Both the market and tax solutions are also equivalent to the competitive contract solution wherein Upton pays Downley $P_G^* G^*$ for the right to dump G^*.

There is a different set of equivalent solutions under polluter rights. For the universal market solution, polluter rights may be represented by initially endowing Upton with G_N pollution rights. In market equilibrium, Upton sells $G_N - G^*$ rights to Downley at price P_G^*. This is equivalent to Downley paying Upton a Pigouvian subsidy, $s = P_G^*$, for each unit that Upton reduces G below the profit maximizing level, G_N. It is also equivalent to the contracting solution wherein Downley pays Upton $P_G^*(G_N - G^*)$ to reduce pollution to G^*.

In general, the core shrinks to a different universal market solution for each specification of property rights.[18] If we assume that changing property rights "does not affect marginal values" (*ala* the Layard and Walters version of the Coase theorem) then the production of the externality is independent of property rights. We have implicitly followed this assumption in our discussion of Figure 1. Typically, however, marginal values will be affected, via both income effects and changes in relative factor prices. In the long run, we would normally expect a shift to polluter rights to be associated with an increase in pollution, although the amount of the change may be small.

As usually presented, the Coase solution holds the equilibrium quantity to be uniquely determined but the division of surplus between contracting parties to be indeterminate. The unaffected-marginal-values assumption is thus necessary to prevent an inconsistency. This assumption becomes unnecessary in the revised Coase theorem, since both quantities and income distribution is determined.

The applicability of the competitive contracting solution may be questioned in situations where there are apparently only two parties involved. But as Coase has noted in verbal discussions of his paper, it is competition prior to the contract that counts. That is, even for one-to-one appropriable externalities there is *ex ante* competition in the sense that the parties involved can choose among various locations, and several parties may be competing for a particular location.[19]

IV. APPLICATIONS AND LIMITATIONS OF CCMPETITIVE CONTRACT THEORY

By abstracting from transactions costs, we are left with a theory which is incapable of explaining many of the rich variations of real world contract. Despite its simplicity, however, we have found the theory capable of explaini a number of interesting patterns observed in sharecropping contracts. In particular, it explains the following *ceteris paribus* observations:

a. Landowner's (percentage) share tends to be higher the better is land quality.

b. Landowner's share tends to be higher the lower is the prevailing real agricultural wage.

c. Landowner's share tends to fall with land saving technological change.

These patterns are documented and explained by Roumasset and James (1979).[20]

One advantage in appealing to the Debreu-Scarf theorem as a cornerstone of the theory of contracts is that we have a list of sufficient postulates under which the theorem is valid. This gives us clues about when the theorem is and is not applicable. Thus, since the core does not shrink in the case of joint externalities (e.g., air pollution) and public goods, we must exercise caution in applying competitive contract theory in such cases.

Another limitation of the theory regards restrictions on production. The Debreu-Scarf theorem requires that the production possibility correspondence is *additive* (Hildenbrand, 1974, p. 212). Additivity is satisfied by constant returns-to-scale. If economies-of-scale prevail, then it is possible that the Walrasian equilibrium does not exist and that the core is empty (Arrow, 1969). If decreasing returns prevail, then the core may contain allocations besides the Walrasian equilibria, regardless of the number of agents.[21]

V. CONCLUSIONS

Abstracting from transactions costs and bargaining power, the equilibrium contracting solution is just the Walrasian equilibrium with universal markets. This cornerstone of contract theory may help clear up some controversies about sharecropping and production externalities.

Applying the proposition to sharecropping, we get Cheung's (1969) competitive theory of share tenancy. But instead of relying on the assumptio that agents are "contract-term-takers (e.g., as in Reid, 1976), the proof involves showing that the core shrinks to the Walrasian equilibrium. In the process of this demonstration, we learn that the core shrinks "fast," i.e., t even for a few landowners, solutions which deviate substantially from the Walrasian equilibrium are not viable.

Applying the proposition to production externalities, we see that the competitive theory of contracts serves as a revised version of, or a substitute for, the Coase theorem. That is, under zero transactions costs and where the production externality situation is characterized by precontract competition, the contracting solution is identical to the market solution where a market exists for the externality good. Furthermore, for any given specification of property rights, the contracting solution is identical to the universal market solution (Lindahl equilibrium) and is also equivalent to a Pigouvian solution. Thus, the efficiency of the contracting solution, the Pigouvian solution, and the market solution cannot be compared on "first-best" grounds as several authors (e.g., Baumol, 1972) have tried to do. As Coase has said, "*the real choice in economic policy is a choice of institutional forms*,"[22] and a complete evaluation inevitably requires consideration of transactions costs and excess burden of alternative arrangements.

For positive analysis, however, even the simple theory of competitive contracts may be sufficient to explain some of the stylized facts of economic organization, e.g., the positive relationship of landowner's share to land quqlity and physiological population density. Explanation of other patterns will require incorporating transactions costs and bargaining power into the analytical framework.[23]

FOOTNOTES

1. Debreu and Scarf (1963) and Hildenbrand (1974). Recently, however, Johansen (1978) has provided an exposition which relies only on calculus.

2. As Hildebrand (1974) has noted, "Walrasian equilibrium" is at once more descriptive and more specific than the more conventional "competitive equilibrium."

3. Weintraub (1977).

4. Debreu and Scarf (1972).

5. Debreu and Scarf also extended the limit theorem to apply to the case of multiple Walrasian equilibria.

6. Hildenbrand also credits Aumann (1964) for making precise the notion that agents will take prices as given if they have no influence on them. That is, Aumann's "equivalence theorem" shows that given reasonable restrictions on preferences, the set of Walrasian equilibria and the core are identical for an atomless exchange economy.

7. This distinction between first-best and second-best has become common only recently (see e.g., Layard and Walters, 1978, sections 6-1 and 6-2). The "theory of the second-best" of the 1950's is "If one of the standard efficiency conditions cannot be satisfied, the other efficiency conditions are no longer desirable." (Layard and Walters, p. 181). In modern terminology, a second-best problem involves solving for efficiency conditions with one or more additional constraints to the basic restrictions on technology, resources and preferences. One such additional constraint has captured the attention of economists recently and has led to a sizeable body of literature known as the theory of optimal taxation. The constraint is that government must generate a fixed amount of tax revenue, but it cannot tax leisure. Second-best efficiency is also

referred to as "constrained Pareto optimality." The relationship to the "theory of the second-best" is straightforward since the reason we cannot satisfy one of the first-best efficiency conditions is that there must be at least one additional constraint.

8. Reid also generalized his model to allow for uncertainty and showed that sharecropping is equivalent to a mixture of wage and rent contracts.

9. Bell and Zusman have also argued for a bargaining approach, but they have, rather arbitrarily, chosen the Nash solution as the appropriate solution concept, assumed that contracts stipulating the amount of labor to be applied have no effect on the amount of labor actually contributed, and also assumed that landowners strike independent bargains with each of their tenants (i.e., that a subgroup of tenants cannot bargain as a unit).

10. For example, in the 6-person game, r_{max} is similarly determined with reference to the 2-landowner, 3-laborer coalition, i.e., the landowner coalition is blocked if r is above the break-even level given by:

$$2^{\frac{1}{2}}3^{\frac{1}{2}} - 3(1 - r) = 2r$$

$$r_{max} = 3 - 2^{\frac{1}{2}}3^{\frac{1}{2}}$$

$$r_{max} = .551$$

$$\text{and } r_{min} = .449$$

11. This convergence is readily checked by substituting any number between 0 and 1 for α. A more formal demonstration exploits the device for Taylor series expansion.

12. For an economy with 2 landowners and 4 tenants, r_{max} is determined by (from $RBC_{1,4}$)

$$1^{\frac{1}{2}}\left(\frac{4}{2}\right)^{\frac{1}{2}} - \frac{4(1 - r)}{2} = r,$$

$$\text{i.e., } r_{max} = 2 - \sqrt{2} = .586$$

r_{min} is determined relative to $RBC_{2,3}$ and is given by

$$2^{\frac{1}{2}}(\frac{3}{2})^{\frac{1}{2}} - \frac{3(1 - r)}{2} = 2r$$

i.e., $r_{min} = .445$.

13. This is readily confirmed by substituting other values for α, besides $\alpha = \frac{1}{2}$ or by substituting α and $1 - \alpha$ in the general expression for r_{max} taking the limit as $n \to \infty$.

14. Layard and Walters (1978, p. 192).

15. Bergstrom (1975, 1976).

16. For a more rigorous demonstration that the core does not shrink to the Lindahl equilibrium for the case of public goods, see Bergstrom (1970) and Muench (1972).

17. That is, Downley is compensated according to the marginal cost he would incur if he invested optimally in pollution avoidance activities (water filters, etc.). Since the compensation is not tied to Downley's actual behavior, it does not cause Downley to underinvest in avoidance activities as Baumol (1972) claims.

18. This result appears to be inconsistent with an example provided by Starrett (1973) wherein the core does not contain the Lindahl equilibrium (universal market solution). The paradox is resolved by noting that Starrett implicitly compares the core under polluter rights with the Lindahl equilibrium under pollutee rights.

19. Harold Demsetz, personal communication.

20. The explanatory models used combine the first-best theory of contracts with reasonable and testable restrictions about the underlying production functions.

21. For an example, see Böhm (1973). One suspects that such examples could be avoided by a complete specification of property rights (to the fruits of production), but that is a matter for further research.

22. Ronald Coase, personal communication.

23. For a preliminary description of such a frameowrk, see Roumasset (1979).

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