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## A THRESHOLD MODEL OF FERTILITY BEHAVIOR

José Encarnación, Jr.\*

### I. Introduction

This paper formulates a model for an explanation of fertility behavior in LDCs. The model, using "natural fertility" and lexicographic utility assumptions, implies threshold values for family income and the wife's educational level such that below the threshold,<sup>1/</sup> fertility increases with more income and more education. Considering that a majority of families in most LDCs probably fall below the threshold, one thus expects a rise in fertility rates during the early phases of economic development as a result of rising incomes and widening of educational opportunities. A further implication concerns the rapidity of the demographic transition -- the time required would depend on the proportion of families below the threshold.

The concept of lexicographical preferences is briefly sketched in Section II.<sup>2/</sup> Section III describes the model, and empirical research

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<sup>1/</sup>We use the singular "threshold" to refer both to income and education thresholds.

<sup>2/</sup>See Fishburn (1974), esp. pp. 1450-53 on "pragmatic modifications and examples," for a review of some applications of the lexicographic principle to the description of choice.

findings consistent with its implications are cited in Section IV.

Concluding remarks are made in Section V.

## II. Lexicographical Preferences

Let  $x$  and  $y$  be elements in a set over which one has preferences. The use of a standard (real-valued) utility function assumes that such preferences can be represented by means of a real-valued function  $u$  such that  $x$  is preferred to  $y$  if and only if  $u(x) > u(y)$ . Whether or not preferences, which are logically prior to utility functions, can be so represented adequately is an empirical question. There are situations where a vector-valued function is needed.

Suppose a sequence of real-valued functions  $u_1, u_2, \dots$ , so that to each  $x$  corresponds the utility vector  $(u_1(x), u_2(x), \dots)$ , such that  $x$  is preferred to  $y$  if and only if the first nonzero difference in the sequence  $u_1(x) - u_1(y), u_2(x) - u_2(y), \dots$ , is positive. Then we say that we have a lexicographical preference ordering of the elements. That is, their preference ordering is defined in terms of the lexicographic ordering of the corresponding vectors. The idea is that, going through the sequence  $u_1, u_2, \dots$ , one determines where  $x$  and  $y$  first differ. If they first differ in regard to  $u_j$ , then  $x$  is preferred to  $y$  if  $u_j(x) > u_j(y)$ .

One may consider the functions  $u_i$  like standard utility functions except that they have a priority ranking among themselves.

Only when all the  $u_i$  that have higher priority (i.e. those with lower indices) are equal for  $x$  and  $y$  can a lower priority one mark the preference direction between them.

The particular case of a lexicographical preference ordering to be used involves "satisfactory" values of the  $u_i$ , viz.  $u_1^*, u_2^*, \dots$ , such that if  $u_i(x) \geq u_i^*$  and  $u_i(y) \geq u_i^*$ , one then turns to  $u_{i+1}$  to compare  $x$  and  $y$ . More precisely, all values of  $u_i$  that are not less than  $u_i^*$  count the same for the purpose of stating the preference ordering. Accordingly we put  $t_i(x) = \min(u_i(x), u_i^*)$ ,  $i = 1, 2, \dots$ , and with each  $x$  associate the vector  $(t_1(x), t_2(x), \dots)$ . The lexicographic ordering of such vectors then defines, as before, the preference ordering.

### III. The Model

We consider  $n$  goods  $X_1, \dots, X_n$ , measured in the same units<sup>3/</sup> but increasing in quality and price with the index. One can therefore write

$$(1) \quad X^i = \sum_{r=i}^n X_r \quad (i = 1, \dots, n)$$

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<sup>3/</sup>Various kinds of food are measurable in units of weight, for example. It would be easy to rewrite the model with many goods measurable only in different units, but that would merely require more notation without adding anything particularly important for present purposes.

to denote the number of units of various goods having quality  $i$  or better. Similarly, consider  $m$  quality-levels of children  $C_1, \dots, C_m$ , and write

$$(2) \quad C^v = \sum_{s=v}^m C_s \quad (v = 1, \dots, m)$$

for the number of children of quality  $v$  or better.

Our assumption on the lexicographical ordering is that

$$(3) \quad u_i = u_i(X^i, C^v) \quad (i = 1, 2, \dots)$$

where

$$(4) \quad v = \max(k(E), g(Y)).$$

$Y$  is family income,  $E$  the wife's educational level,  $g(Y)$  the quality that a family with income  $Y$  wants the children to have, and  $k(E)$  the similar quality if the wife has education  $E$ ;  $k$  and  $g$  are assumed to be monotone increasing functions of  $E$  and  $Y$  respectively.

The rationale underlying (3), in regard to  $X^i$ , is simply that quality improvements are secondary to having enough of the basic necessities for plain subsistence. Regarding  $C^v$ , we are assuming the same argument in all the functions  $u_i$ , and as defined in (4),  $C^v$  depends on  $E$  and  $Y$ . For given  $E$ ,  $k(E)$  serves as a floor for  $v$ , for at low  $Y$  such that  $g(Y) < k(E)$ ,  $v = k(E)$ . We interpret  $k(E)$  to be the quality-level of a child that has (or is planned to have) an educational level  $E$ . In other words, a woman wants her children to reach at least the same level of education that she has had. If  $g(Y) \geq k(E)$ ,

$v = g(Y)$ , and we have a case of income-dependent preferences that appears quite reasonable.

We further assume that "natural fertility" or the capacity number of children a woman can bear,  $CK$ , depends positively on  $E$  and  $Y$ :<sup>4/</sup>

$$(5) \quad CK = f(E, Y).$$

The basis is better nutrition, health and medical (prenatal) care afforded by more income, and the better knowledge of good health practices and nutritional values that more education brings. Finally, we assume that the number of child deaths in a family, or child mortality  $CM$ , depends negatively on  $E$  and  $Y$ :

$$(6) \quad CM = h(E, Y)$$

for reasons opposite those regarding (5).

The fertility behavior of a couple can then be described by the solution to the following problem. Given  $E$  and  $Y$ ,<sup>5/</sup> maximize

$$(7) \quad u_j(X^j, C^V)$$

where  $j$  is the highest index compatible with satisfying the constraints

(8)-(10):

$$(8) \quad u_i(X^i, C^V) \geq u_i^* \quad (i = 1, \dots, j-1)$$

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<sup>4/</sup>Cf. Tabarrah (1971), Encarnación (1973), Easterlin (1975).

<sup>5/</sup>The question might be raised that family income being dependent on the income-earning activities of the children, one cannot take  $Y$  as "given." This point would be relevant for econometric estimation of the relationships involved, as  $Y$  and  $C^V$  would be simultaneously determined in a broader setting of the problem. This does not affect the fact, however, that the maximization problem determines  $C^V$  as a function of  $E$  and  $Y$ .

$$(9) \quad \sum_{r=1}^n p_r X_r + \sum_{s=v}^m q_s C_s = Y$$

$$(10) \quad C^v \leq f(E, Y) - h(E, Y).$$

In the budget constraint (9), the  $p_r$  and the  $q_s$  are prices and costs at the different quality-levels, with  $p_1 < p_2 < \dots$  and  $q_1 < q_2 < \dots$ . By (10), the number of (surviving) children does not exceed the capacity number less child mortality.

It is clear that the solution is determinate and would involve  $X_j$  (quality  $j$  only) but not  $X_{j+1}$  since the latter costs more and the maximand (7) is indifferent (so to speak) between  $X_j$  and  $X_{j+1}$ . On the other hand,  $X_{j-1}$  will generally be needed to satisfy (8) for  $j - 1$ , where  $X^{j-1}$  would include both  $X_{j-1}$  and  $X_j$ . Regarding  $C^v$ , while  $C_{v+1}$  is zero in the solution because of its higher cost,  $C_{v-1}$  does not appear either since all the  $u_i$  have  $C^v$  as argument. Thus  $C^v = C_v$  and a family will have children of the same quality.

As income increases, more  $X_j$  would appear in the solution (replacing<sup>6/</sup> some  $X_{j-1}$ ) and if the increase in  $Y$  is sufficiently large so that  $u_j^*$  becomes attainable,  $u_{j+1}(X^{j+1}, C^v)$  takes the place of  $u_j(X^j, C^v)$  as the maximand and (8) is augmented by the additional condition  $u_j(X^j, C^v) \geq u_j^*$ . In regard to  $C^v$ , an increase in  $Y$  when  $g(Y) < k(E)$  does not affect  $v$ ; the number of children (allowing non-integer values) would thus increase. With  $g(Y) > k(E)$ , an increase in  $Y$  raises  $v$ ; it is then possible for the number of children to remain

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<sup>6/</sup> Every good is thus an "inferior" good relative to a better-quality one.

the same, the additional  $Y$  being absorbed by the cost difference of a higher  $v$  as well as more  $X_j$ .

There are two possibilities about the solution: case A, where (10) is satisfied as an equality; and case B, where (10) is satisfied as a strict inequality. If

$$(A) \quad C_v = f(E, Y) - h(E, Y)$$

then  $(\partial u_j / \partial C_v) / q_v > (\partial u_j / \partial X_j) / p_j$  except fortuitously, so that  $C_v$  would be greater and  $X_j$  less were it not for (A). (Figure 1 illustrates. The budget line  $ZZ'$  shown there corresponds to the amount allocated to  $C_v$  and  $X_j$  implied by the solution to the maximization problem.) In other words, the couple is not getting all the children that they want to have. An increase in income (or a shift in the child mortality function as a result of public health and sanitation programs) would therefore simply increase the number of children (not child quality, since a larger  $C_v$  would be wanted with the same  $Y$  in the absence of (10)). If

$$(B) \quad C_v < f(E, Y) - h(E, Y)$$

the marginal rate of substitution equals the price ratio and we have the usual conditions for an equilibrium. An increase in  $Y$  means more  $X_j$  (if not  $X_{j+1}$ ) and  $C_v$  would be larger if  $g(Y) < k(E)$ ; but if  $g(Y) > k(E)$ , the increase in  $Y$  raises  $v$  and the number of children may remain the same but of higher quality.

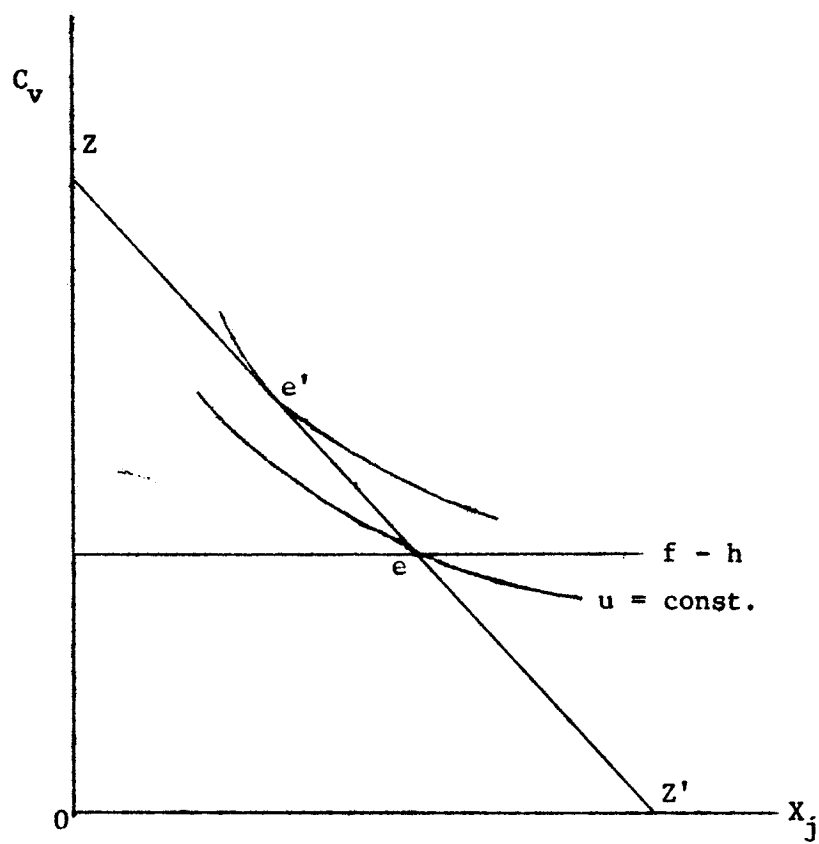


Figure 1

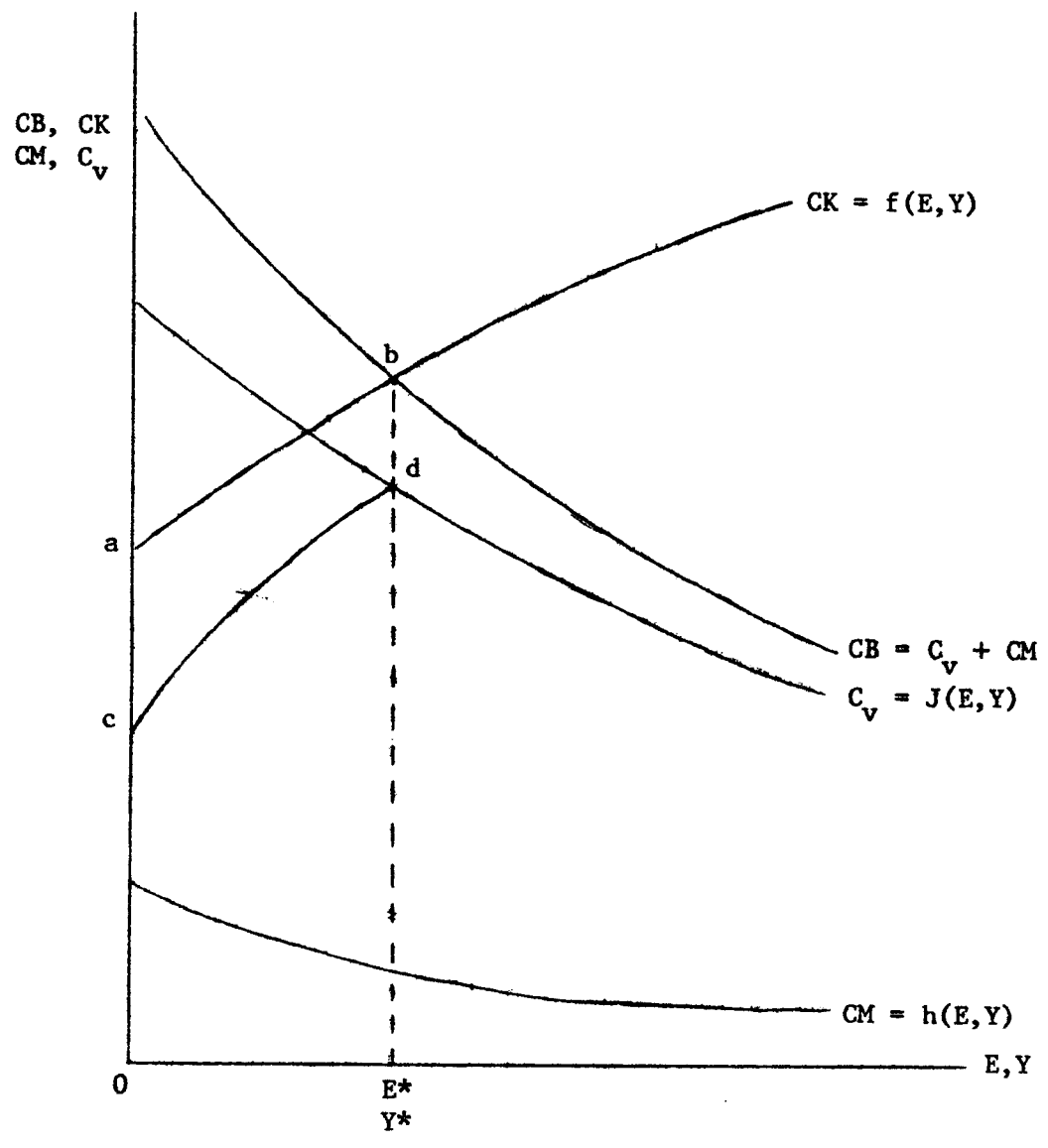


Figure 2

Evidently the difference between the conditions for (A) and (B) to hold is that  $E$  and  $Y$  are sufficiently high in the latter case so that  $C_v$  is not determined by (A). In order to have a simple diagram, suppose for the moment that  $E$  and  $Y$  are perfectly correlated. Then we could have something like Figure 2. The  $CK$  and  $CM$  curves are drawn from (5) and (6). The  $C_v$  curve obtains from solutions of the maximization problem with the constraint (10) omitted:

$$(11) \quad C_v = J(E, Y).$$

This is downward sloping in Figure 2 since  $C_v$  must be less with higher  $E$ , ceteris paribus. What would then be observed for the number of births is the curve  $abCB$ , and for the number of surviving children the curve  $cd\tilde{C}_v$ , where the segment  $cd$  obtains from  $ab$  after deducting  $CM$ . Peak fertility occurs at  $E^*$ ,  $Y^*$ , which may be called threshold values for  $E$  and  $Y$ .

Since  $E$  and  $Y$  are not perfectly correlated, the threshold values can be obtained by the solution of

$$(T1) \quad f(E, Y) = J(E, Y) + h(E, Y)$$

$$(T2) \quad g(Y) = k(E)$$

for  $E$  and  $Y$ . The rationale for (T1) is clear from Figure 2. For (T2), the basis is (4), where there is a value of  $Y$  such that  $g(Y) = k(E)$  for any given  $E$ . We would expect this value of  $Y$  (perhaps the median  $Y$  of families with wives at level  $E$ ) to rise with  $E$ . The locus of points  $(E, Y)$  satisfying (T2) would thus have a positive slope, while the similar locus satisfying (T1) has a negative one, so that

in general the solution  $E^*$ ,  $Y^*$  is unique.  $E^*$  and  $Y^*$  are, of course, not invariant: they would change with shifts in the functions  $f$ ,  $J$ ,  $h$ ,  $g$ , and  $k$ .

#### IV. Implications and Empirical Findings

If the model is broadly correct, fertility will be a nonlinear function of  $E$  and  $Y$  with a maximum at  $E^*$ ,  $Y^*$ , and linear regression estimates of the relationship between fertility and education or income would yield positive, negative, or zero regression coefficients depending on the fraction of families falling below the threshold. While most studies do exhibit (the usually expected) negative coefficients, others also turn up positive ones. Cochrane (1977) has recently reviewed the fertility-education relationship and concludes that "the fairly extensive review of the evidence contained in this monograph shows that the relationship between education and fertility is not always inverse .... there is theoretical and empirical evidence which indicates that education in the poorest regions may increase the biological supply of children" (p. 199).

In Indonesia, Hull and Hull (1977) show that "an inverted U-shaped relation between schooling (hence economic class) and the mean number of children ever born is found in every age group, in both rural and urban areas." In Malaysia, Palmore and Ariffin (1968) find that among women age 35-44, those with 1 to 5 years of schooling have had more children (6.2)

than both those with no schooling (5.8) and those with 6 years or more (4.8). Similar findings, for both urban and rural samples, are reported by Knodel and Pitaktepsombati (1973) for Thailand. In the Philippines, Encarnación (1974) reports regression results indicating an education threshold at about six years of schooling and an income threshold at the statutory minimum wage.

The obvious implication is a fertility rise as subsistence levels improve during the early phases of development. Indeed, Tabbarah (1971) cites a number of studies showing that the Western European experience had been one of rising birth rates before any decline took place, and that a majority of LDCs today have been experiencing higher birth rates. Families above the threshold must outweigh those below if rising incomes and more education are to have the commonly expected fertility-reducing effects of development.

The model also explains the finding in some Southeast Asian countries that the relationship between labor force participation and fertility is negative in the case of urban wives but positive in the case of rural (see Goldstein 1972, Concepción 1973). The reason would simply be that rural families (the majority of which fall below the threshold) with working wives have higher incomes ceteris paribus; hence their fertility would be higher. As for urban wives, we expect working women to have more education and therefore less children.

Some interesting findings on rural migrants' fertility are also easily explained. A study by Oey (1975) shows that Javanese migrants to Lampung (a province in Sumatra) had higher fertility than Javanese nonmigrants (who have the lowest fertility rates in Indonesia) but lower than Lampung natives. Oey remarks that "improved economic conditions... tend to support a Malthusian thesis that fertility increases as the means of subsistence increases." Goldstein's (1971) figures also show that "lifetime migrants" (province of birth different from current residence) in rural, agricultural areas in Thailand had higher fertility than nonmigrants. On the other hand, "5-year migrants" (province of residence 5 years earlier different from current residence) had lower fertility than the natives, which suggests that 5-year migrants (who, on average, would have made their move 2.5 years earlier, or even less if the flow of migrants had increased through time) had not had sufficient time to improve their means of living and thereby increase their fertility.

Finally, the model has an important implication in regard to the fertility effects of a reduction in child mortality and, therefore, for the demographic transition. Schultz (1976) has recently surveyed this literature and concludes that "parents seem to respond to the decline in child mortality by having fewer births [and this] appears to reflect strong behavioral preferences of parents to replace an infant who dies." But as Cassen (1976) remarks, "most studies find that fertility decline does not 'fully compensate' for mortality decline in the short run,"

with the result that the number of surviving children is larger. This number is an average, and much depends on the proportion of families below the threshold.

In figure 2, a downward shift of the CM curve lowers the CB curve to the same extent. Families above the threshold thus match the mortality decline fully with a reduction in fertility. The CK curve remains the same, however, and families below the threshold simply have more surviving children. The responses of below-threshold and above-threshold families are thus quite different. Fertility does not change for the former while the number of children surviving does not change for the latter; the observed changes are average figures from different phenomena.

The relative proportions of such families in a population therefore assume major importance as to the aggregate effect of mortality reductions on fertility. The implication for the demographic transition is then immediate: (i) some countries have had lower mortality for decades but still have high fertility, because of the preponderance of below-threshold families; (ii) some countries have experienced lower mortality and then lower fertility shortly afterwards, because of a large above-threshold majority; and (iii) there is the possibility, with rapidly rising educational levels, of a fertility decline even before a decline in mortality.

## V. Concluding Remarks

The assumptions involved in the model appear to be reasonable and supported by general knowledge. People do want better quality goods if they can afford them, and surely most people want their children to have at least the same level of education that they have had. If income is lower, or cost of children higher, they simply have fewer. As for the "natural fertility" assumption, this looks obvious once stated.

The model explains, I believe, a number of phenomena that would otherwise seem unrelated, and provides a guide to the time requirements of the demographic transition. What is disturbing from a policy viewpoint is that "the population problem" becomes more of a problem than it already is. More than half (as in the Philippines) of the population in most LDCs probably fall below the threshold, and one cannot rely solely on general economic development as a solution.

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