curve is everywhere less than β, agglomeration takes place at the location offering the greatest profit $\hat{\Pi}_n$.

All firms however do not possess identical and homogeneous iso-welfare curves hence, although clustering in certain locations do occur there would still be some firms that are fairly dispersed. But regardless of the shape of the iso-welfare curves of firms provided only that they are convex with respect to the origin, an increase in β would tend to draw firms toward high profit locations. In general, as the "price" β of μ increases, firms would tend to agglomerate at relatively high profit locations. As β decreases on the other hand, firms would tend to disperse. The limiting cases are an infinitely large β so that the constraint approaches the $\hat{\Pi}$-axis, in which case only corner solutions would occur, that is, firms agglomerate at the nth location; and, $\beta = 0$ so that the constraint becomes a horizontal line, in which case $0 \leqq \hat{\Pi}_1 = \hat{\Pi}_2 = \hat{\Pi}_3 = \ldots = \hat{\Pi}_n$ and location is indeterminate. This is interpreted as a tendency toward dispersion over space, all locations being equally profitable. These results are shown graphically in Fig. 3.

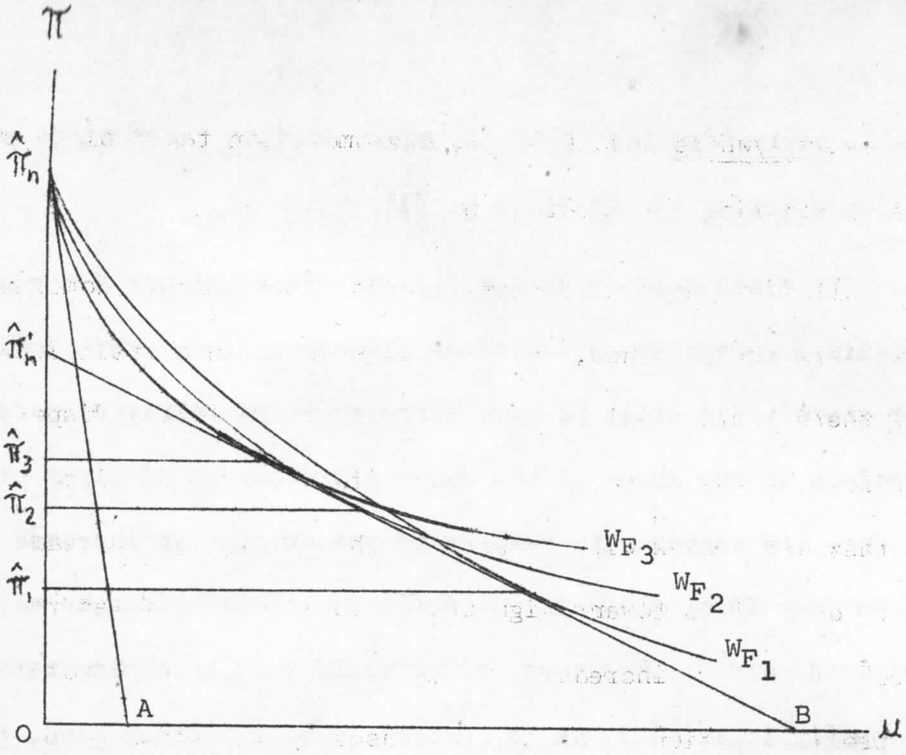

Fig. 3

W_{F_1}, W_{F_2}, W_{F_3} are the iso-welfare curves of firms 1, 2, and 3 respectively. With a very large β depicted by the very steep constraint $\hat{\Pi}_n A$, all three firms locate at the nth site. With a very small β depicted by line $\hat{\Pi}'_n B$, firm 1 chooses location 2, firm 3 chooses location 3.

That an increase in β would lead to a tendency toward agglomeration at relatively high profit locations while a decrease in β leads to dispersion (indicating asymmetric response to changes in β) hinges on the assumption that the shape of the iso-welfare curves of firms have the common feature that it is biased in favor of profit and against extra-economic factors, that is, firms' welfare levels are "profit-intensive". In other words, while it can happen that corner solutions could occur on the $\hat{\Pi}$-axis, that is, the slope of the iso-welfare curves of firms can be everywhere smaller than β, the opposite can not occur - corner solutions can not occur on the μ-axis, that is, the slope of the iso-welfare curves of firms can not be everywhere greater than β. This is what one would generally expect in reality - that firms can not and, as a rule, do not live on μ alone. Besides maximizing profit in the sense of equalizing the slopes of the revenue and cost functions and requiring that profit be positive, firms would also desire to obtain a relatively large positive spread between revenue and cost that they can possibly get, along with whatever non-economic considerations that they may have.

Parenthetically, a way to empirically verify the above proposition on agglomeration (deglomeration) tendencies would be to show two different profit and hence β-situations (say at two time periods) for alternative locations and then compare the changes if any, in the number of firms (in the same industry) or any index of spatial concentration (dispersion) in the two situations. Having done this, the locational interdependence factors in the alternative locations which led to the change in β, and changes in these factors in the two situations could be investigated to establish a casual link between these factors and agglomeration (deglomeration) tendencies.

V

Plant Location Under Conditions of Uncertainty[12/]

In this sketch of an alternative approach to the problem of optimal plant location, the location decision of the firm under condition of uncertainty consists in selecting a probability distribution from a given set of such distributions. Rational behavior then means selecting the best of the available distributions. This means that location decision under uncertainty must be based on a preference ordering over the probability distributions in a set of such distributions. We attempt to construct such preference ordering for a firm confronted with the problem of choice of location among the various possible sites, based on the Bernoulli principle.

For simplicity, we consider only discrete distributions. As in the first approach, the prospective firm is confronted with n economically feasible locations each of which offers as gain the stochastic variable $\hat{\Pi}_j \geq 0$ (negative $\hat{\Pi}$s are out of consideration), the maximum profit that could occur at the ith location, with probability distributions $f_i(\hat{\Pi}_j)$, $i = 1, 2, \ldots, n$; $j = 0, 1, 2, \ldots, m$. In symbols, the firm is confronted with a set D the elements of which are the n probability distributions, i.e.,

$$D = \{f_1(\hat{\Pi}_j), f_2(\hat{\Pi}_j), \ldots, f_n(\hat{\Pi}_j)\}, \; j = 0, 1, 2, \ldots, m \quad (8)$$

Thus for the ith location for example we can interpret $f_i(\hat{\Pi}_o)$, $f_i(\hat{\Pi}_1)$,

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[12/] This section makes much use of the method elaborated by Karl Henrik Borch in his The Economics of Uncertainty. (Princeton, New Jersey: Princeton University Press, 1968), esp. Ch. III.

$f_i(\hat{\Pi}_2)$, ..., $f_i(\hat{\Pi}_m)$ as the probabilities that the site will give the maximum profits $\hat{\Pi}_o$, $\hat{\Pi}_1$, $\hat{\Pi}_2$, ..., $\hat{\Pi}_m$, respectively. It should be noted that as a result of the imperfect character of spatial markets and differences in locational interdependence factors among the locations all of which would imply demand and/or cost variations over space, the range of the $\hat{\Pi}$s that could occur may differ among the various sites. This is taken care of by simply taking the largest range of $\hat{\Pi}_j$ and assigning zero probabilities to those $\hat{\Pi}$s which do not appear where the range is small. It is obvious that

$$\sum_{j=0}^{M} f_i(\hat{\Pi}_j) = 1 \quad \text{for all} \quad i = 1, 2, 3, \ldots, n$$

Since each element of D is associated with a point in economic space, the location of the firm is determined when the firm chooses the best distribution in C. Hence to solve the firm's location problem, we seek a preference ordering over the η elements of the set D. Assuming that this ordering can be represented by a utility function, our objective then is to associate a real number U (f) with each of the η elements (the probability distributions $f_i(\hat{\Pi}_j)$, hereafter called "prospects") of the set D such that

$$U\{f_i(\hat{\Pi}_j)\} > U\{f_k(\hat{\Pi}_j)\}$$

if and only if $f_i(\hat{\Pi}_j) \succ f_k(\hat{\Pi}_j)$. Mathematically, the problem is to find a mapping from the space of all discrete probability distributions or the prospects in D to the real line. To do this, we employ the axioms laid down by Borch.[13]

Axiom 1. To any probability distribution $f_i(\hat{\Pi}_j)$ in the set D, there corresponds a certainty equivalent $\bar{X}_i$.

In symbols, Axiom 1 says $(1, \bar{X}_i) \sim f_i(\hat{\Pi}_j)$ (" $\sim$" denotes equivalence relation.)

Set D includes all binary type distributions in which the only two possible outcomes are

$\hat{\Pi}_M$ with probability p

0 with probability $(i - p)$

If $(p, \hat{\Pi}_M)$ stands for such binary distribution, we have from Axiom 1 that for any p, there is an X_p so that

$$(1, X_p) \quad (p, \hat{\Pi}_M)$$

Axiom 2, As p increases from 0 to 1, X_p will increase from 0 to $\hat{\Pi}_M$.

Axiom 3. $f_i(\hat{\Pi}_j)$ and $f_i^*(\hat{\Pi}_j)$, the equivalent binary distribution of $f_i(\hat{\Pi}_j)$, have the same certainty equivalent.

With Axiom 1 we determine the certainty equivalent of each of the $\hat{\Pi}$s with their respective probabilities $f_i(\hat{\Pi}_o)$, $f_i(\hat{\Pi}_1)$, ..., $f_i(\hat{\Pi}_m)$. With Axiom 2 we form the equivalent binary type distribution of the original prospects. Axiom 3 together with Axiom 2 allows replacement of $\hat{\Pi}_j$, $j = 0, 1, 2, \ldots, m = M$, except $\Pi_o = 0$ and $\hat{\Pi}_m = \hat{\Pi}_M$, in the ith location with the equivalent binary form $(p_j, \hat{\Pi}_M)$ to give a modified

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[13] Ibid., pp. 25-26.

prospect

$$f_i^* (\hat{\Pi}_o = 0) = f_i (0) + f_i (\hat{\Pi}_1) (1 - p_1) + f_i (\hat{\Pi}_2) (1 - p_2) + \ldots + f_i (\hat{\Pi}_M) (1 - p_M)$$

$$f_i^* (\hat{\Pi}_1) = 0 \tag{9}$$

$$f_i^* (\hat{\Pi}_2) = 0$$

. . . .

$$f_i^* (\hat{\Pi}_M) = p_1 f_i(\hat{\Pi}_1) + p_2 f_i (\hat{\Pi}_2) + \ldots + p_M f_i (\hat{\Pi}_M)$$

In this manner we obtain for the ith location a prospect of the type $(P_i, \hat{\Pi}_M)$ which has the same certainty equivalent as the original prospect $f_i (\hat{\Pi}_j)$. For the ith location, P_i is determined by the last equation in (10), i.e.

$$P_i = \sum_{j=0}^{M} P_j f_i (\hat{\Pi}_j) \tag{10}$$

Applying this method to each of the prospects $f_i (\hat{\Pi}_j)$, $i = 1, 2, \ldots, n$, in the set D, we obtain a complete preference ordering over the elements of D and hence of the corresponding locations. Thus for two arbitrary distributions in D, $f_i (\hat{\Pi}_j)$ and $f_k (\hat{\Pi}_j)$, we can determine the corresponding binary prospects $(P_i, \hat{\Pi}_M)$ and $(P_k, \hat{\Pi}_M)$ and their certainty equivalents. The ordering is then that $f_i (\hat{\Pi}_j)$ is preferred to $f_k (\hat{\Pi}_j)$ if and only if $P_i > P_k$ (or equivalently if $f_i (\hat{\Pi}_j)$ has the greater certainty equivalent). And since P_i is associated with the ith point in economic space, the firm chooses this location.

To represent this preference ordering by a utility function, we define

$$U\{f_i(\hat{\Pi}_j)\} = P_i = \sum_{j=0}^{M} P_j f_i(\hat{\Pi}_j), \quad (i = 1, 2, 3, \ldots, n) \qquad (11)$$

As a final point in this discussion, we note that since the certainty equivalent of a prospect varies positively as its probability and independently with $\hat{\Pi}_j$ the distribution and hence the location that is chosen does not necessarily have to offer the greatest positive spread between revenues and costs among all the possible locations.

In this approach to plant location determination, there are two sources of support for the firm's rational behavior in the choice of site: the first arises from the fact that the best (in terms of the certainty equivalent) of the available distributions is chosen; the second is that the distribution and hence the location that is chosen has a $\hat{\Pi}$ which is the result of equalizing the slopes of the revenue and cost functions i.e. of the profit maximizing behavior of the firm.

VI

A Note on General Equilibrium of Location and Pareto Optimality in Production Over Space

The objective of this section is simply to sketch an approach to general equilibrium of location and Pareto optimality in production over space based on the first of the two alternative approaches to plant location problem that have been presented. Consider two firms A and B at contiguous locations i and j respectively. The equality of the firms' delivered prices determines the market areas, i.e.

$$P_i = P_j$$

$$P_i^A + t_i\, d_i = P_j^B + t_j\, d_j \qquad (12)$$

$$d_j - \frac{t_i}{t_j}\, d_i = \frac{P_i^A - P_j^B}{t_j}$$

defines the boundary between areas tributary to two geographically competing markets for homogeneous goods, where P_i^A and P_j^B are the firms' factory prices; t_i and t_j are transport rates which are given; d_i and d_j are distances. The determination of market areas M_A and M_B is illustrated graphically in Fig. 4.

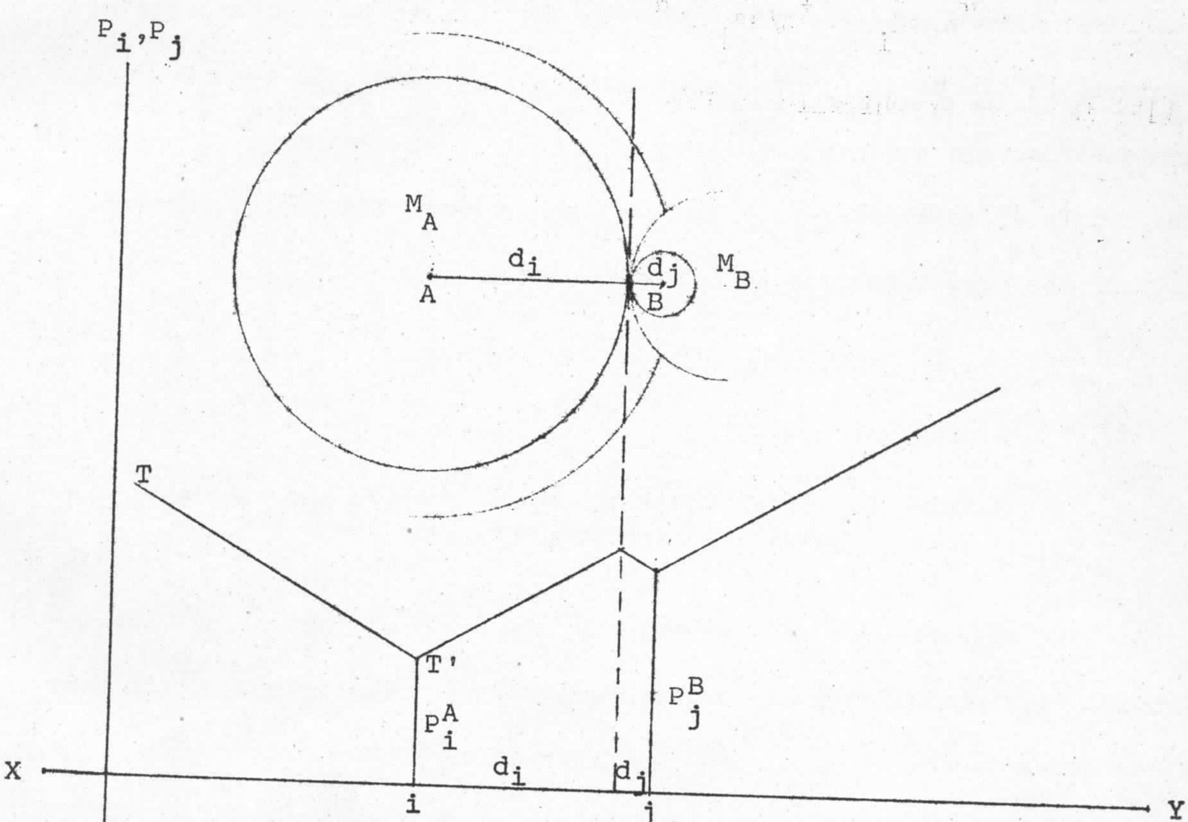

Fig. 4

The slope of TT' is equal to t_i.

With these delivered prices, profits are maximized at the respective locations. The firms' having chosen locations i and j means that

$$\text{For firm A:}\quad \frac{\partial f_{A}/\partial \mu_A}{\partial f_{A}/\partial \hat{\Pi}_i} = -\frac{d\hat{\Pi}}{d\mu_A} = \frac{\beta}{\alpha} \qquad (13)$$

$$\text{For firm B:}\quad \frac{\partial f_{B}/\partial \mu_\beta}{\partial f_{B}/\partial \hat{\Pi}_j} = -\frac{d\hat{\Pi}}{d\mu_B} = \frac{\beta}{\alpha} \qquad (14)$$

And since β is <u>common to all firms</u> (in the same industry), and $\alpha = 1$, and taking the firms as sharing the total profits for both locations that is, $\hat{\Pi}_T = \hat{\Pi}_i + \hat{\Pi}_j$, we have

$$\frac{\partial f_{A}/\partial \mu_A}{\partial f_{A}/\partial \hat{\Pi}_i} = \frac{\partial f_{B}/\partial \mu_B}{f_{B}/\partial \hat{\Pi}_j} = \frac{\beta}{\alpha} \qquad (15)$$

(15) is shown graphically in Fig. 5.

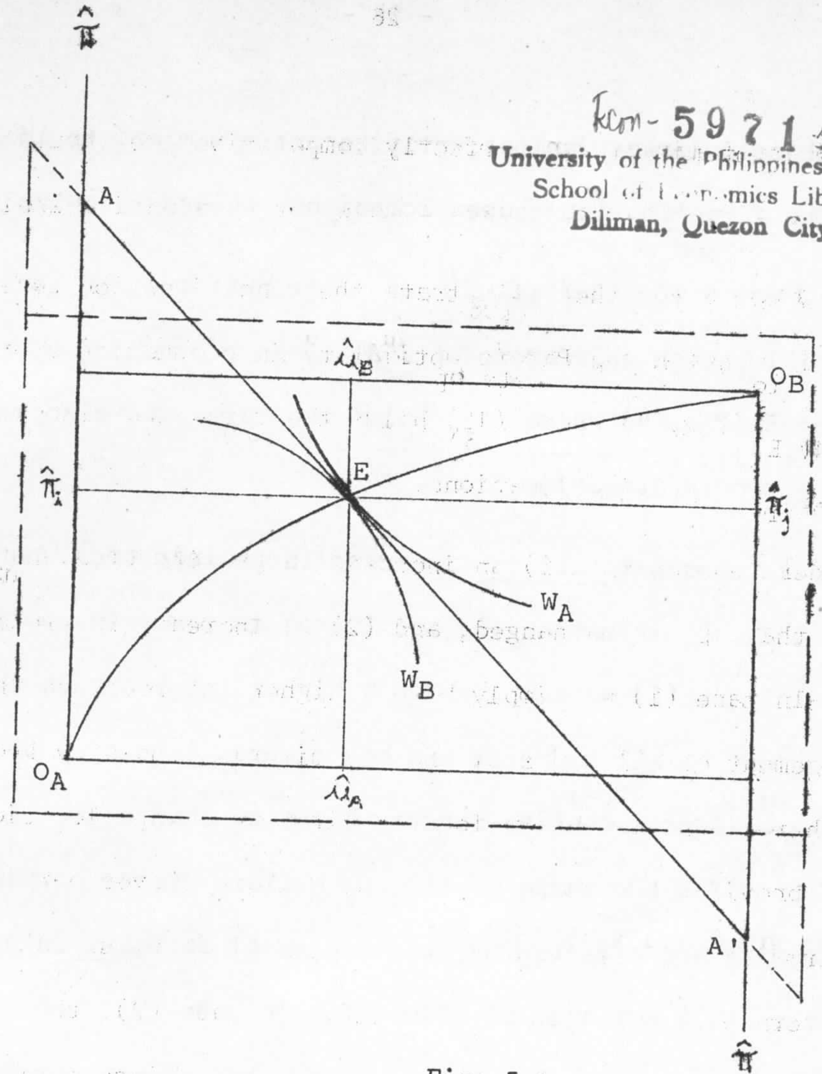

Fig. 5

Since firm A is maximizing profit at the ith location, we have

$$\frac{\partial Q_i/\partial X_{i1}}{\partial Q_i/\partial X_{i2}} = \frac{r_{i1}}{r_{i2}} \tag{16}$$

where r_{i1} and r_{i2} are the prices of inputs X_{i1} and X_{i2} respectively.

Similarly, for firm B,

$$\frac{\partial Q_j/\partial X_{j1}}{\partial Q_j/\partial X_{j2}} = \frac{r_{j1}}{r_{j2}} \tag{17}$$

If the input market is "perfectly competitive" we should have $r_{i1} = r_{j1}$ and $r_{i2} = r_{j2}$. Eq. (18) however does not necessarily imply this.

Figs. 3 and 4 together illustrate the conditions for general equilibrium of location and Pareto optimality in production over space since at point E (Fig. 4) where (15) holds the firms are also maximizing profits at their respective locations.

Consider two cases: (1) an increase in profits (for whatever reason) such that β is unchanged, and (2) an increase in profits that affects . In case (1) we simply have a higher intercept on the $\hat{\Pi}$-axis and an enlargement of all sides of the box diagram (shown by broken lines, Fig. 4) so there is no incentive for the firms to change locations. Furthermore, provided the shape of the iso-welfare curves particularly of potential firms is not affected by this manner of increase in profits, location pattern will not also be affected. In case (2), the constraint AA′(Fig. 5) becomes steeper than before. Here two things could occur: either the firms (existing and potential) move to high profit locations but total profits which the firms share increase i.e., the <u>width</u> of the box diagram increases and we still have equality of the slopes or, the firms remain in their respective locations which means that there has to be a change in the shape of their iso-welfare curves analogous to the "change in taste" in consumer theory. The tendency toward agglomeration (deglomeration in the case where β decreases) is thus present in case 2 where there is a change in the slope of the constraint AA′.

VII

Conclusion

The theory that has been presented takes into account extra-economic factors in the firm's choice-of-location problem without impairing the notion of rational behavior in the context of existing dimensionless theory of the firm. Moreover, the theory is free from highly unrealistic assumptions as spatially homogeneous and uniformly distributed resources, homogeneous plain, uniform population densities, uniform transport costs, instantaneous costless relocation, etc.[14] which have burdened existing location theories. Besides purporting to explain plant site determination and location pattern, the theory may also serve as an explanation of capital movement over space. Although locational interdependence factors have not been examined in detail, the influence of these in location pattern has not been ignored but is captured in the behavior of the constraint to which firms react. Thus location pattern is systematically linked with foregone profit, that is, with "locational opportunity cost."

As a final point, it should be noted that theories of plant location would perhaps be applicable only to firms that cater essentially

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[14] See for example August Losch, The Economics of Location, tr. by William H. Woglom with Wolfgang F. Stolper. (New Haven: Yale University Press, 1954); Harold Hotelling, "Stability in Competition," Economic Journal, 39 (1929), pp. 41-57; Walter Isard, Location and Space-Economy, (Cambridge, Mass.: The MIT Press, 1956).

localized markets, that is to say, the markets of which are not as large as the national market - not to mention international market. Where the firm is large as in the case of steel or transport equipment firms in highly industrialized economies, and the market of which is the whole country and/or the world, demand can be taken as more or less given so that least-cost locations would be the optimal plant sites.[15] In this case, so-called extra-economic factors would have virtually no significance although conceivably, they may still be present in the firm's location decision problem (for example, providing employment to certain "depressed areas.") Clearly in this instance, it makes no difference whether profit maximization is taken to mean the behavior of equalizing the slopes of the revenue and cost functions or the attainment of the widest positive gap between revenue and cost among locations.

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[15] This is perhaps one reason for international capital movement.

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