

Institute of Economic Development and Research  
SCHOOL OF ECONOMICS  
University of the Philippines

Discussion Paper No. 75-1      February 5, 1975  
C. 2

A THEORY OF PLANT LOCATION

by

CASIMIRO V. MIRANDA, JR., 1935-

NOTE: IEDR Discussion Papers are preliminary versions circulated privately to elicit critical comment. References in publications to Discussion Papers should be cleared with the author(s).

## A THEORY OF PLANT LOCATION

### I

The absence of a satisfactory theory of optimal plant location owes much to the misunderstanding of the meaning of the principle of profit maximization as it applies to plant location decisions. For in location theory, profit maximization is confused with the behavior of firms to seek the site which offers the greatest positive spread between revenues and costs among all possible locations.<sup>1/</sup> This of course does not make sense both mathematically and economically for it implies that firms are considering "absolutes" not "relatives", and are therefore unduly concerned with revenue and cost levels in their search for location.

Indeed the theory of plant location has developed along two contending rather than complementary lines in regard to this, one emphasizing the search for the least-cost site by abstracting from demand, the other emphasizing demand, by abstracting from cost.<sup>2/</sup> Consequently,

---

<sup>1/</sup> See for example Harry W. Richardson, Regional Economics. (New York: Praeger Publishers, 1969), Ch. 4, esp. pp. 90-100; also D.M. Smith, "A Theoretical Framework for Geographical Studies of Industrial Location", Economic Geography, 42 (April 1966), pp. 95-113; Leon N. Moses, "Location and the Theory of Production," Quarterly Journal of Economics, 72 (1959), pp. 259-272; Arthur Smithies, "Optimum Location in Spatial Competition", Journal of Political Economy, 49 (June 1941), pp. 423-439; Melvin L. Greenhut, "Integrating the Leading Theories of Plant Location", Southern Economic Journal, 18 (April 1952), pp. 526-538. In this article, Greenhut concludes that the "theories of the least-cost location and the inter-dependent location are, despite their differences, quite similar; both emphasize the search for the site which offers the greatest spread between total costs and total revenues."

<sup>2/</sup> Melvin L. Greenhut, op. cit., pp. 526-527.

and what is perhaps more central to the problem is that, if profit maximization is interpreted strictly in the above sense in the theory of optimal plant location, the generally observed phenomenon of firms locating at "less than maximum profit locations" or alternatively, the absence of observed clustering of all firms (in the same industry) at the one location that offers the greatest positive spread between revenue and cost relative to all other possible locations, cannot be explained.<sup>3/</sup>

As a result of this failure to emerge with a satisfactory theory of optimal plant site, profit maximization as a rational behavior of the firm has been questioned and an increasing attention has been given to so-called "non-economic" considerations of firms in their choice of location.<sup>4/</sup> This of course, suggests a trade-off between profits and "non-economic" considerations - whatever these may contain.

Properly understood, profit maximization simply refers to the behavior of firms to equalize the slopes of the revenue and cost functions, and this they do by finding the combination of factors that minimize cost and the scale of output that equalizes the marginal cost and the price of the product.<sup>5/</sup>

<sup>3/</sup> See references in footnote 1.

<sup>4/</sup> Harry W. Richardson, op. cit., pp. 90-100; Charles Tiebout, "Location Theory, Empirical Evidence, and Economic Evolution," Papers and Proceedings of the Regional Science Association, 3 (1957), p. 81; Melvin L. Greenhut, Plant Location in Theory and Practice (Chapel Hill: University of North Carolina Press, 1956), pp. 175-176; Melvin L. Greenhut, "Observation of Motives to Plant Location," Southern Economic Journal, 18 (October 1951) pp. 225-228.

<sup>5/</sup> Oskar Lange, "On the Economic Theory of Socialism," Review of Economic Studies, IV, Nos. 1 and 2 (Oct. 1936 and Feb. 1937).

✓ The objective of this study is to sketch a theory of plant location which will show that if profit maximization is taken to mean simply as the behavior of firms to equalize marginal revenue (or price in a perfectly competitive market) and marginal cost, then it is possible to generate a simple but general explanation of observed plant locations. In addition, the result contributes to the analysis of locational interdependence factors influencing spatial concentration (dispersion) of firms and, more significantly perhaps, a satisfactory theory of general equilibrium of location and Pareto optimality in production (and possibly consumption) over space can also be worked out. Lastly, one result of the approach which may be worth noting is the integration of location theory with the orthodox dimensionless micro-theory of the firm by showing that "extra-economic" factors in the choice of site are not inconsistent with notions of rational behavior in the context of the orthodox theory of the firm, hence doubts concerning the profit motive of firms in the choice of location dispelled - an important result since it allows retention of conventional resource allocation efficiency criteria.

## II

In this study we envision a firm whose choice-of-location problem involves essentially two stages: First, consideration of the probability of all possible sites; second, actual choice of site is made.

To begin with, let there be at a point in time a finite number of possible locations which a firm is initially faced with. (Mathematically

there is an infinite number of points over economic space, but here we consider only economically feasible points.) If profit maximization simply means that whoever it is that makes the decision (we refer to him here as the entrepreneur or the firm) is "equalizing the slopes of the revenue and cost functions", then at each and every point in economic space, there is for a firm a  $\hat{\Pi}(\hat{X}) \geq 0$ . Or alternatively, every point in economic space can be represented as  $\hat{\Pi}(\hat{X})$  where  $\hat{X}$  is a vector of inputs that maximize profits. Needless to say, this means that at every point in economic space there is for a firm a revenue function and a cost function. This  $\hat{\Pi}(\hat{X})$  is the "maximized" profit that is, the result of "solving" for the maximum of the profit equation,

$\hat{\Pi}$  = Total Revenue - Total Cost. A firm confronted with the problem of choosing a site among alternative locations "solves" one profit equation for each of the  $n$  locations that it is considering. Thus all in all a firm solves  $n$  profit equations and comes up with  $n$   $\hat{\Pi}$ s. For the  $i$ th location, for example, a prospective firm faced with the problem of choice of plant site first "solves" the profit equation and obtains  $\hat{\Pi}_i(\hat{X}_i)$ :

$$\begin{aligned}\hat{\Pi}_i &= P_i Q_i - C_i - T_i, \quad (i = 1, 2, 3, \dots, n) \\ &= (P_i^F + t_i d_i) Q_i(X_i) - r_i X_i - t_i d_i Q_i(X_i)\end{aligned}\quad (1)$$

where:

$P Q$  = total revenue

$r X$  = total (input) cost  $C$

$P^F = t d$  = delivered price  $P$

$P^F$  = factory price

$Q = Q(X)$  = production function

$X$  = vector of inputs

$r$  = vector of input prices

$T$  = total transport cost =  $t d Q(X)$

$t$  = transport rate

$d$  = distance

Some remarks on (1) are appropriate at this juncture. The main initial concern of a firm confronted with a location problem is to see what the  $\hat{\Pi}$  is that obtains at each of the  $n$  possible locations. This means that although the firm gives attention to the various components of the revenue and cost functions to the extent that they affect profit, they are not however its main concern. Perhaps the firm becomes more concerned over the behavior of revenue and cost functions as such only after decision on which site to locate has been made and the plant set up, since at this stage of the life of the firm survival becomes the overriding objective. At the outset however, the profit equation simply summarizes the effect of the components and determinants of revenue and cost, and the profit that the firm obtains after it "maximizes" the equation is what it considers in its choice of site.

The function of delivered price  $P_i$  in (1) is, together with the delivered prices of other firms, to determine the market area. There is no apparent reason for transport cost to received special attention in location theory any more than input costs. The significant role of

transport cost in location theory is not as the principal object of minimization but as a determinant of the market area. It is in this perspective that the role of transport cost is viewed here. With respect to the input terms it matters less for a firm where and how they can be made available, and as regards transport costs of inputs these are assumed to be reflected by input prices. All this means allowing spatial cost variations.

Having solved for each of the  $n$   $\hat{\pi}_i$ s, the firm is now faced with the following set of all nonnegative  $\hat{\pi}_i$ s arranged in order of magnitude (negative  $\hat{\pi}_i$ s are out of consideration) where  $\hat{\pi}_1 =$  maximum profit obtaining at location  $i$ , ( $i = 1, 2, \dots, n$ ):

$$0 \leq \hat{\pi}_1 \leq \hat{\pi}_2 \leq \hat{\pi}_3 \leq \dots \leq \hat{\pi}_n \quad (2)$$

With  $m$  firms, we have  $m$  sets of (2). Because of locational interdependence factors and, allowing for differences in the way firms would combine the inputs, the firms do not necessarily obtain identical profits at the same location, that is, at the  $i$ th location for example,

$$\hat{\pi}_1^1 \geq \hat{\pi}_1^2 \geq \hat{\pi}_1^3 \geq \dots \geq \hat{\pi}_1^m$$

Since the study is concerned with how a firm - not all firms taken together - solves its location decision problem, we focus attention on just one of the  $m$  sets of maximized profits, the set of  $\hat{\pi}_i$ s shown in (2). It is conceivable that some of the equalities in (2) may hold. Realistically however, the chances are very slim that even at any two different sites of similar physical features exactly the same maximized

Profits occur, because of the imperfect character of spatial markets and differences in locational interdependence factors among the possible locations. Consequently, and to have a mapping of the profits on the real line and as we shall see, for uniqueness of solution to the location decision problem we assume that

$$0 \leq \hat{\Pi}_1 < \hat{\Pi}_2 < \hat{\Pi}_3 < \dots < \hat{\Pi}_n \quad (3)$$

The consequent question then is: which of these  $\hat{\Pi}$ s and hence location will the firm choose? Two approaches to this question will be explored. One approach explicitly takes into account so-called non-economic factors in the choice of location by positing a trade-off between these factors and the maximized profits in (3) and on the basis of this considers an objective function of the firm; the other approach envisions the firm as making a decision under conditions of uncertainty into which is lumped all non-economic and locational interdependence factors, and posits a preference ordering over the probability distribution in economic space according to their respective certainty equivalents.

### III

Consider the first approach. As pointed out elsewhere, existing location theories are unable to settle the observed location of firms at "less than maximum profit" sites where "maximum profit" refers to the greatest positive difference between revenues and costs among the various possible locations. For this reason, recent studies advance such motives

as "personal preferences and constraints not closely related to any calculus of money cost, revenue, or profit."<sup>6/</sup>; or "limited objectives, or psychic income"<sup>7/</sup> as considerations vis-a-vis profits in location decisions. In a study by Tiebout particularly, these extra-economic factors in the firm's location decision were given empirical justification.<sup>8/</sup> It is evident that all these point to the existence of a trade-off between profits and extra-economic factors (whatever these are) in location decisions. With this, and with the aid of the axioms in consumer theory which build up the utility function<sup>9/</sup>, we have a justification for assuming an analogous objective function for the firm confronted with the problem of choice of location, which we will call the firm's welfare function.<sup>10/</sup>

Why has not the orthodox dimensionless theory of the firm assigned a welfare function to the firm? The answer is precisely because the firm

---

<sup>6/</sup> Edgar M. Hoover, An Introduction to Regional Economics, First ed. New York: Alfred A. Knopf, 1971) pp. 60-62.

<sup>7/</sup> Harry W. Richardson, op. cit., pp. 90-100.

<sup>8/</sup> Charles Tiebout, op. cit., pp. 74-86.

<sup>9/</sup> For which, see Peter Newman, The Theory of Exchange (New Jersey: Prentice-Hall, Inc., 1965). Ch. 2.

<sup>10/</sup> If this does not suffice, we can call forth Arrow's possibility theorem since the board of directors in the case of a corporation; the partners in the case of a partnership; the individual in the case of a single-proprietorship, who make the decisions, play the role of "dictators" and the organizational structures of these types of firms can satisfy or be made to satisfy Arrow's axioms [for which see James M. Henderson and Richard E. Quandt, Micro-economic Theory. Second ed. (New York: McGraw-Hill Book Co., 1971); pp. 284-286].

in a spaceless setting, is not faced with a choice among profit alternatives and extra-economic considerations so that no trade-off exists. In the theory of the firm in which spatial aspects are absent the firm has but a singular objective namely, to maximize profit by which is meant both the behavior of equalizing slopes and the search for the greatest positive gap between revenue and cost, hence no conflict between these notions arises. Here lies the synthesis of the orthodox micro-theory of the firm and location theory. For the firm's welfare function is relevant only insofar as it is deciding where to locate. Once decision is made and the plant actually set up at the chosen site, the welfare function, although it may still be there, drops out of importance and the firm can be viewed simply as having that singular objective of maximizing profit in the sense of both equalizing slopes and, attaining the greatest spread between revenue and cost at the chosen site where it must operate for the duration of its life.

Now, let  $\mu$  represent all locational considerations of a firm other than profit. Then on the basis of the existence of a trade-off between  $\hat{\Pi}$  and  $\mu$ , we write the firm's welfare function as

$$W_F = f(\hat{\Pi}, \mu) \quad (14)$$

where:

$$\hat{\Pi} \geq 0, \mu \geq 0,$$

$$\frac{\partial W_F}{\partial \hat{\Pi}} > 0, \text{ and } \frac{\partial W_F}{\partial \mu} > 0$$

The constraint is taken to be of the linear form:

$$Z = \alpha \hat{\Pi} + \beta \mu \quad (5)$$

We interpret  $Z$  as the maximum monetary value of total income consisting of money income from profits  $\alpha \hat{\Pi}$  and the monetary value of "psychic" income  $\beta \mu$ . Defined in this manner, the monetary value of total income  $Z$  is equal to the greatest profit that a firm can obtain among the various sites, that is, the  $\hat{\Pi}$ -intercept of (5) is  $\hat{\Pi}_n$ .

$\alpha$  maybe interpreted as the average of the weights which firms give to profit. In other words  $\alpha$  is taken as the average of the indices of "attitudes" - which vary from 0 to 1 - of firms toward profit. At most  $\alpha = 1$  when firms take profit simply for what it is. The existence of extra-economic factors however suggests that  $0 \leq \alpha \leq 1$ , disregarding negative attitude towards profit. Since extra-economic factors are already explicitly treated separately, we get rid of  $\alpha$  in the constraint by simply taking profit as the consideration thus making  $\alpha = 1$ .

The existence of  $\mu$  brings up the idea of foregone profit (locational opportunity cost). Thus  $\beta$  maybe interpreted as some average profit differential among locations in the industry. Hence what we have referred to as the "monetary value of psychic income"  $\beta \mu$  is the profit foregone by a firm if  $\mu > 0$ . By interpreting  $\beta$  in this manner we have in effect assumed that  $\beta$  is common to all firms. In addition, having linked  $\beta$  with profits obtaining among the possible sites, to the extent that locational interdependence factors giving rise to agglomeration (deglomeration) and the economies (diseconomies) associated with it

would affect the firms' production functions and hence costs,  $\beta$  will be affected through the  $\hat{\Pi}$ s. Thus  $\beta$  may change due to locational interdependence factors. This will be dealt with later.

From (4) and (5) the choice-of-location problem of a firm is simply the maximization problem:

$$\begin{aligned} &\text{maximize } W_F = f(\hat{\Pi}, \mu) \\ &\text{subject to } Z = \alpha\hat{\Pi} + \beta\mu \end{aligned} \quad (6)$$

Since the solution to this problem gives a  $\hat{\Pi} = \hat{\Pi}_1$ , the firm's location is also known; that is, in solving (6) for  $\hat{\Pi}$  we also solve for the firm's location. Graphically the solution to (6) is shown by the tangency of the constraint AC to the highest attainable  $W_F$  - curve which is  $W_F^0$ . This tangency occurs at point B in Fig. 1 where we have

that  $\frac{\partial f / \partial \mu}{\partial f / \partial \hat{\Pi}} = \beta$ .

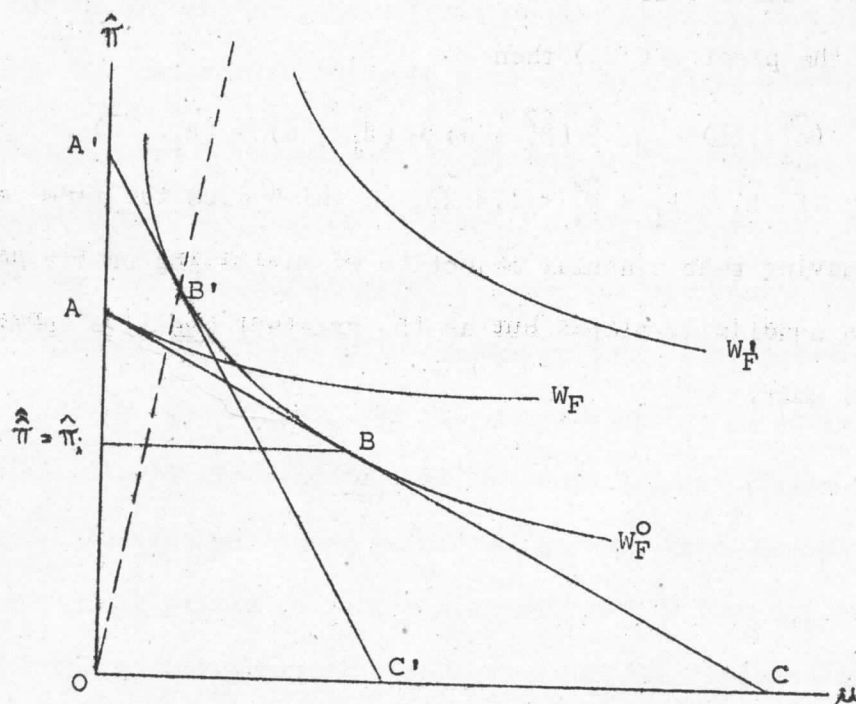


Fig. 1

As shown by the graphical solution, it does not necessarily mean that the firm locating at the  $i$ th location is not maximizing profit since  $\hat{\Pi} = \hat{\Pi}_i$ , the maximum profit obtaining at the  $i$ th location. Since  $\hat{\Pi}_i < \hat{\Pi}_n = OA$ , we have explained theoretically the phenomenon of firms locating at "less than maximum profit locations" where "maximum profit" refers to the widest gap between revenues and costs among the sites. Moreover as Fig. 1 shows, the behavior of firms to search for the location offering the highest positive profit among the various alternative sites, is not entirely ruled out but maybe taken as just a special case in that these firms possess iso-welfare curves of the shape of  $W_F$  in which case a so-called corner solution occurs at point A where  $\hat{\Pi}_n$  obtains. Thus in general, the non-economic considerations of firms in their location decision problem are not inconsistent with the notion of economic rationality. Lastly, if  $\hat{\mu} = \bar{\mu}$ , a constant (after the site has been chosen and the plant set up) then

$$(\hat{\Pi}_i^k, \bar{\mu}) \succ \dots \succ (\hat{\Pi}_i^2, \bar{\mu}) \succ (\hat{\Pi}_i^1, \bar{\mu}) \succ (\hat{\Pi}_i, \bar{\mu}) \quad (7)$$

if and only if  $\hat{\Pi}_i < \hat{\Pi}_i^1 < \hat{\Pi}_i^2 < \dots < \hat{\Pi}_i^k$ , in which case the firm can then be viewed as having that singular objective of maximizing profit now taken not only as equalizing slopes but as the greatest positive spread between revenue and cost.

IV

A Note on Locational Interdependence

As mentioned earlier, the profit equation offers a convenient summary of all the combined effects of revenue and cost. The location decision of firms will be affected only to the extent that revenue and cost factors influence profit. Therefore, locational interdependence factors that influence location pattern - that is, whether firms will tend to agglomerate or disperse over space - come into focus only as they affect profit through revenue and/or cost. Schematically the relationship between locational interdependence factors and location pattern is shown below:

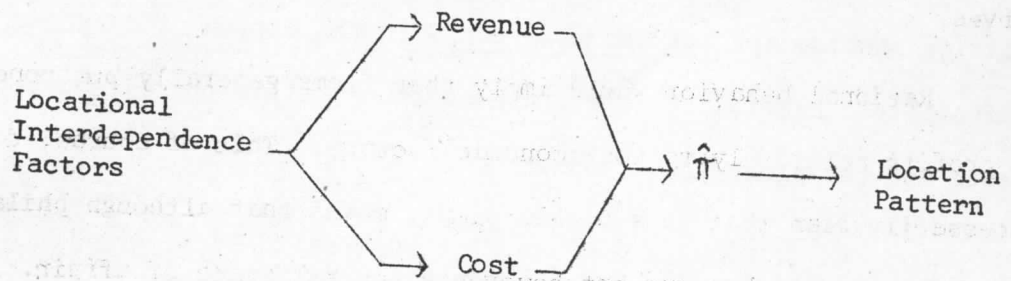


Fig. 2

We will not go into a detailed discussion of the various locational interdependence factors (demand elasticities, relationship between freight rate and selling price, shapes of the cost curves, spatial cost variations, etc.<sup>11/</sup>). Rather, on the basis of the schematic

<sup>11/</sup> Melvin L. Greenhut, *Plant Location in Theory and Practice*. (Chapel Hill: University of North Carolina Press, 1956), esp. Chs. II and VI.

relationship shown in Fig. 2, we will investigate the influence of locational interdependence factors on location pattern through the  $\hat{\Pi}$ s.

In Fig. 1, we see that the  $\hat{\Pi}$  and hence the location that is chosen by the firm together with  $\mu$ , is determined largely by two things namely, the shape of the firm's iso-welfare curves and the relative weights of  $\hat{\Pi}$  and  $\mu, \frac{\beta}{\alpha}$ . From its definition and since the  $\hat{\Pi}$ s are non-negative,  $\beta \geq 0$ .  $\beta$  varies positively as the profit differential among the locations. Since  $\alpha = 1$ , the slope of the constraint will depend mainly on  $\beta$ . Having interpreted  $\beta$  in such a way as to link it with the  $\hat{\Pi}$ s, agglomeration (dispersion) tendencies can be investigated by simply looking into the behavior of  $\beta$  together, of course, with the firm's iso-welfare curves.

Rational behavior would imply that firms generally put more weight to profits relatively to non-economic factors. This of course, does not necessarily mean that  $\mu = 0$ . It simply means that although philanthropic firms do exist, they are not however a general state of affair. We state this technically in the assumption that the welfare levels of firms are "profit-intensive." With this, we proceed to look into the effects of changes in  $\beta$ .

For any  $\beta > 0$ , if firms generally have identical and homogeneous iso-welfare curves of the shape of (say)  $W_F^0$ , agglomeration would tend to take place at such location as the  $i$ th where  $\hat{\Pi}_i$  obtains. Allowing  $\beta$  to vary but within certain limit, if firms generally possess identical iso-welfare curves of the shape of  $W_F$ , that is, the slope of the iso-welfare